

PLASMA DYNAMICS



## XVIII. PLASMAS AND CONTROLLED NUCLEAR FUSION\*

### A. Waves and Radiation

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### RESEARCH OBJECTIVES

The major goal of this group is to generate a basic understanding of various types of oscillations and waves in ionized gases that are relevant to problems in thermonuclear fusion and space research. Our research is a continuation of last year's work with a major item (4) added to our program.

1. Investigations of electromagnetic radiation emitted spontaneously by the plasma, because of thermal and nonthermal fluctuations of the free charges. At present, particular attention is being paid to emission generated as a result of many-body (collective) interactions between the charges at frequencies near the electron plasma frequency.

2. Studies of the dispersion characteristics of small-amplitude stable and unstable waves. We are now looking at the properties of ion sound waves in two collisionless plasma facilities built during the past two years.

3. Investigation of nonlinear plasma phenomena is one of our major goals for the next few years. In this connection, we are particularly interested in the properties of large-amplitude waves, particle trapping, nonlinear Landau damping, parametric coupling of three or more waves, coupling of waves with particles, and turbulence.

4. Studies of fluctuation, particle losses, and turbulence in magnetic multipole geometries. For this purpose, we are constructing a steady-state linear quadrupole (SLIM-1) to be operated in the Francis Bitter National Magnet Laboratory.

G. Bekefi

#### 1. STEADY-STATE LINEAR MULTIPOLE (SLIM-1)

A linear quadrupole is under construction at the present time. This device (see Fig. XVIII-1) is intended for use as a plasma source for the study of the behavior of a low- $\beta$  plasma in a simple two-dimensional magnetic field geometry. It will be operated on a continuous, rather than pulsed, basis in order to insure that the plasma is in an equilibrium configuration and to facilitate the collection of information on low-frequency

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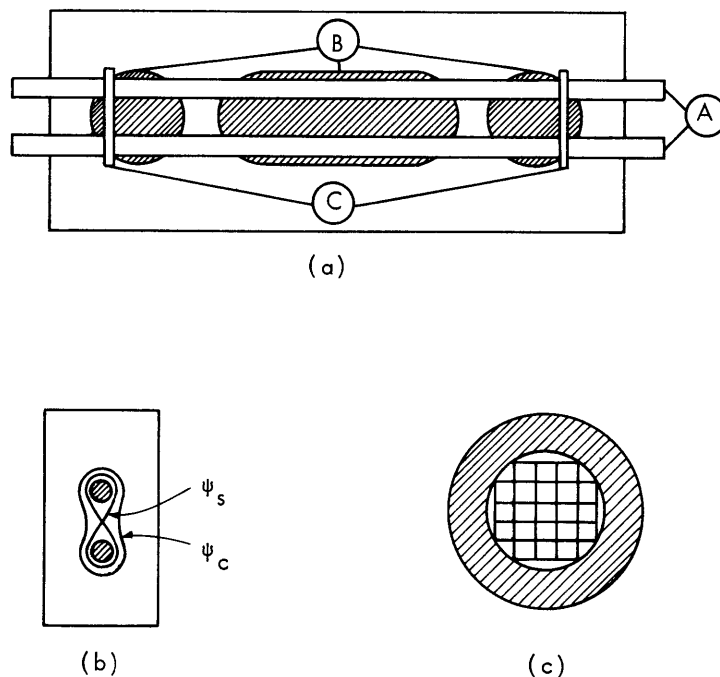


Fig. XVIII-1. (a) Side view showing quadrupole bars (A), typical access ports (B), and bar supports (C).  
 (b) End view showing two magnetic field lines;  $\psi_s$  labels the separatrix, which passes through the axis of the device;  $\psi_c$  labels the "critical" field line, for which  $\delta \int dl/B = 0$ .  
 (c) Cross section of conducting bar showing placement of 21 square, water-cooled, copper conductors inside stainless-steel pipe.

phenomena. The device will be located at the Francis Bitter National Magnet Laboratory in order to take advantage of the high-current power supply that is available there. It is anticipated that the power consumption will be approximately 2 MW at the highest current.

The basic parameters of the device are the following.

Length between supports	$\leq 120$ cm	adjustable
Separation between bars	19.4-32.7 cm	adjustable
Current per bar	85,000 A	maximum

The magnetic field strength at maximum current will vary along the separatrix (the field line passing through the axis,  $\psi_s$ ) from 0 at the axis to 4800 G behind the bars; along  $\psi_c$  (the line defined by  $\delta \int dl/B = 0$ ) the field strength will vary from a minimum of 1300 G

to a maximum of 4200 G behind the bars. For a hydrogen plasma with an ion temperature of 0.25 eV, this gives approximately 22 ion Larmor orbit diameters between  $\psi_s$  and  $\psi_c$ .

The quadrupole bars are constructed from 2 5/8 in. OD, 1/2 in. wall stainless-steel pipe, in order to minimize the deflection of the bars when they are energized. With this construction the maximum deflection will be less than 0.025 in. The current will be carried by 21 square, hollow, water-cooled, copper conductors placed inside each steel pipe. The vacuum chamber enclosing the quadrupole bars is a rectangular box with many large ports designed for maximum accessibility and flexibility.

The plasma will be produced by electron-cyclotron resonance with a slotted cylinder for coupling the RF power into the plasma. The slotted cylinder (3 cm in diameter) will be concentric with the quadrupole axis at one end of the device. The use of ECR in this configuration is made possible by the fact that, close to the axis, the lines of constant  $|B|$  are circular. The magnetic field strength at the surface of the coupling structure varies between 230 G and 520 G, depending upon the bar separation. The electron-cyclotron frequencies for these values are 640 MHz and 1450 MHz, and power supplies giving approximately 50 W cw in this frequency range are readily available. The anticipated plasma parameters are  $T_e \sim 10$  eV,  $T_i \sim 0.25$  eV,  $n \sim 10^{11}$  cm<sup>-3</sup>. The plasma is  $\sim 10\%$  ionized, but the ionization percentage can be increased by using differential pumping techniques.

The initial experimental work will be a study of the equilibrium plasma configuration, with special emphasis on the understanding of the convective cell structures that have been observed in other multipole devices. We shall also investigate the spectrum of low-frequency fluctuations in this device, and correlate these observations with the extensive theoretical work that has been done recently on the stability of plasmas in two-dimensional magnetic field geometries.

P. A. Politzer

## 2. PLASMA COLLECTIVE EFFECTS AND RADIATION EMISSION FROM ROTATING COLLAPSED STARS

### Introduction

Among the most striking features of pulsar emission are the sharpness of the period and, in many cases, the constancy of the pulse characteristics. These facts strongly support the idea that the emitting region is remarkably small and stable.<sup>1</sup> Since the discovery of the two pulsars PSR0833 and NP0532 in remnants of supernovae,<sup>2,3</sup> it has been generally accepted that pulsars are associated with collapsed objects, which are most likely to be neutron stars.<sup>4</sup> Particularly significant for NP0532 is its optical identification with the star that Baade and Minkowski suggested<sup>5</sup> is the collapsed core of the Crab Nebula, following the original supernova theory advanced by Baade and Zwicky.<sup>6</sup>

On the basis of evidence available at present from laboratory plasmas, we propose a plasma mechanism to account for the most evident physical characteristics of pulsar emission, including the production of x rays and the acceleration of high-energy particles. We shall make special reference to the best known pulsar (NP0532) in the Crab Nebula and also suggest an application of the same model to x-ray sources.

