

AN INTRODUCTION TO UNCERTAINTY  
IN MEASUREMENT USING THE GUM  
(GUIDE TO THE EXPRESSION OF  
UNCERTAINTY IN MEASUREMENT)

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