
ASYMMETRIC DOUPOLY COMPETITION WITH INNOVATION SPILLOVER AND INPUT CONSTRAINTS

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Abstracts. This paper subjects to examine how technology spillover affects input competition and how input constraints impact firm innovation by a two-stage game model and theoretic analysis. The results show that with low spillover, the high cost firm can capture more input than the low cost firm through cost-reducing innovation. Adding input increases firms' innovation, but it cannot improve the disadvantaged firm's state under input constraint. Compared with non-cooperative innovation, cooperative innovation reduces innovation difference and firm size difference. The research implications are that disadvantage firms could take innovation spillover and capacity constraints as a competition strategy to obtain competition advantage and regulators should stimulate cooperative innovation to higher social welfare. The major value of this paper is that it combines capacity constraints and innovation investment originality.

Keywords: cost reducing innovation, input constraints, asymmetric competition, technology spillover, game theory, strategic effects.

JEL Classification: C72, D24, L13.

Introduction

Capacity constraints are common in many industries. Arenas and cinemas have only limited number of seats, and airports can only land limited number of airplanes. Firms sustain capacity constraints such as finance constraints (Love 2003; Whited 2006; Sena 2006; Carvajal *et al.* 2012), cash flow constraints (Kaplan, Zingales 2000), space constraints (Lester 2011), energy constraints (Veit *et al.* 2011), or information constraints (Ramadorai 2013). Capacity constraints change firms' competition strategy. On one hand, constraints restrict performance and development of the firm. On the other hand, firms can also obtain strategic advantages from constraints (Riordan 2003; Khanna, Schroder 2010). For example, Deng and Yano (2006) showed that the firm with tight capacity constraints will adopt more aggressive price.

In recent years, capacity constraints have received much attention (e.g. Sena 2004; Whited 2006; Adilov 2012; Van Den Berg *et al.* 2012; Ramadorai 2013). All these studies can be classified into four areas generally. Firstly, some have investigated the effects of capacity constraints on prices. For example, Ishibashi (2008) studied collusive price equilibrium with capacity constraints and showed that capacity constraints heighten collusive leadership price. More interestingly, Arnold and Saliba (2011) implied capacity constraints yield price dispersion and high constrained firm charges high price. Secondly, some prior researches focused on the relationships between capacity constraints and output or industrial structure. Nie and Chen (2012) investigated duopoly competition with input constraints. They declared that input constraints reduce market size difference and price difference. Esó *et al.* (2010) obtained similar results and they concluded that capacity constraints lead to industry symmetry.

Part of the existent studies highlighted the effects of capacity constraints on other competition strategies such as merger and collusion. Froeb *et al.* (2003) studied the effects of mergers among firms facing capacity constraints. Taking dynamic aspects of competition into account, Compte *et al.* (2002) investigated the effects of capacity constraints on mergers and collusion. Van Den Berg *et al.* (2012) discussed capacity constraints under dynamic Cournot competition and considered the effects of capacity constraints on equilibrium while Froeb *et al.* (2003) and Chowdhury (2009) analyzed capacity constraints under Bertrand competition. These studies show that capacity constraints induce industrial symmetry. Fourthly, there are also a few studies of capacity constraints in particular industries such as service providers (Burkart *et al.* 2012; Wang, Dragahi 2013), retail stores (Murray *et al.* 2012) and air transportation (Evans, Schaefer 2011). Genc and Reynolds (2010) identified how capacity constraints affect output equilibrium with wholesale electricity markets, and they also found that capacity constraints result in symmetry. Besides, Inderst and Wambach (2001) analyzed competitive insurance markets with limited capital.

For one thing, among the existing researches of capacity constraints, none takes innovation (or R&D investment) into account despite its obvious significance. For example, energy-dependent firms should invest in innovation to promote its efficiency because of the scarcity of energy resources. Different from other studies, we consider innovation behavior with capacity constraints. The main purpose of this paper is to examine how technology spillover affects input competition and how input constraints affect firm innovation. We find that the conclusion drawn by prior studies such as Esó *et al.* (2010) and Nie and Chen (2012) that capacity constraints increase symmetry is not necessarily robust. For another thing, different from the major studies about cost reducing innovations of D'Aspremont and Jacquemin (1988) and Suzumura (1992), we investigate innovation with asymmetric firms and obtain some interesting conclusions. Firstly, spillover has a major effect on duopoly competition. Large spillover aggravates the high cost firm's status. But if spillover is small, the high cost firm can even obtain a competitive edge through innovation. Including innovation, we find input constraints increase firm symmetry only when spillover is low. Secondly, innovations of both firms increase as the input constraint bound increases. But increase in input constraint bound does not change the differences of innovation and firm size, because competitive firms

divide increments equally. Finally, cooperative innovation mitigates innovation difference and firm size difference.

