# New physics search in the LHCbera 

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$W$ e present theoretical and experim ental preparations for an indirect search for new physics (NP) using the rare decay $\bar{B}_{d}$ ! $\bar{K}^{0}+$. W e design new observables $w$ ith very sm all theoretical uncertainties and good experim ental resolution.

## 1. Introduction

A the start of the LH C we are confronted $w$ ith the experim ental fact that alldata on avour observables from Babar, B elle, C LEO and also from D 0 and CDF are consistent $w$ ith the Standard M odel (SM ) predictions [1]. This im plies that generic new physics (NP) contributions in K K $m$ ixing for exam ple guide us to a new-physics scale of $10^{3} \quad 10^{6} \mathrm{TeV}$ depending if the new contributions enter at loop-or tree-level. This is in strong contrast to the working hypothesis of the LHC that there is NP "around the comer" at 1 TeV in order to stabilise the H iggs boson m ass. Therefore, any NP at the 1 TeV scale has to have a non-generic avourstructure and we have to understand why new avour-changing neutral currents (FCNC) are suppressed. R are decays and C P violating observables allow an analysis of th is avour problem.
The crucial problem in the new physics search w ith in avour physics is the optim alseparation of NP e ects from hadronic uncertainties. It is well know $n$ that inclusive decay $m$ odes are dom inated by partonic contributions; non-perturbative corrections are in general rather sm all [2,3]. A lso ratios of exclusive decay $m$ odes such as asym $m e-$ tries are well suited for the new-physics search. Here large parts of the hadronic uncertainties partially cancel out; for exam ple, there are CP asym $m$ etries that are govemed by one w eak phase only; thus the hadronic $m$ atrix elem ents cancel out com pletely. It is the latter opportunity which represents the generalstrategy follow ed by LH C b
for the construction of theoretically clean observables.

In this letter we brie $y$ discuss the theoretical and experim entalpreparations for an indirect $N P$ search using the rare decay $\bar{B}_{d}$ ! $\bar{K} 0+$ based on the QCDf/SCET approach [4]. QCD corrections are included at the next-to-leading order level and also the im pact of the unknow $n=m b$ corrections is $m$ ade explicit.

The exclusive decay $\bar{B}_{d}!\overline{\mathrm{K}} 0+$ was rst observed at Belle [5]. It o ers a rich phenom enology of various kinem atic distributions beyond the $m$ easurem ent of the branching ratio. W e note that som e experim ental analyses of those angular distributions are already presented by the B factories [ $6,7,9,10]$. Those experim ental results already have a signi cant im pact on the $m$ odel-independent constraints $w$ ithin the $m$ inim al avourviolation approach [8].

Large increase in statistics at LHC.b [11,12,13] for $\bar{B}_{d}$ ! $\overline{\mathrm{K}} 0+\mathrm{w}$ ill m ake m uch higher precision $m$ easurem ents possible. T here are also great opportunities at the future (Super-)B factories in th is respect [14,15,16,17].

P reviously proposed angular distributions and CP violating observables in $\overline{\mathrm{B}}{ }_{\mathrm{d}}!\overline{\mathrm{K}} 0+$ are review ed in Ref. [23], and m ore recently QCDf analyses of such angular distributions [24,25] and C P violating observables [26], based on the N LO results in $R$ ef. [27], w ere presented.

## 2. QCD factorization, SCET

$R$ egarding the hadronic $m$ atrix elem ents of exclusive modes, the m ethod of QCD-im proved factorization (QCDf) has been system ized for non-leptonic decays in the heavy-quark lim it. $T$ his $m$ ethod allow sfor a pertunbative calculation of QCD corrections to naive factorization and is the basis for the up-to-date predictions for exclusive rare B decays in general [18].