### Plasma Emission Model

Following the arguments proposed by previous authors,<sup>4</sup> we assume that the condensed stars associated with pulsars are strongly magnetized and rapidly rotating, as a consequence of gravitational collapse and of angular momentum and magnetic flux conservation. We use the rotation as the timer for the pulsation mechanism and as the principal energy source for the emission process. In order to be consistent with the observation of periodically spaced radiation pulses, the magnetic field is assumed not to be symmetric about the axis of rotation, for example, a dipolar or quadrupolar type with its principal axis not aligned with the axis of rotation. The star is assumed to be surrounded by a relatively dense plasma supported by the magnetic field as indicated by the approximate equilibrium equation<sup>7</sup>

$$e\mathbf{E}_\perp \left( Z - \frac{n_e}{n_i} \right) + eZ(\mathbf{u}_{i\perp} - \mathbf{u}_{e\perp}) \times \mathbf{B} + m_i(\mathbf{g}_\perp - \mathbf{u}_i \cdot \nabla \mathbf{u}_{i\perp}) = 0. \quad (1)$$

Here  $n_e$  and  $n_i$  are the electron and ion densities,  $Z$  is the ion charge number,  $m_i$  is the mass,  $\mathbf{g}_\perp$  is the perpendicular component of the acceleration of gravity,  $\mathbf{B}$  is the magnetic field,  $\mathbf{E}_\perp$  is the electric field (as seen in an inertial frame) transverse to  $\mathbf{B}$ , and  $(\mathbf{u}_{i\perp} - \mathbf{u}_{e\perp})$  is the relative velocity of the ions with respect to the electrons in the same direction. The charge separation  $(Z - n_e/n_i)$  is very small and, by virtue of the high magnetic field, its effect, which will tend to prevail over that of the gravitational and centrifugal force, is compensated by a very slow drift of the ions relative to the electrons.

We now recall that, according to Baym, et al.,<sup>8</sup> in a highly condensed star superconductivity can occur; therefore, the magnetic field cannot diffuse across it, but co-rotates rigidly. Then, if we consider the region inside the light cylinder, where  $\omega_0 r < c_0 - r$  is the radial coordinate, the effect of electron inertia is negligible, and the electrons are rigidly tied to the magnetic field lines and to the star's rotation.

We shall use the indices  $\perp$  and  $\parallel$  to denote components perpendicular and parallel to the magnetic field. Therefore in an inertial frame there will be an electric field  $E_{\perp}$  given by

$$\underline{E}_{\perp} \approx -\underline{u}_0 \times \underline{B}, \quad (2)$$

where  $\underline{u}_0 = \omega_0 \times \underline{r}$  is the rotation velocity. In view of the expected low values of  $J_{\perp}$  (the transverse current density) the resistive contribution of the term  $\eta_{\perp} J_{\perp}$  is considered to be small,  $\eta$  indicating the electrical resistivity. In general there will also be a parallel electric field  $E_{\parallel}$  in a co-rotating frame, so that the Poisson's equation is

$$-\nabla_{\parallel} E_{\parallel} = \nabla \cdot (\underline{u}_0 \times \underline{B}) + 4\pi e n_i \left( Z - \frac{n_e}{n_i} \right), \quad (3)$$

where  $e(Zn_i - n_e)$  is a residual charge separation whose magnitude will not affect appreciably our considerations. This electric field generates a current along the B lines and is related to it by a large anomalous<sup>9-11</sup> resistivity,  $\eta_{an}$ , that is mainly due to interaction between particles and plasma collective modes rather than to electron-ion collisions,

$$\underline{E}_{\parallel} = \eta_{an} \underline{J}_{\parallel}, \quad (4)$$

where  $\underline{J}_{\parallel} = -en_e \underline{u}_{e\parallel}$  (the ion current is negligible). We shall discuss the evaluation of anomalous resistivity eventually and recall that this arises because of the strong plasma turbulence that is excited when the electron flow velocity  $u_{e\parallel}$  exceeds certain critical values.

First, we notice, as pointed out by Goldreich and Julian,<sup>12</sup> that in the case of the Crab Nebula pulsar  $\omega_0 = 2 \times 10^2$  rad/sec, and B at the star surface is likely to be  $\approx 10^{12}$  G, so that at this surface ( $r = R \sim 20$  km) we have

$$E_{\perp} \approx 4 \times 10^{12} \text{ V/cm}. \quad (5)$$

It is evident that, even if  $E_{\parallel}$  is a small fraction of  $E_{\perp}$ , particles can be accelerated along the magnetic field lines by a field that is much larger than that of gravitation.

Second, it is important to realize that mildly relativistic electrons ( $\gamma = \epsilon/mc^2 \approx 1$ ) lose their perpendicular (to  $\underline{B}$ ) energy by cyclotron radiation with maximum power at

frequencies

$$\omega \approx \Omega_e = 1.76 \times 10^7 B \text{ sec}^{-1}.$$

The time scale of energy loss from an electron is

$$\tau \approx 2.58 \times 10^8 B^{-2} \text{ sec.}$$

For a value of  $B \approx 10^{12}$  G, we have  $\Omega_e = 1.76 \times 10^{19} \text{ sec}^{-1}$  (x-ray band) and  $\tau \approx 2.58 \times 10^{-16} \text{ sec}$ . Clearly, this energy loss does not contradict Eq. 1 and does not force the plasma back to the stellar surface, contrary to the conclusions given by Chiu and Occhionero.<sup>13</sup>

Third, it is possible to transfer transverse energy again to the electrons via plasma collective effects maintained by the parallel electric field. This process is well known in plasma physics, and has been the object of recent theoretical works.<sup>10, 14</sup> In the presence of a plasma wave with frequency  $\omega$ , a particle-wave resonance occurs that leads to the energy exchange<sup>10, 15</sup>

$$\hbar\omega + \Delta\epsilon_p = 0, \quad (6)$$

where  $\hbar\omega$  is the energy of the excited mode, and  $\Delta\epsilon_p$  is the exchanged particle energy. Now,  $\Delta\epsilon_p = \Delta\epsilon_{\parallel} + \Delta\epsilon_{\perp}$ , with  $\Delta\epsilon_{\parallel} = m_e \underline{v}_{\parallel} \cdot \Delta\underline{v}_{\parallel}$  and  $\Delta\epsilon_{\perp} = n_o \hbar\Omega_e$  ( $\Omega_e$  = electron gyro frequency), and  $n_o$  is an integer. The momentum balance equation is

$$\hbar\underline{k} + \Delta\underline{p} = 0, \quad (7)$$

where  $\underline{k}$  is the mode vector, and  $\Delta\underline{p}$  is the exchanged particle momentum. Again, we have  $\Delta\underline{p} = \Delta\underline{p}_{\parallel} + \Delta\underline{p}_{\perp}$ , with  $\Delta\underline{p}_{\parallel} = m_e \Delta\underline{v}_{\parallel}$ , and  $\Delta\underline{p}_{\perp}$  is taken up by the magnetic field. The resonance condition is then

$$\omega + n_o \Omega_e - k_{\parallel} v_{\parallel} = 0. \quad (8)$$

For  $\omega \approx \omega_{pe}$  (the electron plasma frequency) and

$$\omega_{pe} < \Omega_e, \quad (9)$$

Eq. 8 then shows that the actual energy exchange with the wave is negligible.