The rest of this paper is organized as follows. Based on some assumptions, the basic model is established in Section 1. Then we analyze the model under different conditions in Sections 2 and 3. Some discussions of the main conclusions are expounded in the last Section.

1. Setup

Two firms, denoted $i \in \{1, 2\}$, with a constrained input and asymmetric costs produce perfectly substituting products. Denote the only input by r_i for $i \in \{1, 2\}$. Furthermore, $R > 0$ represents the total capacity in this industry or $r_1 + r_2 \leq R$. Each firm transforms one unit input into one unit output efficiently, but with asymmetric costs¹. The two firms produce homogenous products or product substitutability $\gamma = 1$ ². And both of them launch R&D investments with spillover $\beta \in [0, 1]$ to reduce their costs before output decisions, which means we consider a two-stage model³. Besides, we assume spillovers are symmetric between producers.

Let p denote the price, q_1 and q_2 the outputs of the two firms. As in Liu and Wang (2013), Sacco and Schmutzer (2011) or Liu *et al.* (2011), the inverse demand function of our study is given by the following:

$$p = A - q_1 - q_2. \tag{1}$$

Both firms produce with the only input r_i efficiently, which means:

$$q_i = r_i. \tag{2}$$

Denote firm 1 the low cost firm and firm 2 the high cost firm. Then the cost function with cost reducing innovation is given as the following:

$$C_i = (c_i = x_i - \beta x_j)r_i. \tag{3}$$

In (3), x_i and x_j are the R&D investments of firm i and j for $i, j \in \{1, 2\}$. Without loss of generality, we assume $c_1 = c_0$ and $c_2 = c_0 + \tau$ are the initial unit costs of firms 1 and 2, where the constant $\tau > 0$ represents the cost difference or market power⁴. Cost of investment is quadratic, which is also employed by D’Aspremont and Jacquemin (1988), and Sacco and Schmutzer (2011). So the input-constrained producer i ’s profit maximization problem is:

$$\begin{aligned} \text{Max}_{r_i} \quad & \pi = (A - r_i - r_j)r_i - (c_i = x_i - \beta x_j)r_i - \frac{1}{2}x_i^2, \\ \text{S.T.} \quad & r_i + r_j < R. \end{aligned} \tag{4}$$

¹ Considering that cost different already represents efficiency difference, we ignore the transform efficiency difference θ_i , $i \in \{1, 2\}$ of Nie and Chen (2012).

² Many studies investigated the relationship between substitutability and innovation (e.g. Holmes *et al.* 2012; Nie and Chen 2012; Sacco and Schmutzer 2011; Aghion *et al.* 2005). Like D’Aspremont and Jacquemin (1988), we only consider the situation that $\gamma = 1$.

³ Firms invest cost reducing innovation at the first stage and then make output decision at the second stage.

⁴ See in Nie and Chen (2012).

To ensure that both firms' inputs and investments are positive, the following assumption is necessary.

Assumption: $\tau < c_0$ and $R \leq A$.

This assumption means that the cost difference between firms should not be too large or the high cost firm will be forced to exit the market and that input constraints should be less than the largest market size. If input constraint is tight, then $r_i + r_j = R$. From equation (1), we get $r_i + r_j < A$. Then, we know that $R \leq A$.

The two firms play a two-stage game. At the first stage, they decide their best R&D investments simultaneously and then decide their output simultaneously at the second stage. All equilibrium solutions are obtained by backward induction. In what follows, we analyze our model both under non-cooperative innovation and undercooperative innovation⁵.