A quantum eld theoretical fram ew ork was proposed \{ known under the name of softcollinear e ective eld theory (SCET) \{ which allows for a deeper understanding of the QCDf approach [19,20]. In contrast to the heavy-quark e ective theory (HQET), SCET does not correspond to a local operator expansion. HQET is only applicable to $B$ decays, when the energy transfer to light hadrons is sm all, for exam ple to B ! D transitionsatsm allrecoil to the D m eson. HQET is not applicable, when some of the outgoing, light particles have $m$ om enta of order $m_{b}$; then one faces a $m$ ulti scale problem that can be tackled w ithin SCET .

There are three scales: a) = few QCD the soft scale set by the typicalenergies and $m$ om enta of the light degrees of freedom in the hadronic bound states; b) $\mathrm{m}_{\mathrm{b}}$ the hard scale set by the heavy-b-quark $m$ ass and also by the energy of the nal-state hadron in the $B-m$ eson rest fram $e$; and $c$ ) the hard-collinear scale $\mathrm{hc}_{\mathrm{c}}=\mathrm{P} \frac{\mathrm{m}_{\mathrm{b}}}{} \mathrm{ap}-$ pears through interactions betw een soft and energetic $m$ odes in the initial and nal states. The dynam ics of hard and hard-collinear $m$ odes can be described pertunbatively in the heavy-quark $\lim$ it $\mathrm{m}_{\mathrm{b}}$ ! 1 . Thus, SCET describes B decays to light hadrons with energies much larger than their $m$ asses, assum ing that their constituents have $m$ om enta collinear to the hadron $m$ om entum.

However, we emphasize that within the QCDf/SCET approach, a general, quantitative $m$ ethod to estim ate the im portant $=m_{b}$ corrections to the heavy-quark lim it is m issing which has im portant phenom enological consequences.

A careful choice of observables needs to be $\underline{m}$ ade to take fulladvantage of the exclusive decay $\overline{\mathrm{B}}_{\mathrm{d}}!\overline{\mathrm{K}} 0+$, as only in certain ratios such
as CP and forw ard-backw ard asym $m$ etries, the hadronic uncertainties canceloutm aking such ratios the only observables that are highly sensitive to NP.
W ithin the QCDf/SCET approach one nds crucial form factor relations [21] which sim plify the theoretical structure of various kinem atical distributions such that, at least at the leading order (LO ) level any hadronic uncertainties cancel out. A well-known example of this is the zero-crossing of the forw ard-backw ard asym $m e-$ try. In [4] new observables of this kind in the $\bar{B}_{d}!\overline{\mathrm{K}} 0+$ decay were proposed which have very sm all theoretical uncertainties and good experim ental resolution. T he only di erence to the forw ard-backw ard asym $m$ etry is that $w$ ithin these new observables the hadronic form factors cancel out for all values of the dilepton $m$ ass.

## 3. Theoretical prelim inaries

The decay $\bar{B}_{\mathrm{d}}$ ! $\overline{\mathrm{K}}{ }^{0+}$, with $\overline{\mathrm{K}} 0$ ! $\mathrm{K} \quad+$ on the $m$ ass shell is com pletely described by four independent kinem atic variables, the lepton-pair invariant $m$ ass squared, $q^{2}$, and the three angles 1, k, . Summ ing over the spins of the nal particles, the di erential decay distribution of $\bar{B}_{d}!\overline{\mathrm{K}} 0 \curlywedge$ ィ can bewritten as [28,29,30,31]:

$$
\frac{d^{4}-\overline{B_{d}}}{{d q^{2}}^{d_{1}} d_{k} d}=\frac{9}{32} I\left(q^{2} ; \text { п к }^{\prime} ;\right) \sin { }_{1} \sin k
$$

w ith

$$
\begin{align*}
I= & I_{1}+I_{2} \cos 2_{1}+I_{3} \sin ^{2}{ }_{1} \cos 2 \\
& +I_{4} \sin 2_{1} \cos +I_{5} \sin { }_{1} \cos \\
& +I_{6} \cos { }_{1}+I_{7} \sin { }_{1} \sin \\
& +I_{8} \sin 2{ }_{1} \sin +I_{9} \sin ^{2}{ }_{1} \sin 2: \tag{1}
\end{align*}
$$