Fourth, since a plasma wave with  $\omega \approx \omega_{pe}$  exists (that is, is not heavily damped) only if  $\omega_{pe} > k_{\parallel} v_{the}$ ,  $v_{the}$  being the electron thermal velocity, we see that the electrons participating in the resonance (8) have to be superthermal. Therefore the electron distribution  $f$  has to possess a sizeable tail of superthermal electrons, as is expected in the presence of longitudinal fields  $E_{\parallel}$  larger than the critical runaway field  $E_{run}$



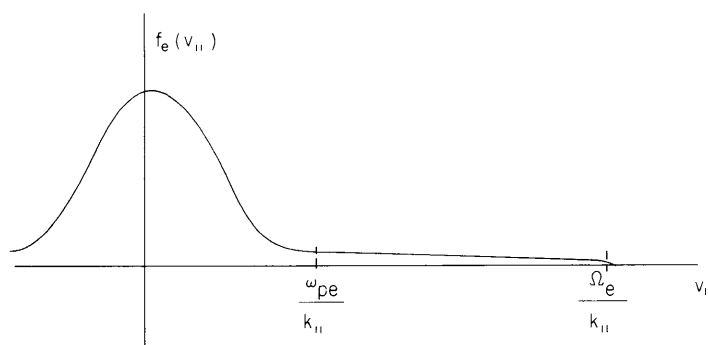


Fig. XVIII-2. Example of a typical "runaway" distribution, with regions of particle-wave resonance indicated.

(see Fig. XVIII-2). We recall that  $E_{\text{run}}$  is the field for which the electron flow velocity, as limited only by electron-ion collisions, would reach the electron thermal velocity. So a typical "runaway" distribution is a Maxwellian with a long tail of relatively few fast electrons. This is a common situation for laboratory plasmas in a strong magnetic field and with an electric field  $E_{\parallel}$  applied along it.<sup>16</sup> Notice that in our case  $E_{\parallel}/E_{\text{run}}$  will be a function of  $r$ , and we expect that the regime where  $(\partial f/\partial v_{\parallel})_{v_{\parallel}} = \Omega_e/k_{\parallel} \neq 0$ , as indicated in Fig. XVII-2, also be localized in space.

The result of the interaction described above is a decrease of longitudinal energy of the electrons, and an increase of their transverse energy.

In addition to, or instead of this mechanism, pitch angle scattering of the fast particles can be provided by simple collisional effects, which, given the high density values under consideration, are bound to play an important role.

The transverse energy is then emitted in the form of cyclotron radiation, and the power output is evaluated by considering the fraction  $n_E/n$  of superthermal electrons, whose total density is  $n_s \gg n_E$ , which at any one time are provided with perpendicular energy.

Therefore

$$w = 3.18 \times 10^{-15} B^2 n_E \text{ erg}/(\text{sec} \cdot \text{cm}^3) \quad (10)$$

is the emitted power density at  $\omega \sim \Omega_e$ . Notice that a ratio of  $n_s/n \approx 10^{-2}$  is usually obtained for  $E_{\parallel} \approx E_{\text{run}}$  in laboratory plasmas.<sup>17</sup> For the Crab Nebula pulsar the total power emitted is  $W \sim 10^{36} \text{ erg sec}^{-1}$ , mainly in the x-ray frequencies. With  $B \sim 10^{12} \text{ G}$  and assuming that the emitting region is a shell of thickness  $\delta$  above the stellar surface ( $R = 20 \text{ km}$ ), we would need

$$n_E \approx 6.6 \times 10^{12} \delta^{-1} \text{ cm}^{-3}; \quad (11)$$

that is,  $n_E \approx 6.6 \times 10^{11} \text{ cm}^{-3}$  if  $\delta \approx 10 \text{ cm}$ , as it was suggested on the basis of dispersion measurements of radio pulses.<sup>18</sup>

Referring to the Maxwellian part of the distribution, we shall assume an electron temperature  $T_e \approx 10^6 \text{ }^\circ\text{K}$ , in agreement with a commonly accepted estimate of the temperature at the surface of a neutron star,<sup>19</sup> and a density  $n \approx 10^{20} \text{ cm}^{-3}$  so that a cutoff at the plasma frequency of the optical radiation is possible, in order to explain the high peaks of pulsed radiation observed in this band. Under these conditions, the classical resistivity caused by ion-electron collision is

$$\eta_{cl} = 1.65 \times 10^{-7} [T_e (\text{keV})]^{-3/2} \ln \Lambda \quad \Omega\text{-cm}, \quad (12)$$

where  $\ln \Lambda = 32.8 - 1.15 \log_{10} n + 3.5 \log_{10} T_e (\text{keV})$ . For the chosen values of  $n$  and  $T_e$ , we obtain  $\ln \Lambda \approx 6.3$  and  $\eta_{cl} \approx 3.3 \times 10^{-5} \Omega\text{-cm}$ . We notice that the corresponding runaway critical field is

$$E_{run} \approx \eta_{cl} n_e e v_{the} \approx 3 \times 10^6 \text{ V/cm}, \quad (13)$$

and in our case we expect  $E_{||} \gtrsim E_{run}$ . Notice that for  $E_{||} \ll E_{\perp}$  and  $n_e \approx 10^{20} \text{ cm}^{-3}$  the relative charge separation appearing in Eqs. 1 and 2 is of order  $10^{-9}$ . Under these conditions, it is well-known<sup>20</sup> that the actual plasma resistivity can be orders of magnitude higher than that quoted above, as a result of wave-particle interactions that transfer parallel energy and momentum into transverse energy and momentum and of other processes<sup>11</sup> associated, for instance, with ion-sound or two-stream instabilities. By considering only the former process, we can argue that the actual resistivity is of the order of

$$\eta_{an} \approx \eta_{cl} f\left(\frac{\Omega_e}{\omega_{pe}}\right), \quad (14)$$

where  $f$  is a fast-growing function of  $\Omega_e/\omega_{pe}$ .<sup>10</sup> So, if  $B \approx 10^{12} \text{ G}$  and  $n_e \approx 10^{20} \text{ cm}^{-3}$ ,  $\Omega_e/\omega_{pe} \approx 3 \times 10^4$ , and there is little difficulty in building up a large resistivity and then accepting the existence of a large electric field  $E_{||}$  at equilibrium. Notice that the condition  $\nabla \cdot \underline{J} = 0$  inferred from the equilibrium gives  $\nabla_{||} E_{||} = J_{||} \nabla_{||} \eta_{an} - \eta_{an} \nabla_{\perp} \cdot \underline{J}_{\perp}$ , and in view of its dependence<sup>10, 11</sup> on  $B$ , the term  $\nabla_{||} \eta_{an}$  is certainly  $\neq 0$ .