2. Non-cooperative innovation competition

The Lagrangian function of firm i's profit maximization problem is:

$$L_i = (A - r_i - r_j)r_i - (c_i - x_i - \beta x_j)r_i - \frac{1}{2}x_i^2 + \lambda_i(R - r_i - r_j). \tag{5}$$

Case 1: Non-binding constraint. Non-binding constraint means $r_1 + r_2 < R$ and $\lambda_1 = \lambda_2 = 0$. We analyze the model by backward induction. Given innovation levels at the first stage, solving function (5) we get the equilibrium outputs in the second stage, given as:

$$r_1 = \frac{A - c_0 + \tau + (2 - \beta)x_1 + (2\beta - 1)x_2}{3}, \tag{6}$$

$$r_2 = \frac{A - c_0 - 2\tau + (2\beta - 1)x_1 + (2 - \beta)x_2}{3}. \tag{7}$$

Substituting (6) and (7) into (5), we obtain the optimal R&D investments at the first stage, given as:

$$x_1 = \frac{(4 - 2\beta)[(A - c_0)(1 - 6\beta + 2\beta^2) - (3 - 4\beta + 2\beta^2)\tau]}{(1 - 6\beta + 2\beta^2)(5 - 2\beta + 2\beta^2)}, \tag{8}^6$$

$$x_2 = \frac{(4 - 2\beta)[(A - c_0)(1 - 6\beta + 2\beta^2) + 2(1 + \beta)\tau]}{(1 - 6\beta + 2\beta^2)(5 - 2\beta + 2\beta^2)}. \tag{9}$$

From (8) and (9) we draw the following conclusions.

⁵ The second-mover firm will be forced to exit the market by the first-mover firm under input constraints Stackelberg competition if $\gamma = 1$. So we ignore leader-follower competition.

⁶ Denote ε as a small constant. Then there exists a small range $\left[\frac{3 - \sqrt{7}}{2} - \varepsilon, \frac{3 - \sqrt{7}}{2} + \varepsilon \right]$ makes $x_1 < 0$. That means if $\beta \in \left[\frac{3 - \sqrt{7}}{2} - \varepsilon, \frac{3 - \sqrt{7}}{2} + \varepsilon \right]$, then x_1 should be equal to 0 and $x_i = 0$ is beyond our study, so $\beta \notin \left[\frac{3 - \sqrt{7}}{2} - \varepsilon, \frac{3 - \sqrt{7}}{2} + \varepsilon \right]$.

Proposition 1. Denote $\bar{\beta}_1$ as a constant given in the appendix. (i) When $\beta < \bar{\beta}_1$, x_1 decreases while x_2 increases with τ . (ii). When $\beta > \bar{\beta}_1$, x_1 increases while x_2 decreases with τ^7 .

Proof: See the appendix.

Remarks. Proposition 1 shows that the spillover of innovation plays a major role in innovation competition. If spillover is small, the cost disadvantaged firm can mitigate its disadvantage by innovation. The larger the cost disadvantage, the stronger the innovation incentive for the disadvantaged firm. But if the spillover is larger than $\bar{\beta}$, the relationship between cost difference and innovation of the duopoly will reverse. Note that if $\tau = 0$, our conclusions are the same as D’Aspremont and Jacquemin (1988), which means their study is a special case of ours.

Combining (8) with (9), we obtain the investment difference Δx (or $|x_1 - x_2|$) and total investments Σx (or $|x_1 + x_2|$) easily, as given by:

$$\Delta x = \frac{(4 - 2\beta)\tau}{6\beta - 1 - 2\beta^2}, \tag{10}$$

$$\Sigma x = \frac{2(2 - \beta)(2A - \tau - 2c_0)}{5 - 2\beta + 2\beta^2}. \tag{11}$$

Then we achieve the following proposition.

Proposition 2. (i) $x_1 < x_2$ if $\beta < \bar{\beta}_1$ and $x_1 > x_2$ if $\beta > \bar{\beta}_1$. (ii) τ enlarges Δx ,

$$\left\{ \begin{array}{l} \frac{\partial \Delta x}{\partial \beta} > 0, \quad 0 \leq \beta < \frac{3 - \sqrt{7}}{2} \\ \frac{\partial \Delta x}{\partial \beta} < 0, \quad \frac{3 - \sqrt{7}}{2} < \beta \leq 1 \end{array} \right. \text{ and } \left\{ \begin{array}{l} \frac{\partial^2 \Delta x}{\partial \beta \partial \tau} > 0, \quad 0 \leq \beta < \frac{3 - \sqrt{7}}{2} \\ \frac{\partial^2 \Delta x}{\partial \beta \partial \tau} < 0, \quad \frac{3 - \sqrt{7}}{2} < \beta \leq 1 \end{array} \right. \cdot \text{(iii) } \frac{\partial \Sigma x}{\partial \tau} < 0, \quad \frac{\partial \Sigma x}{\partial \beta} < 0$$

and $\frac{\partial^2 \Sigma x}{\partial \tau \partial \beta} > 0$.

Proof: See the appendix.