The $I_{i}$ depend on products of the seven com plex $K$ spin am plitudes, $A_{\text {? } L=R}, A_{k L=R}, A_{0 L=R}, A_{t}$ $w$ ith each of these a function of $q^{2}$; the explicit form ulae are given in the appendix. $A_{t}$ is related to the tim e-like com ponent of the virtual K , which does not contribute in the case ofm assless leptons and can be neglected if the lepton $m$ ass is $s m$ all in com parison to the $m$ ass of the lepton pair. We will consider this case in our present analysis.

The six complex $K$ spin amplitudes of the $m$ assless case are related to the well-know n helicity am plitudes (used for exam ple in [29,30,32]):
$A_{? ; k}=\left(\begin{array}{ll}H_{+1} & H \\ 1\end{array}\right)=\bar{P} \overline{2} ; \quad A_{0}=H_{0}:$
The crucial theoretical input we use in our analysis is the observation that in the lim it where the initial hadron is heavy and the nal $m$ eson has a large energy [21] the hadronic form factors can be expanded in the sm all ratios ecd $=m_{b}$ and $Q C D=E$, where $E$ is the energy of the light $m$ eson. N eglecting corrections of order $1=m_{b}$ and
$s$, the seven a priori independent B! K form factors reduce to two universal form factors ? and $k$ [21,22] and one nds that the spin am plitudes at leading order in $1=\mathrm{m}_{\mathrm{b}}$ and s have a very sim ple form :

$$
\begin{align*}
& A_{? \mathrm{~L} ; \mathrm{R}}=\mathrm{P}_{2} \mathrm{Nm}_{\mathrm{B}}\binom{1}{\mathrm{~S}} \quad\left(\mathrm{C}_{9}^{(\mathrm{e})} \quad \mathrm{G}_{0}\right) \\
& +\frac{2 m_{b}^{b}}{s}\left(C_{7}^{(e)}+C_{7}^{0}\left(e^{)}\right) \quad ?\left(E_{K}\right) ;\right. \\
& A_{k L ; R}=P_{2} N_{m_{B}}(1 \quad s) \quad\left(C_{9}^{(e)} \quad G_{0}\right) \\
& +\frac{2 m_{b}}{s}\left(C_{7}^{(e)} \quad G^{0}(e)\right) \quad ?\left(E_{K}\right) ; \\
& A_{0 L ; R}=\frac{N m_{B}^{P}}{2 m_{K}} \bar{S}(1 \quad s)^{2}\left(C_{9}^{(e)} \quad G_{0}\right) \\
& +2 \mathrm{~m}_{\mathrm{b}}\left(\mathrm{C}_{7}^{(\mathrm{e})} \quad \mathrm{G}^{0}{ }^{(\mathrm{e})}\right) \quad \mathrm{k}_{\mathrm{k}}\left(\mathrm{E}_{\mathrm{K}}\right) \text {; } \tag{3}
\end{align*}
$$

with $S=q^{2}=m_{B}^{2}, \hat{m}_{i}=m_{i}=m_{B}$. Here we neglected term $s$ of $O\left(\mathrm{~m}_{\mathrm{K}}^{2}\right)$. It is important to $m$ ention that the theoretical sim pli cations are restricted to the kinem atic region in which the energy of the $K$ is of the order of the heavy quark $m$ ass, i.e. $q^{2} \quad m_{B}^{2}$. M oreover, the in $u$ ences of very light resonances below 1 G eV question the QCD factorization results in that region. Thus, we will con ne our analysis of all observables to the dilepton $m$ ass in the range $1 \mathrm{GeV}^{2} \quad \mathrm{q}^{2} \quad 6 \mathrm{GeV}^{2}$.
4. C onstruction of theoretically clean observables

By inspection one nds that the distribution functions $I_{i}$ in the di erential decay distribution (see Eq. [12)) are invariant under three sym $m$ etry transform ations which are given explicitly in the appendix (see Eqs. 13-15)). This im plies that only 9 of the 12 K spin am plitudes are independentand that they can be xed by an fullangular $t$ to the 9 independent coe cients of the di erential decay distribution. A nother direct consequence is that any observable based on the di erential decay distribution has also to be invariant under the sam e sym $m$ etry transform ations.