As in laboratory experiments, there will also be a few "fugitive" particles that are able to undergo a full acceleration process and remain unaffected by classical and effective collisions because of plasma collective phenomena.<sup>16</sup> So the presence of electric fields  $E_{||}$  would also provide a mechanism for particles to attain very high energies and contribute to the high-energy tail of cosmic rays.<sup>21</sup>

Application to the Pulsar Problem

We point out once more that the emission and acceleration process previously described is energetically sustained by the star's rotation, which is the largest energy source. As many authors have proposed,<sup>4</sup> in the Crab Nebula pulsar the rotational energy is likely to amount to  $\sim 10^{48}$  erg, if  $\omega_0 = 2 \times 10^2$  rad/sec and  $I$  (moment of inertia)  $\approx 10^{44}$  gm/cm<sup>2</sup>. Therefore the present emission rate could be maintained for  $\sim 10^5$  y. We also notice that all of the conditions required for the occurrence of our mechanism are satisfied quite close to the stellar surface. This will then ensure the perfect co-rotation of the emission region and therefore the stability of the pulse period and shape, which are the most important requirements for any pulsar theory.

Four more points have to be discussed: the origin of the pulses, the spectrum of NP0532, the lack of optical and x radiation from pulsars other than NP0532, and the slowing down of the period.

1. We have assumed that the considered magnetic configuration does not have its poles aligned on the rotation axis. Thus the emission process that we have outlined is expected to be more efficient in the magnetic polar regions where the magnetic field lines converge. We may recall that magnetic stars show evidence of enhanced activity at the polar regions. Therefore we would expect to observe a stronger signal when these polar regions point toward the Earth during the star's rotation. Bohm-Vitense,<sup>22</sup> in fact, has shown how a dipolar structure can reasonably explain the presence of double pulses and intermediate pulses, when rotation enables us to see both polar regions. By means of a quadrupolar structure, we might also explain composite pulses. We also recall that  $\Omega_e \gg \omega_{pe}$ , so that x-ray frequencies can freely be transmitted through the plasma and we expect to observe the emission from the whole star with enhancement at the poles. In fact, for the Crab Nebula pulsar<sup>23</sup> the detected x-ray pulses are only  $\sim 5\%$  above the nebular background, which we consider to be significantly contributed by the direct star emission; also, primary and secondary pulses are comparable in intensity, are quite wide, and not well separated.

2. The proposed cyclotron radiation from superthermal electrons in a strong magnetic field can explain the total power emitted by NP0532 in the x rays. The spectrum attributable to this process alone is expected to have a maximum in the x-ray region, steeply decreasing at optical and radio frequencies, and more slowly decaying in the  $\gamma$ -ray region. On the other hand, present observations suggest an over-all spectrum with two distinct maxima: one at radio frequencies, and another at optical or x-ray frequencies.<sup>24</sup> For this we notice first of all that nonlinear mode-mode coupling<sup>15</sup> would give nonlinear decay toward lower frequencies than the x-band, with sufficient efficiency to explain the level of power emitted in the optical and radio spectra. This process is ineffective for extending the spectrum in the  $\gamma$  region, so that no comparable power

emission should be observed in this band. In fact the higher harmonics of the cyclotron mechanism give a distribution several orders of magnitude below the maximum.

With reference to the radio emission we notice that another mechanism of emission is to be related with the low-frequency (less than  $\omega_{pe}$  and the ion gyro-frequency) modes that are induced by the flowing current  $J_{\parallel}$ . There are several modes of this kind, longitudinal or transverse, with different directions of propagation relative to the magnetic field, which have been theoretically investigated and correlated with the experiments. For instance, one of these is due to a current gradient<sup>25</sup>  $dJ_{\parallel}/dr$ , and another, of greater interest, with frequency  $\omega \approx k_{\parallel}\sqrt{KT_e/m_i}$  and of ion sound-wave type, occurs for  $u_{e\parallel} > \sqrt{KT_e/m_i}$ , is longitudinal, propagates along the magnetic field lines, and is associated with electron thermal conductivity or Landau damping in the same direction. Modes of this kind can in turn couple to transverse waves with typical frequencies and wavelengths falling in the observed radio spectrum. So we expect a strong preferential emission in the direction of the magnetic axis, in accordance with the observation of the optical and radio high narrow peaks of NP0532.<sup>25, 27</sup>

3. Thus far, the Crab Nebula pulsar is the only observed pulsar emitting over a wide range of frequencies. All attempts to detect optical radiation from other pulsars have failed. From energy considerations and from results concerning the slowing down of the period it is quite reasonable to conclude that only relatively young pulsars have x and optical radiation, and that this phase has quite a short lifetime. Moreover, since rotation is the source of the emitted energy, we may consider the period as a measure of the pulsar age. The period of PSR0833 is only approximately three times longer than that of NP0532; nevertheless, PSR0833 has only radio pulses. Therefore, although there is no reason to believe that the two pulsars were born with the same physical characteristics, a sharp aging process would have to be considered, followed by a long radio phase. For this purpose, we recall that the existence of the plasma collective effects providing particles with transverse energy was related to the existence of a "runaway" regime; that is, to the presence of a sizeable tail of superthermal electrons with non-zero derivative for  $v_{\parallel} = \Omega_e/k_{\parallel}$  (Fig. XVIII-2), where  $k_{\parallel}\lambda_{De} < 1$ , with  $\lambda_{De}$  the Debye length. On the other hand, we know that the appearance of this tail is a very sharp function of  $E_{\parallel}/E_{run}$ , typically of the form<sup>16</sup>

$$n_s \approx n_e e^{-E_{run}/E_{\parallel}},$$

where  $E_{run}$  is given by Eq. 13. We shall assume that this is still so for our model, and that  $E_{run}$  is replaced by  $(E_{run})_{eff} > E_{run}$  to take account of the effects of plasma turbulence and interparticle collisions. Therefore, radiation emission by our mechanism will take place when  $E_{\parallel}/(E_{run})_{eff}$  is larger than or of the order of 1.

To explain the transition from the case of NP0532 to that of PSR0833, we assume that

for the latter star, because of its initial conditions and of aging, which implies slowing down of rotation and probably decrease of  $T_e$ , the ratio  $E_{\parallel}/(E_{\text{run}})_{\text{eff}}$  is no longer sufficient to maintain an adequate population of fast particles that make the particle-wave resonance represented by Eq. 8 prevail over ordinary Landau damping. This corresponds to the resonance  $\omega + k_{\parallel}v_{\parallel} = 0$ , which generally involves a considerably larger number of particles (Fig. XVIII-2). Then we are left with the low-frequency modes that are excited by the current  $J_{\parallel}$  produced by the electric field  $E_{\parallel}$ , whose typical frequencies are in the radio band, and whose propagation is strongly correlated with the magnetic field direction. So, the resulting transverse waves could be linearly polarized and the highly regular rotation of the plane of polarization during a pulse of PSR0833 (Vela) could be related to the rigid rotation of the magnetic field lines.<sup>28</sup>

4. As we have indicated, the plasma emission is not the only energy loss of the star; in fact, the most efficient loss is probably the magnetic multipole radiation. In fact, we can evaluate the dipole<sup>29, 30</sup> radiation to be  $\sim 10^{38}$  erg/sec for a value of  $B_{\text{surf}} \sim 10^{12}$  G. With reasonable assumptions on the physical parameter of the star, these forms of radiation could explain quite well the lengthening of the Crab Nebula pulsar period, but they seem to be in contradiction with recent experimental work on the second derivative of the period.<sup>31</sup> If these results are confirmed, a quadrupole radiation would appear to be more suitable.<sup>32</sup> Therefore we suggest a rotating magnetic quadrupole radiation emission, which seems to be compatible with a simple geometry and reasonable magnetization densities. We notice that this type of emission is characterized by very low frequencies, and is therefore completely absorbed by the nebula or the interstellar gas surrounding the star.