Remarks. Proposition 2 shows that the high cost firm innovates more if spillover is small, while the low cost firm innovates more if spillover is large. In other words, small spillover stimulates the high cost firm’s innovation while high spillover stimulates the low cost firm’s innovation. But the increase of spillover inhibits total innovation which is consistent with reality. Externality is harmful for innovation, so our society needs patent protection to protect the profits of innovating firms. On the one hand, cost difference, which also is regarded as market power (Nie and Chen 2012), enlarges innovation difference. On the other hand, cost difference reduces total innovation. These results show that monopoly inhibits innovation. Part (ii) of Proposition 2

⁷ $\bar{\beta}_1 = \frac{3 - \sqrt{7}}{2}$ is the root of $1 - 6\beta + 2\beta^2 = 0$.

illustrates that cost difference boosts the relationship of spillover and innovation difference because $sign\left\{\frac{\partial \Delta x}{\partial \beta}\right\} = sign\left\{\frac{\partial^2 \Delta x}{\partial \beta \partial \tau}\right\}$ and part (iii) of Proposition 2 shows that cost difference attenuates the relationship of spillover and total innovation because $sign\left\{\frac{\partial \Sigma x}{\partial \beta}\right\} = -sign\left\{\frac{\partial^2 \Sigma x}{\partial \beta \partial \tau}\right\}$.

Substituting (8) and (9) back into (5) and solving obtains the final expressions of the equilibrium input of the two firms is given by:

$$r_1 = \frac{3(1 - 6\beta + 2\beta^2)(A - c_0) - 3(3 - 4\beta + 2\beta^2)\tau}{(1 - 6\beta + 2\beta^2)(5 - 2\beta + 2\beta^2)}, \tag{12}$$

$$r_2 = \frac{3(1 - 6\beta + 2\beta^2)(A - c_0) + 6(1 + \beta)\tau}{(1 - 6\beta + 2\beta^2)(5 - 2\beta + 2\beta^2)} \tag{13}$$

and the input difference is given by:

$$\Delta r = \left| \frac{3\tau}{6\beta - 1 - 2\beta^2} \right|. \tag{14}$$

Differentiating expressions (12)–(14) with respect to τ and β , we acquire the following conclusions.

Proposition 3. (i) $r_1 < r_2$ if $\beta < \bar{\beta}_1$ and $r_1 > r_2$ if $\beta > \bar{\beta}_1$. (ii) When $\beta < \bar{\beta}_1$, $\frac{\partial r_1}{\partial \tau} < 0$, $\frac{\partial r_2}{\partial \tau} > 0$ and $\frac{\partial \Delta r}{\partial \tau} < 0$. When $\beta > \bar{\beta}_1$, $\frac{\partial r_1}{\partial \tau} > 0$, $\frac{\partial r_2}{\partial \tau} < 0$ and $\frac{\partial \Delta r}{\partial \tau} > 0$. (iii) $\frac{\partial \Delta r}{\partial \beta} < 0$ and $\frac{\partial^2 \Delta r}{\partial \beta \partial \tau} < 0$.

Proof: See the appendix.

Remarks. It is a general conclusion that the cost advantaged firm produces more than its cost disadvantaged competitors. But interestingly, Proposition 3 shows that if innovation spillover is small, the disadvantaged incumbent can improve its status or even obtain an advantage by innovation. Proposition 3 also indicates that the relationship between cost difference and output depends on innovation spillover. High innovation spillover is beneficial to the high cost firm because spillover decreases input difference. More interestingly, the increase of cost difference reduces the effect of spillover on input difference.

Case 2. Tight constraint. Tight constraint means $r_1 + r_2 + R$ and $\lambda_1 > 0$, $\lambda_2 > 0$. To simplify the analysis, we further assume that $\lambda_1 = \lambda_2$, which means the shadow prices of inputs of the two firms are the same. From Equation (5), we have:

$$r_1^* = \frac{R + \tau + (1 - \beta)(x_1^* - x_2^*)}{2}, \tag{15}$$

$$r_2^* = \frac{R - \tau - (1 - \beta)(x_1^* - x_2^*)}{2}. \tag{16}$$

Comparing (15) and (16) with (6) and (7), we find that without innovation ($r_1^* - r_2^*$) is exactly equal to $(r_1 - r_2)$. Substituting r_1^* and r_2^* into (5), we have:

$$x_1^* = \frac{(\beta - 4\beta^2 + \beta^3)R + (A - c_0)(1 - 5\beta + 5\beta^2 - \beta^3) - (2 - 3\beta + 4\beta^2 - \beta^3)\tau}{(1 + \beta^2)(1 - 4\beta + \beta^2)}, \quad (17)^8$$

$$x_2^* = \frac{(\beta - 4\beta^2 + \beta^3)R + (A - c_0)(1 - 5\beta + 5\beta^2 - \beta^3) + (1 + 2\beta - \beta^2)\tau}{(1 + \beta^2)(1 - 4\beta + \beta^2)}. \quad (18)$$