B esides thism andatory criterium there are further criteria required for an interesting observable. [S im plicity:] A sim ple functional dependence on the 9 independent $m$ easurable distribution functions; at best it should depend only from one or two in the num erator and denom inator of an asym $m$ etry. [C lean liness:] At leading order in $=\mathrm{m}_{\mathrm{b}}$ and in s the observable should be independent of any form factor, at best for $\mathrm{allq}^{2}$. A lso the in uence of sym $m$ etry-breaking corrections at order $s$ and at order $=m b$ should be $m$ inim al. [Sen sitivity:] The sensitivity to the $\mathrm{C}_{7}{ }^{( }{ }^{(e)}{ }^{\text {) }} \mathrm{W}$ ison coe cient representing NP w ith another chirality than in the SM should be $m$ axim al. [P recision:] The experim ental precision obtainable should be good enough to distinguish di erentN P m odels.

In the lim it where the $\overline{\mathrm{K}}{ }^{0} \mathrm{~m}$ eson has a large energy, only tw o independent form factors occur in $A_{0 L=R}$ and in $A_{\text {? } L=R}$ and $A_{k L=R}$. C learly, any ratio of two of the nine $m$ easurable distribution functions proportional to the sam e form factor ful 1 the criterium of sym $m$ etry, sim plicity, and theoretical cleanliness up to $=m$ b and s corrections. H ow ever, the third criterium, a sensitivity to a special kind of NP and the subsequent requirem ent of experim ental precision, singles out particular combinations. In [4] we focused on new right-handed currents. O ther NP sensitivities $m$ ay single out other observables as w ill be analysed in a forthcom ing paper [33].


Figure 1. For $A_{T}^{(2)}$, theoretical errors (top), experim ental errors (bottom) as a function of the squared dim uon $m$ ass, see text for details.

## 5. R esults

The rst surprising result is that the previously proposed quantity $A_{T}^{(1)}$ [29],
$A_{T}^{(1)}=\frac{+}{+\quad+}=\frac{2<\left(A_{k} A_{?}\right)}{\left\langle A_{?} \mathcal{J}^{2}+2 A_{k} J^{2}\right.}:$
 $m$ ost im portant criterium of sym $m$ etry while it has very attractive new physics sensitivity [24,25]. $T$ herefore, it is not possible to extract $A_{T}^{(1)}$ from the full angular distribution which is constructed after sum $m$ ing over the spins of the nal particles. Because it seem s practically not possible to $m$ easure the helicity of the nalstates on a event-by-event basis, $A_{T}^{(1)}$ cannot be $m$ easured at either LHCb or at a Super-B factory with electrons or $m$ uons in the nalstate.

O ne nds that the well-known quantities, the forw ard-backw ard asymmetry $A_{F B}$ and the


Figure 2. $\mathrm{A}_{\mathrm{T}}^{(3)}$, as in F ig.1.



Figure 3. $A_{T}^{(4)}$, as in Fig.1.


Figure 4. $A_{F B}$, as in Fig.1.