### Discussion

We summarize our conclusions as follows.

1. A sequence of plasma processes, which have been observed through laboratory experiment and investigated theoretically, can account for pulsar and x-ray emission on the basis of the large rotational energy and the high values of magnetic field that are associated with a collapsed star.
2. The considered process for radiation emission occurs close to the star, within the "light-speed cylinder" ( $R_c = c/\omega_0$ ), where the particles are rigidly tied to the rotating magnetic field lines. This is important in explaining the rigorous synchronism of the pulse sequence for the pulsar emission and the regular pattern of the radiation polarization.
3. The frequency range of maximum emission, for a given rotation frequency, depends on the intensity of the magnetic field, and, therefore, is related to  $\Omega_e$  - the electron gyro frequency -<sup>33</sup> and to the ratio  $E_{\parallel}/E_{\text{run}}$ , where  $E_{\text{run}}$  is the critical runaway

electric field, which controls the magnitude of the superthermal tail in the electron distribution function. So, for the Crab Nebula pulsar, if  $B \sim 10^{12}$  G, as indirect theoretical results seem to suggest, the maximum of the spectrum can be expected to be in the x-ray region ( $\Omega_e \approx 10^{18}$  r/s), and no comparable radiation should be detectable in the  $\gamma$  rays.

4. The optical radiation emission is considered to be strongly correlated with the x emission, and to result, for instance, from a nonlinear process of mode-mode coupling (frequency decay) that leads to emission at lower frequencies and with considerably smaller energy output. This is consistent with the fact that for the Crab Nebula pulsar, the ratio of energy emitted at x-ray frequencies to that emitted at lower frequencies is large. The emission in the radio band can also be contributed by a similar process, but the contribution of plasma low-frequency modes, excited by the current longitudinal to the magnetic field, has also to be considered. We also notice that the thick plasma surrounding the collapsed star provides absorption below the plasma frequency for propagation across the magnetic field. Hence, referring to the Crab Nebula pulsar, we expect little absorption for the x rays and considerable absorption for the optical and upper radio bands, which we consider to result from a process of frequency decay. Thus we can explain why the x-ray pulses are only  $\sim 5\%$  above the general background that we consider has been significantly contributed by the star emission, are wider than the optical pulses, and are accompanied by a secondary pulse of the same magnitude. On the contrary, the optical and radio pulses, at the upper end of the radio spectrum, are quite narrow and well above the star background. The lower end of the radio spectrum could instead be attributed to the low-frequency modes, so that the progressive pulse widening as the frequency decreases may be justified.

5. The sharp transition from a Crab Nebula type of pulsar to a Vela type of pulsar, characterized by a shift of the maximum of emission from x rays to radio waves, is interpreted as being due to a decrease of the value of  $E_{\parallel}/E_{\text{run}}$ , which causes the high-energy tail of superthermal electrons to be depleted and the mildly relativistic cyclotron emission in the x-ray region to stop. Under these conditions, only low-frequency modes, driven by the current longitudinal to the magnetic field, survive. Hence, we can expect a strongly enhanced radio emission in the direction of the magnetic axes so that the plane of polarization is strongly referred to the magnetic lines. Thus, during the rotation, this plane sweeps across the signal, as observed in PSR0833.

6. We recall that the Crab Nebula appears as an extended x-ray source, whose power is  $\sim 10$  times larger than the pulsed component, and also as an optical and radio source. Many efforts have been made to explain this emission on the basis of synchrotron mechanism by highly relativistic electrons in the nebular magnetic field.<sup>34</sup> The main difficulty in all of the models proposed thus far is the requirement of a continuous injection of high-energy electrons. For this our model provides a relatively small population of electrons which, under the influence of  $E_{\parallel}$ , are able to escape the effects of

interparticle collisions and plasma collective phenomena attaining very high energies ( $\gamma \gg 1$ ). These fast electrons are then slowed down by synchrotron emission while travelling across the nebula and contribute to the extended radiation emission that is observed.

Additional processes of particle acceleration connected with the rotation of a multipolar magnetic field are likely to take place around and outside the "light-speed cylinder" ( $R_c = c/\omega_0$ ); in this case, both mildly and highly relativistic accelerated electrons can be supplied. So the over-all effect of these two principal acceleration mechanisms would be to provide particles for the synchrotron emission from the Crab Nebula on a very large spectrum.

7. The particles that are able to escape the effects of interparticle collisions and plasma collective effects, undergoing almost free acceleration by  $E_{\parallel}$  close to the star and not affected by the nebula, will then contribute to the high-energy tail of the cosmic-ray spectrum. Hence the combined high values of rotation and magnetic field of a collapsed star can provide a mechanism for high-energy particle acceleration, with the consequence that a sizeable cosmic-ray anisotropy should be present at very high energies.

8. We point out the possibility of interpreting other x-ray sources<sup>34</sup> in terms of rotating collapsed bodies. Observational results on the optical counterpart of Scorpio X1 do, in fact, suggest that this is actually a neutron star or a white dwarf.<sup>35</sup> Again for Scorpio X1 radio emission is also detected.<sup>36</sup> Our mechanism could then be correctly applied to the interpretation of point x sources; they would differ from NP0532 mainly in the absence of the regular pulse pattern. This could be due to lack of observations of special regions of enhanced emission, either because no magnetic polar regions are present or we have never swept by the polar beams. We would then observe the steady emission of x rays and, depending on the plasma density around the star, also radio and optical emission. The absence of a nebula in the vicinity of the star would in these cases make the source pointlike to observation.

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B. Diffusion and Turbulence

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RESEARCH OBJECTIVES

1. Differential Cross-Section Measurements

Techniques developed in magnetic-mirror experiments can be applied to the rapid and accurate measurement of differential scattering cross sections. The main feature in this experiment is the use of the mirror fields, rather than slits or edges, as angle-discriminating elements. The result is the ability to use the total azimuthally integrated scattering from a line source as the scattering signal, thereby improving both sensitivity and the signal-to-noise ratio. The experiment is in progress.

L. C. Pittenger, L. M. Lidsky

2. Toroidal Electron Trap

Our original experiment for measuring the lifetime of electrons circulating in a toroidal magnetic trap has been completed. The technology needed to inject electrons into the trap and to measure their lifetime has been perfected. We are attempting to see if this experiment can be improved by correcting the causes of the long-term magnetic field drifts that limited electron lifetime to  $\lesssim 200$  transits. If sufficient lifetime can be achieved, this apparatus will be used for the study of waves propagating on electron beams.