So the innovation difference and total innovation are:

$$\Delta x^* = \left| \frac{(3 - \beta)\tau}{1 - 4\beta + \beta^2} \right|, \quad (19)$$

$$\Sigma x^* = \frac{2\beta R + 2(1 - \beta)(A - c_0 - \frac{1}{2}\tau)}{1 + \beta^2}. \quad (20)$$

Then the optional inputs and the difference of inputs are given by:

$$r_1^* = \frac{(1 - 4\beta + \beta^2)R - 2\tau}{2(1 - 4\beta + \beta^2)}, \quad (21)$$

$$r_2^* = \frac{(1 - 4\beta + \beta^2)R + 2\tau}{2(1 - 4\beta + \beta^2)}, \quad (22)$$

$$\Delta r^* = \left| \frac{2\tau}{1 - 4\beta + \beta^2} \right|. \quad (23)$$

From (17)–(23), we obtain the following conclusions.

Proposition 4. All propositions under non-binding constraint hold in tight constraint situation.

Proof: From the proofs of Propositions 1 to 3 we can get the conclusions of Proposition 4 by straightforward calculations⁹.

Remarks. Proposition 4 shows that all the conclusions above are robust. There also exists $\bar{\beta}_2$ such that $r_1^* > r_2^*$ when $\beta < \bar{\beta}_2$, and $r_1^* < r_2^*$ when $\beta > \bar{\beta}_2$ ¹⁰. But $\bar{\beta}_1 < \bar{\beta}_2$, which means comparing with non-tight constraints, in tight constraints circumstance the cost disadvantaged firm has more chance to plunder more input or market share than its competitor. In other words, the high cost firm benefits from input constraints. Different from innovation, inputs of the firms are only decided by R, τ and β . Maximum market size A and basic cost c_0 have no effect on input.

Proposition 5. $\frac{\partial x_1^*}{\partial R} > 0$, $\frac{\partial x_2^*}{\partial R} > 0$, $\frac{\partial \Sigma x^*}{\partial R} > 0$ and $\frac{\partial r_1^*}{\partial R} = \frac{\partial r_2^*}{\partial R} = \frac{1}{2}$.

⁸ Denote ε a small constant, then there exists a small range $[2 - \sqrt{3} - \varepsilon, 2 - \sqrt{3} + \varepsilon]$ in which $x_1 < 0$. Which means if $\beta \in [2 - \sqrt{3} - \varepsilon, 2 - \sqrt{3} + \varepsilon]$, then x_1 should be equal to 0. $x_i^* = 0$ is out of our study, so $\beta \notin [2 - \sqrt{3} - \varepsilon, 2 - \sqrt{3} + \varepsilon]$.

⁹ For more details of the proof, please see Table 1 in the appendix.

¹⁰ $\bar{\beta}_1 = \frac{3 - \sqrt{7}}{2} \approx 0.177$ and $\bar{\beta}_2 = 2 - \sqrt{3} \approx 0.268$, so we get $\bar{\beta}_1 < \bar{\beta}_2$ easily. $\bar{\beta}_2$ is the root of $1 - 4\beta + \beta^2 = 0$.

Proof: Differentiating (17)–(20) with respect to R, we obtain, $\frac{\partial x_1^*}{\partial R} = \frac{\partial x_2^*}{\partial R} = \frac{\beta}{(1 - 4\beta + \beta^2)} > 0$, $\frac{\partial \Sigma x^*}{\partial R} = \frac{2\beta}{1 + \beta^2} > 0$ and $\frac{\partial r_1^*}{\partial R} = \frac{\partial r_2^*}{\partial R} = \frac{1}{2}$.

Conclusions are therefore achieved and the proof is complete.

Remarks. Proposition 5 reveals that, as R increases, innovation of firms, total innovation and output of the industry increase. And both the increase rate of innovation and output of the firms are the same. More interestingly, innovation of the firm has nothing to do with R if spillover is equal to 0 or no innovation spillover. Furthermore, any incremental input will be divided by the two firms equally, which means increasing input resources of the constrained industry cannot improve the status of the cost disadvantaged producer or firm size difference has nothing to do with the increase of R.