F igure 5. $\mathrm{F}_{\mathrm{L}}$, as in F ig.1.
longitudinal $K$ polarization $F_{L}$ ful 11 the sym $m$ etry but they include larger theoretical uncertainties due to the fact that the form factors do not cancel at leading order level for all dilepton $m$ asses. M oreover, the sensitivity to right-handed currents is $m$ arginal as it is show $n$ below,
$A_{F B}=\frac{3}{2} \frac{<\left(A_{k L} A_{? L}\right)<\left(A_{k R} A_{? R}\right)}{\left\{A_{0} \jmath^{2}+\left\{A_{k}\right\}^{2}+2 A_{?}\right\}^{2}}$
$w$ here for $i ; j=0 ; k ;$ ?
$A_{i} A_{j} \quad A_{i L}\left(q^{2}\right) A_{j L}\left(q^{2}\right)+A_{i R}\left(q^{2}\right) A_{j R}\left(q^{2}\right) ;$

In contrast, the follow ing three observables,


$A_{T}^{(4)}=\frac{z_{0 L} A_{? L} \quad A_{0 R} A_{? R} j}{Z_{0 L} A_{k L}+A_{O R} A_{k R} j} ;$
are theoretically clean for alldilepton $m$ asses and also show a very high sensitivity to right-handed currents.

In the follow ing gures the results on the observables, $F_{L}, A_{F B}, A_{T}^{(2)}, A_{T}^{(3)}$, and $A_{T}^{(4)}$ are ilustrated: For all the observables the theoretical sensitivity is plotted on the top of each gure. $T$ he thin dark line is the central NLO result for the SM and the narrow inner dark (orange) band that surrounds it corresponds to the NLO SM uncertainties due to both input param eters and perturbative scale dependence. Light grey (green) bands are the estim ated $=m_{b} \quad 5 \%$ corrections for each spin am plitude while darker grey (green) ones are the $m$ ore conservative $=m b$ 10\% corrections. T he curves labelled (a) \{ (d) correspond to four di erent benchm ark points in the M SSM for righthanded currents (form ore details see [4]). T he experim ental sensitivity for a dataset corresponding to $10 \mathrm{fb}^{1}$ of LH Cb data is given in each gure on the bottom, assum ing the SM. H ere the solid (red) line show $s$ the $m$ edian extracted from
the $t$ to the ensemble of data and the dashed (black) line show s the theoretical input distribution. The inner and outer bands correspond to 1 and 2 experim ental errors.

The observables $A_{T}^{(3)}$ and $A_{T}^{(4)} \circ$ er sensitivity to the longitudinalspin am plitude $A_{\text {oL ; }}$ in a controlled way com pared to the old observable $\mathrm{F}_{\mathrm{L}}$ : the dependence on both the paralleland perpendicular soft form factors $k(0)$ and ? ( 0 ) cancels at LO.A residual of this dependence $m$ ay appear at NLO , but as show n in F igs. 2 and 3, it is basically negligible. It is also rem arkable that for $A_{T}^{(3)}$ and $A_{T}^{(4)}$ at low $q^{2}$ the im pact of this uncertainty is less im portant than the uncertainties due to input param eters and scale dependence. The observables $A_{T}^{(3)}$ and $A_{T}^{(4)}$ also present a di erent sensitivity to $C_{7}$ via their dependence on $A_{0 L} ; R$ com pared w ith $A_{T}^{(2)}$. This may allow for a particularly interesting cross check of the sensitivity to this chirality ipped operator $\mathrm{O}_{7}^{0}$; for instance, new contributions com ing from tensor scalars and pseudo-scalars w ill behave di erently am ong the set of observables.

A nother rem arkable point that becom es clear when com paring the set of clean observables $A_{T}^{(2)}$, $A_{T}^{(3)}$ and $A_{T}^{(4)}$ versus the old observables $F_{L}$ and $A_{F B}$ concems the potential discovery of $N P$, in particular of new right-handed currents. There are large deviations from the SM curve from the ones of the four supersym $m$ etric benchm ark points. A large deviation from the $S M$ for $A_{T}^{(2)}$, $A_{T}^{(3)}$ or $A_{T}^{(4)}$ can thus show the presence of righthanded currents in a w ay that is not possible w ith $F_{L}$ or $A_{F B}$. In the latter cases the deviations from the $S M$ prediction of the sam e four representative curves are $m$ arginal.