P. M. Margosian, L. M. Lidsky

3. Incoherent Scattering – Anisotropic Velocity Distribution

We plan to use incoherent scattering techniques to measure the distribution of plasma electron velocities in the directions parallel and perpendicular to the confining magnetic field. Experiments show that the electron temperature in the HCD plasma is anisotropic and that the degree of anisotropy depends on several plasma parameters. Our first aim is to determine whether thermalization is caused by classical (Coulomb) scattering or by wave-particle interactions. A hollow-cathode arc plasma source has been built specifically for this experiment with greater than usual emphasis placed on diagnostic access. A 100-MW Q-switched laser has been provided and tested.

G. K. McCormick, L. M. Lidsky

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\* This work is supported by the U.S. Atomic Energy Commission (Contract AT(30-1)-3980).

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4. Coherent Scattering from Steady-State Plasmas

We are attempting to observe coherent scattering of  $10.6 \mu$  radiation from the moderate density steady-state plasma produced by the hollow-cathode discharge source. Our goal is the comparison of the experimentally measured and theoretically predicted scattered spectra in order to determine the spectrum of plasma density fluctuations, that is, to measure plasma turbulence. We are using a 100-W  $N_2$ - $CO_2$ -He laser as a radiation source and cryogenic Ge detectors.

Measurements of signal-noise ratios for the separate parts of this system have been completed. It appears that final S/N ratios of 6 are achievable. We hope to observe the detailed structure of the plasma-frequency satellites.

K. R. S. Chen, L. M. Lidsky

5. Superconducting Magnet Design Studies

It is apparent that moderately high field (50-100 kG) steady-state plasma confinement experiments will require the use of superconducting coils. We are beginning a study of the applicability of superconductivity technology to toroidal plasma systems; in particular, to the realization of a high-field, high-shear stellarator. As an adjunct to this, we have designed and are building a single element of a linear quadrupole pair.

A. E. Wright, L. M. Lidsky

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### C. Plasma Diagnostics

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Prof. B. L. Wright  
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Dr. P. A. Politzer

#### RESEARCH OBJECTIVES

The aims of this group are to perfect and refine existing methods of studying the properties of plasmas and, in particular, to devise novel techniques. At present, we are concentrating on the effects of intense high-frequency electric fields acting on excited atoms. Under suitable conditions these fields can give rise to optical satellites in the neighborhood of quantum transitions that are normally forbidden by standard selection rules.

G. Bekefi

#### 1. OPTICAL SATELLITES INDUCED IN A PLASMA THROUGH THE ACTION OF INTENSE, HIGH-FREQUENCY ELECTRIC FIELDS

An intense, high-frequency electric field acting on an excited atom can generate optical satellites in the neighborhood of quantum transitions that are normally forbidden by the ordinary selection rules. The electric field may be created spontaneously by the charges of the plasma, or it can be produced by an external source.

In the first part of this report we describe experiments in which the electric field is due to the plasma electrons; in the second part, we describe preliminary work done with the view of observing satellites induced by the electric field of a high-power CO<sub>2</sub> laser beam focused on the plasma.

#### Satellite Excited by Thermal Electron Oscillations

In a sufficiently dense plasma a normally forbidden optical transition, adjacent to an allowed line, radiates as a result of the strong electric field ( $E_R$ ) generated by the random motions of the charges. Collective plasma oscillations cause the spectrum of the field to be sharply peaked at some frequency  $\omega_0$ . If this "collective" field ( $E_p$ ) is strong enough, its presence manifests itself by the appearance of optical satellites,<sup>1</sup> flanking the forbidden line and separated from it in frequency by  $\pm\omega_0$ .

Here we report observations of a satellite induced by longitudinal electron-plasma

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oscillations. Satellites that are due to unstable oscillations in helium have been observed by Kunze and Griem.<sup>2</sup> Our satellite, on the other hand, is excited by the relatively weak thermal fluctuations of the electrons, and under these conditions it has been surmised<sup>1, 2</sup> that the satellite may be difficult to resolve.

We produce the plasma<sup>3</sup> by discharging a capacitor (50 J) through a thin (~0.05 mm OD) lithium wire, thereby causing it to explode. The plasma lasts for ~7  $\mu$ s during which period time resolved measurements of electron density and temperature are performed.

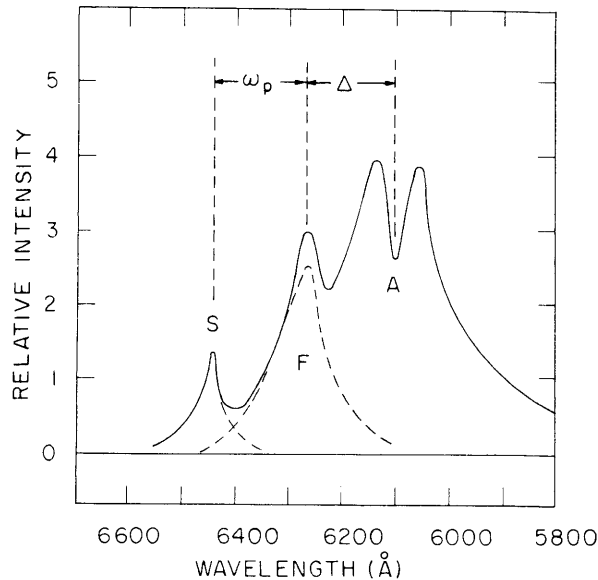


Fig. XVIII-3. Section of time-integrated spectrum emitted from an exploded lithium wire (after subtraction of the continuum), as viewed at right angles to the wire axis. A represents the allowed 2P-3D line of LiI; F, the 2P-3P forbidden line; and S, the plasma-induced satellite. Dashed curves represent the unfolding of the partially overlapping lines.

Typical values are,  $10^{17} \lesssim N \lesssim 3 \times 10^{18} \text{ cm}^{-3}$ ,  $6 \times 10^3 \lesssim T \lesssim 2 \times 10^4 \text{ K}$ . The time-integrated spectrogram shown in Fig. XVIII-3 illustrates the main features, namely the allowed 2P-3D line of LiI exhibiting self-reversal; the 2P-3P forbidden line, and a satellite. The other satellite presumably lies within the linewidth of the allowed line. The separation  $\omega_o \approx \omega_p = (N e^2 / m \epsilon_o)^{1/2}$  between satellite and forbidden line corresponds to an electron density  $N \approx 2 \times 10^{18} \text{ cm}^{-3}$ . By using independent measurements of  $N$  with temporal and spatial resolutions<sup>3-5</sup> it can be determined that such an electron density exists at the axial region of the wire approximately 2  $\mu$ s after the initiation of the explosion. Theoretical calculations will show that it is indeed then and there that the satellite should be predominantly emitted. In a less dense plasma produced by a weaker

explosion of the wire, the satellite moves nearer the forbidden line, as expected. The other satellite is still not resolved, however, from the allowed line.

The ratio of the integrated line intensity of the satellite to that of the forbidden line with second-order perturbation theory is given by<sup>1</sup>

$$R = \frac{1}{2} \frac{E_p^2}{E_R^2} \frac{\Delta^2}{(\Delta \pm \omega_0)^2}, \quad (R \ll 1), \quad (1)$$

where  $\Delta$  is the frequency separation (in r/s) between the forbidden and allowed lines, and  $E_p^2$  is the time-averaged field squared. When the amplitude of plasma oscillations is large or  $R$  is not much smaller than 1 (which is the case here) the calculations should be carried to a higher order. This has been done recently,<sup>6</sup> and been shown to slightly enhance the satellite farther from the allowed line. On the other hand, the satellite closer to the allowed line becomes weaker, and this is why it has not been observed here in weak explosions, nor in other experiments<sup>6</sup> in which its existence would be expected from second-order theory.