In the experim entalplots we nd a good agree$m$ ent betw een the central values extracted from the ts and the theoretical input. A ny deviations seen are sm all com pared to the statistical uncertainties. The experim ental resolution for $F_{L}$ is very good but w ith the sm alldeviations from the SM expected this is not helpfulin the discovery of new right-handed currents. C om paring the theoretical and experim ental gures for the other observables it can be seen that in particular $A_{T}^{(3)}$


Figure 6. Belle (black/blue) and BaBar (grey/red) data points on $F_{L}$ and on $A_{F B}$ with SM predictions and weighted SM averages over the bin $q^{2} 2\left[1 \mathrm{GeV}^{2} ; 6 \mathrm{GeV}^{2}\right]$
show great prom ise to distinguish betw een NP m odels.
$F$ inally, let us $m$ ention that the old observables $F_{L}$ and $A_{F B}$ are already accessible to the BaB ar[10,34] and Belle [35] experim ents. The rst $m$ easurem ents are show $n$ in $F$ ig. 6 w ith the SM predictions and the weighted SM averages over the bin $q^{2} 2\left[1 \mathrm{GeV}^{2} ; 6 \mathrm{GeV}^{2}\right]$. A 11 the present data is com patible $w$ ith the $S M$ predictions. For exam ple, the rst m easurem ent of the B abar collaboration on $F_{L}$ in the low $-q^{2}$ region is given as an average over the b in $\mathrm{q}^{2} 2\left[4 \mathrm{~m}^{2} ; 6: 25 \mathrm{G} \mathrm{EV}^{2}\right]$ :
$\mathrm{F}_{\mathrm{L}}\left(\left[4 \mathrm{~m}^{2} ; 6: 25 \mathrm{GeV}^{2}\right]\right)=0: 35 \quad 0: 16 \quad 0: 04$; (10)
while the theoretical average, w eighted over the rate, using the bin, $q^{2} 2\left[1 \mathrm{GeV}^{2} ; 6 \mathrm{GeV}^{2}\right]$, based on our results is given by:
$F_{\mathrm{L}}\left(\left[1 \mathrm{GeV}^{2} ; 6 \mathrm{GeV}^{2}\right]\right)=0: 86^{+0: 04} 0:$

Here，one should keep in $m$ ind that the spectrum below $1 \mathrm{GeV}^{2}$ is theoretically problem atic due to the in uence of very light resonances；m oreover the rate and also the polarisation $\mathrm{F}_{\mathrm{L}}$ are chang－ ing dram atically around $1 \mathrm{GeV}^{2}$ ．Therefore，we strongly recom $m$ end to use the standard bin from $1 \mathrm{GeV}^{2}$ to $6 \mathrm{GeV}^{2}$ in all future m easurem ents．

## 6．Sum m ary

The full angular analysis of the decay $\bar{B}_{d}$ ！ $\overline{\mathrm{K}} 0+$ at the LHCb experim ent O ers great opportunities for the new physics search．New observables can be designed to be sensitive to a speci c kind of NP operator w ith in the model－ independent analysis using the e ective eld the－ ory approach．The new observables $A_{T}^{(2)}, A_{T}^{(3)}$ and $A_{T}^{(4)}$ are show $n$ to be highly sensitive to right handed currents．C learly，theoretical progress on the $=m_{b}$ corrections would enhance their sensi－ tivity signi cantly and would be highly desirable in view of a possible upgrade of the LHCb exper－ im ent．M oreover，we have shown that the previ－ ously discussed angular distribution $A_{T}^{(1)}$ cannot bem easured at either LH Cb or at a Super－B fac－ tory．

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A ppendix
W e add here the explicit form ula for the distri－ bution functions and their sym $m$ etries：