Since we surmise that thermal plasma oscillations produce the optical satellite,  $E_p^2$  can be determined by equating the energy density in the electric field with the thermal energy density of collective plasma oscillations<sup>7</sup>:

$$\frac{1}{2} \epsilon_0 E_p^2 \approx \int_0^{L_D^{-1}} \frac{KT}{2[1+(kL_D)^2]} \frac{4\pi k^2 dk}{(2\pi)^3} = 0.0054 KT/L_D^3. \quad (2)$$

Here,  $K$  is Boltzmann's constant,  $k$  the wave number of oscillation, and  $L_D = (KT\epsilon_0/Ne^2)^{1/2}$  the electron Debye length. The integral is cut off at  $L_D^{-1}$  because the oscillations are heavily Landau-damped when  $kL_D^{-1} \gtrsim 1$ .

The proper value of  $E_R^2$  to be used in Eq. 1 should be obtained by averaging the field squared over the Holtmark distribution of quasi-static random field. This distribution, with charged-particle correlation (or screening) taken into account, was calculated by Mozer and Baranger<sup>8</sup> for different values of the ratio  $r$  of the mean distance between ions and the Debye length. At the relevant time of the explosion, namely  $2 \mu\text{s}$ ,  $r$  equals  $\sim 0.8$ , and the resulting distribution has a maximum at  $E = 0.7E_0$ ,  $E_0$  being the Holtmark normal field given by  $E_0 = 2.6 \text{ eN}^{2/3}/4\pi\epsilon_0$ . The integration when computing this average should be cut off at the field where linear Stark effect starts to dominate (without this cutoff the integral diverges and has no physical meaning). The field at which this happens is approximately given by  $E = em\Delta/2hn^2 4\pi\epsilon_0$ , where  $n$  is the principal quantum number of the upper state. For  $N = 2 \times 10^{18} \text{ cm}^{-3}$  we obtain for the cutoff field  $E = 0.9E_0$ . The average  $E_R^2$  is then  $0.4E_0^2$ . From Eqs. 1 and 2 and the preceding

discussion we can finally write the ratio of line intensities as

$$R \approx \frac{0.48}{N_D^{1/3}} \frac{\Delta^2}{(\Delta + \omega_0)^2} \quad (3)$$

with  $N_D = (4/3)\pi L_D^3 N$  as the number of particles in the Debye sphere. For our plasma  $N_D \approx 1.7$ ,  $\Delta/(\Delta + \omega_0) \approx 0.5$ , and therefore  $R \approx 0.1$ .

The forbidden line undergoes self-absorption. By comparing its half-width with that of the allowed line (Fig. XVIII-3) and knowing the optical depth of the latter<sup>9</sup> (which is 3.2), the optical depth of the forbidden line turns out to be close to 1. This means that it was stronger before absorption by approximately a factor  $\exp \approx 2.7$ . The ratio  $R$  of Fig. XVIII-3 is  $\sim 0.35$ , and after self-absorption is accounted for becomes 0.13, which agrees with the theoretical value 0.1. Note that the forbidden line and satellite exist simultaneously, both being due to second-order effects, and their ratio is very insensitive to the value of  $N$  or  $T$ . The foregoing calculations show that part of the time the linear Stark effect prevails. This should give rise to an additional satellite about the allowed line which will about coincide with the forbidden line. The present satellite will then become weaker by almost a factor of 2 which, however, is almost exactly offset by the correction of third-order approximation.<sup>6</sup>

The width of the satellite is attributed mainly to the frequency spread of the plasma oscillations and the temporal changes in the wire; the width that is due to the ordinary Stark effect is considerably smaller.<sup>10</sup> The spectrum of  $E_p^2(\omega)$  is given by<sup>12</sup>

$$\frac{1}{2} \epsilon_0 E_p^2(\omega) d\omega = \frac{KT}{4\pi^3} \frac{d\omega}{\omega} \int_0^{L_D^{-1}} k^2 dk \operatorname{Im} \left[ \frac{1}{D(\omega, k)} \right], \quad (4)$$

where  $D$  is the dielectric response function for longitudinal oscillations. This spectrum is roughly bell-shaped; it has zero value for  $\omega < \omega_p$ , a maximum at  $\omega \approx 1.1 \omega_p$ , and, because of Landau damping, it is virtually zero for  $\omega \gtrsim 1.4 \omega_p$ . The full width at half-power points is  $\omega \approx 0.3 \omega_p$  which for our plasma corresponds to  $\sim 50 \text{ \AA}$ . To compute the width of the satellite from  $E_p^2(\omega)$ , we sum the contributions over successive intervals of time, in the range 1.5-7  $\mu\text{s}$  (the duration of the optically thin plasma). At very early times (1-2  $\mu\text{s}$ ), when the plasma is highly ionized and the density of neutral atoms is low, the contribution to the time-integrated light intensities is small. At late times the electron density is low; thus we find that the major contribution to the satellite comes from a short time interval near 2  $\mu\text{s}$ . The net computed linewidth, with the temporal changes<sup>11</sup> of  $\Delta$  also taken into account, is  $\sim 60 \text{ \AA}$ , whereas the width obtained from Fig. XVIII-3 is  $\sim 40 \text{ \AA}$ .

The appearance of a thermally excited satellite is due mainly to the special



properties of this plasma. It has a low temperature and it is dense. Therefore, the number of particles in the Debye sphere ( $N_D$ ) is very small compared with that found in most other gaseous plasmas. From Eq. 2 we compute the ratio of electrical energy ( $\epsilon_0 E_p^2/2$ ) to the particle kinetic energy ( $3NKT/2$ ) and obtain the value  $0.015/N_D$ , which means that in our plasma nearly 1% of the total energy is in the form of electrical energy of collective fluctuations. The corresponding field strength  $E_p \approx 2.2 \times 10^5$  V/cm is about equal to the average random field  $E_R$ . Furthermore, for the satellite and forbidden line to be of appreciable intensity, the electron density must be high; however, it must not be so high as to cause strong merging of the allowed and forbidden lines. Such merging has not occurred in our plasma, despite the high electron density, because of the large separation ( $\approx 150 \text{ \AA}$ ) between these two lines. At the time  $2 \mu\text{s}$  when the satellite is emitted (as inferred from its position) the current happens to about complete its first period and be small.<sup>3</sup> It is inconceivable that microinstabilities associated with the current will be then operative. This further supports the assertion that the observed satellite is thermally excited.

#### Satellites Induced by Laser Radiation

Suppose the electric fields  $E_R$  and  $E_p$  are too weak to produce a measurable forbidden line or satellites, as would be the case in a relatively tenuous plasma; and furthermore, that the plasma is acted upon by an intense, essentially monochromatic electric field of strength  $E_{\text{ext}}$  and frequency  $\omega_0$ . Then a forbidden line will not be excited (contrary to the case considered above) but, under suitable conditions, satellites may appear whose intensity is proportional to  $E_{\text{ext}}^2$  and whose frequency is  $\pm\omega_0$  relative to the forbidden line. Theory shows<sup>13</sup> that the ratio of the intensity of a satellite to the intensity of the allowed line is of the form

$$S \approx \frac{1}{10} \frac{e^2 a_0^2 R_{\ell\ell'}}{4\pi\epsilon_0 \hbar^2} \frac{E_{\text{ext}}^2}{(\Delta \pm \omega_0)^2}. \quad (5)$$

Here  $a_0$  is the Bohr radius, and  $R_{\ell\ell'}$  is an integral over the radial wave function as defined by Baranger and Mozer.<sup>1</sup>

We produce  $E_{\text{ext}}$  by focusing a high-power, Q-switched  $\text{CO}_2$  laser onto a small sodium plasma. The electric field at the focus of the lens (focal length 1.5 in.) is estimated to be of the order of  $10^5$  V/cm at a wavelength of  $10.6 \mu$ . The optical lines that we chose to examine are the 3P-3D allowed line ( $8186 \text{ \AA}$ ) and the 3P-3D forbidden line ( $7520 \text{ \AA}$ ) of NaI. Substituting appropriate values in Eq. 5, we find for the stronger satellite ( $\Delta - \omega_0$ ) that  $10^{-3} \lesssim S \lesssim 10^{-2}$ . Thus far, no satellites have been observed, on account of the lack of sensitivity of the present experimental arrangement.

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By using suitable optical discrimination within the focus, this technique could be a useful method of probing the electric field intensity of high-frequency, high-power, laser sources.

Much of this work was done at the Hebrew University of Jerusalem, Jerusalem, Israel, by two of the present authors (B. Y. and G. B.)

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## XVIII. PLASMAS AND CONTROLLED NUCLEAR FUSION\*

### D. Fusion-Related Studies

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### RESEARCH OBJECTIVES

#### 1. Economics of Toroidal Reactors

There is experimental and theoretical evidence that the maximum allowable  $\beta$  will be a strong function of the aspect ratio of the torus. Studies of the economics of toroidal reactors are being undertaken using realistic assumptions for the functional dependences of  $\beta$  on the aspect ratio.

R. A. Blanken

#### 2. Feasibility of Mirror Reactors

The crucial problems of synchrotron radiation have been handled very approximately in previous studies. A re-examination of the economics of mirror reactors using more realistic models for the synchrotron radiation process is therefore contemplated.

R. A. Blanken

#### 3. Fusion-Fission Reactors

Calculations show that a fusion reactor of moderate size could be built that would yield tritium breeding ratios in excess of 1.35. This, in effect, would free at least 0.10-0.15 neutrons per fusion reaction to be used for other purposes. A possible use is as input to a marginal fast breeder or advanced converter fission reactor. This symbiosis would act to eliminate the major weaknesses of both power-producing schemes, that is, the low power density of fusion reactors and the long doubling time of breeder reactors.

L. M. Lidsky

#### 4. Gamma-Ray Transport in Fusion Reactor Blankets

We have begun a design study for a thermal neutron-fed 14-MeV neutron generator plate to be used with the MITR fission reactor. The convecter plate will be optimized for studies of gamma-ray transport in thermonuclear blanket models. Our goal is the

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generation of data for comparison with the very sketchy computer studies that have been done thus far.

R. L. McCrory, Jr., A. Pant, L. M. Lidsky

## 5. Stability of Relativistic Electron Layers

During the past year we completed our study of the long-wavelength modes of gyrating electron layers in a cold plasma. In the early part of 1970, we plan to use the model that we have developed to investigate possible instability modes around the cold plasma upper hybrid frequency. Beyond this, we do not plan any further work in this area.

R. J. Briggs

### 1. STABILITY OF A RELATIVISTIC ELECTRON LAYER

In a previous report,<sup>1</sup> we formulated the dispersion relation for a cylindrical layer of relativistic electrons embedded in a cold plasma. We found that a beam parametrized by  $\zeta = 4\%$ ,  $R/\tau = 20$ ,  $\gamma_0 = 9$  can be rendered stable by a high-density plasma background. Here,  $\zeta$  is the "loading factor,"  $\tau$  the beam thickness,  $R$  the mean radius of the E layer,  $\gamma_0$  the usual relativistic mass factor, and the plasma is assumed to be uniform everywhere in the container. The critical plasma density increases with  $\ell$ , the azimuthal wave number. It is noted, according to our data, that the beam is stable (or unstable) depending on whether it has an inductive (or capacitive) wave admittance at the beam edges. (The admittance,  $b_+ + b_-$ , is termed inductive when it is negative and capacitive when it is positive.)

Further investigation of the dispersion relation indicates that the critical plasma density as a function of  $\ell$  for  $\zeta = 6\%$  and  $10\%$  is identical with that for  $\zeta = 4\%$ . The explanation of this follows. A careful consideration of the expressions for  $b_{\pm}$  shows<sup>2</sup> that the admittance is infinitely inductive when the plasma density  $\omega_p$  is slightly above the critical density  $\omega_{pc}$  and is infinitely capacitive when  $\omega_p$  is slightly less than  $\omega_{pc}$ . In other words, the transition from the inductive to the capacitive regime is "sharp." Accordingly, the Nyquist plot encounters an abrupt change as  $\omega_p$  transverses  $\omega_{pc}$ . This critical plasma density  $\omega_{pc}$  is independent of  $\zeta$ , since the wave admittance, whose transition defines  $\omega_{pc}$ , is independent of beam thickness and beam density. This explains why the  $\omega_{pc}$  vs  $\ell$  plot is the same for different values of  $\zeta$  and has a remarkable resemblance to the result of Briggs,<sup>2</sup> which was obtained from a model with zero thickness.

We have also analyzed cases in which the background plasma is confined only within the beam. Thus a vacuum is formed in the region between the beam and the tank walls, and in such cases the admittance at the beam edge is a function of the vacuum geometry and is independent of plasma density. We have studied the Nyquist plots for various azimuthal modes ( $\ell$ ) as a function of plasma density for the geometries (i)  $a/R = 0$ ,  $b/R = 1.4$ , and (ii)  $a/R = 2/3$ ,  $b/R = 1.4$ .<sup>3</sup> The beam parameters are:  $\zeta = 4\%$ ,  $\gamma_0 = 9$ ,  $R/\tau = 20$ .

The result is summarized as follows. For case (i), the beam is stable or unstable depending on whether the admittance is inductive or capacitive. For case (ii), where the structure is always capacitive, we found that the lower azimuthal modes ( $\ell = 1$  and  $2$ ) can be stabilized by a high enough background plasma density. The higher modes,  $\ell \geq 3$ , cannot be stabilized, even if the plasma density is taken as high as  $N_0 \sim 10^{14}/\text{cm}^3$ , which indicates that the beam stability is predominantly determined by the wave admittances at the beam edges. Indeed, we can show that the dispersion relation with background plasma included reduces to that of a thin layer in vacuum as we let  $\tau \rightarrow 0$ .

The details of the foregoing discussions may be found in a thesis,<sup>4</sup> submitted to the Department of Electrical Engineering, M. I. T., in partial fulfillment of the requirements for the degrees of Master of Science and Electrical Engineer.

Y. Y. Lau, R. J. Briggs

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