In them assless lim it，the distribution functions $I_{i}$ depend on products of the six com plex K spin am plitudes，$A_{\text {？L＝}}, A_{k L=R}, A_{\text {OL }=R}$ ：

$$
\begin{aligned}
& \mathrm{a} \sin ^{2} \mathrm{k}+\mathrm{b}_{\mathrm{cos}}{ }^{2} \mathrm{k} \text {; }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 动 } 0 \text { 号 } \cos ^{2} k+(L!R) \\
& \mathrm{C} \sin ^{2} \mathrm{~K}+\mathrm{d} \cos ^{2} \mathrm{~K} \text {; }
\end{aligned}
$$

$$
\begin{align*}
& \mathrm{e} \sin ^{2} \text { к; } \\
& I_{4}=\frac{1}{\overline{2}}<\left(A_{0 L} A_{k L}\right) \sin 2_{K}+(L!R) \\
& \mathrm{f} \sin 2_{\mathrm{K}} \text {; } \\
& I_{5}={ }^{\mathrm{P}} \overline{2}<\left(\mathrm{A}_{0 \mathrm{~L}} \mathrm{~A}_{\text {? }}\right) \sin 2_{K} \quad(\mathrm{~L}!\mathrm{R}) \\
& g \sin 2_{k} \text {; } \\
& I_{6}=2<\left(A_{k L} A_{\text {? } L}\right) \sin ^{2} K(L!R) \\
& \mathrm{h} \sin ^{2} \mathrm{k} \text {; } \\
& I_{7}={ }^{\mathrm{P}} \overline{2}=\left(\mathrm{A}_{0 \mathrm{~L}} \mathrm{~A}_{\mathrm{kL}}\right) \sin 2_{\mathrm{K}} \quad(\mathrm{~L}!\mathrm{R}) \\
& j \sin 2 \text { к; } \\
& I_{8}=\frac{1}{P_{\overline{2}}}=\left(A_{0 L} A_{?_{L}}\right) \sin 2_{K}+(L!R) \\
& \mathrm{k} \sin 2_{\mathrm{K}} \text {; } \\
& I_{9}=\quad=\left(A_{k L} A_{? L}\right) \sin ^{2} K+(L!R) \\
& m \sin ^{2}{ }_{k} \text { : } \tag{12}
\end{align*}
$$

Taking into account $\mathrm{a}=3 \mathrm{c}$ and $\mathrm{b}=\mathrm{d}$ ，we are left w ith 9 independent param eters which can be xed experim entally in a full angular $t$ ．
The distribution functions are invariant un－ der the follow ing three independent sym $m$ etry transform ations of the spin am plitudes as one easily veri es，using the explicit form ulae given above：（1）a global phase transform ation of the L－am plitudes

$$
\begin{align*}
A_{? L}^{0} & =e^{i^{L}} A_{? L} ; \\
A_{k L}^{0} & =e^{i^{L}} A_{k L} ; \\
A_{0 L}^{0} & =e^{L^{L}} A_{0 L} ; \tag{13}
\end{align*}
$$

（2）a global transform ation of the $R$－am plitudes

$$
\begin{align*}
A_{? R}^{0} & =e^{i^{R}} A_{? R} ; \\
A_{k R}^{0} & =e^{i_{R}^{R}} A_{k R} ; \\
A_{0 R}^{0} & =e^{i_{R}} A_{0 R} ; \tag{14}
\end{align*}
$$

and (3) a continuous L \$ R rotation

$$
\begin{align*}
A_{? L}^{0} & =+\cos A_{? L}+\sin A_{? R} \\
A_{? R}^{0} & =\sin A_{? L}+\cos A_{? R} \\
A_{0 L}^{0} & =+\cos A_{0 L} \sin A_{0 R} \\
A_{0 R}^{0} & =+\sin A_{O L}+\cos A_{0 R} \\
A_{k L}^{0} & =+\cos A_{k L} \sin A_{k R} \\
A_{k R}^{0} & =+\sin A_{k L}+\cos A_{k R}: \tag{15}
\end{align*}
$$

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