Proposition 6. Total innovation, innovation difference and input difference under different

circumstances satisfy the following relationships: $\Sigma x^* > \Sigma x$, $\left\{ \begin{array}{l} \Delta x^* < \Delta x, \quad 0 \leq \beta < \frac{5 - \sqrt{7}}{4} \\ \Delta x^* > \Delta x, \quad \frac{5 - \sqrt{7}}{4} < \beta \leq 1 \end{array} \right.$

and $\left\{ \begin{array}{l} \Delta r^* < \Delta r, \quad 0 \leq \beta < \frac{12 - \sqrt{109}}{7} \\ \Delta r^* > \Delta r, \quad \frac{12 - \sqrt{109}}{7} < \beta \leq 1 \end{array} \right.$.

Proof: See the appendix.

Remarks. Esó *et al.* (2010) indicated that capacity constraints lead to industrial symmetry and our conclusion is the same as theirs if spillover β is equal to 0. But Proposition 6 implies that if duopoly competes with innovation and innovation has spillover, capacity constraints do not necessarily result in symmetry. Furthermore, capacity constraints even increase industrial asymmetry if innovation spillover is high.

3. Cooperative innovation

Prior studies have showed that cooperative innovation makes things different (D’Aspremont and Jacquemin 1988; Suzumura 1992; Erkal, Piccinin 2010). Next we analyze our model under cooperative innovation. Cooperative innovation means firms decide their optimal equilibrium innovations based on the joint profit maximization problem. Two firms make innovation decisions based on joint profit maximization at the first stage and then decide their outputs separately at the second stage. Here we also use backward induction to solve the problem. Denote the joint profit function as $\pi_1 + \pi_2 = f[r_1, r_2, x_1, x_2], i = (1, 2)$. $r_i(x_i, x_j)$ and x_i are the output and innovation of firm $i, i = (1, 2)$. Then optional equilibrium innovation is given by:

$$x_i = \arg \max (\pi_1 + \pi_2) f[r_1, r_2, x_1, x_2], i = (1, 2). \tag{24}$$

Case 1. Non-binding constraints. Solving (24), we get:

$$x_1^c = \frac{2[(1 - 3\beta - 2\beta^2 + 2\beta^3)(A - c_0) - (4 - 7\beta + 2\beta^2)\tau]}{7 - 32\beta + 28\beta^2 - 4\beta^4}, \tag{25}$$

$$x_2^c = \frac{2[(1 - 3\beta - 2\beta^2 + 2\beta^3)(A - c_0) + (3 - 4\beta + 2\beta^2)\tau]}{7 - 32\beta + 28\beta^2 - 4\beta^4}, \tag{26}$$

$$\Delta x^c = \left| \frac{2(1 - \beta)\tau}{1 - 4\beta + 2\beta^2} \right|, \tag{27}$$

$$\Sigma x^c = \frac{2(1 + \beta)(2A - 2c_0 - \tau)}{7 - 4\beta - 2\beta^2}. \tag{28}$$

In the model of D'Aspremont and Jacquemin (1988), x_1^c is equal to x_2^c and $\Delta x^c = 0$ because they did not consider cost difference τ . If $\tau = 0$, we also have $x_1^c = x_2^c$ and $\Delta x^c = 0$, and our conclusions are the same as D'Aspremont and Jacquemin (1988).

Substituting (25) and (26) into (6) and (7), we have:

$$r_1^c = \frac{3(1 - 4\beta + 2\beta^2)(A - c_0) + (5 - 8\beta + 2\beta^2)\tau}{7 - 32\beta + 28\beta^2 - 4\beta^4}, \tag{29}$$

$$r_2^c = \frac{3(1 - 4\beta + 2\beta^2)(A - c_0) - 2(1 + 2\beta - 2\beta^2)\tau}{7 - 32\beta + 28\beta^2 - 4\beta^4}, \tag{30}$$

$$\Delta r^c = \left| \frac{\tau}{1 - 4\beta + 2\beta^2} \right|. \tag{31}$$

Case 2. Tight constraints. Optimal equilibrium innovations are given by:

$$x_1^{c*} = \frac{(1 - 3\beta - 2\beta^2 + 2\beta^3)R - 2(1 - \beta)\tau}{2 - 8\beta + 4\beta^2}, \tag{32}$$

$$x_2^{c*} = \frac{(1 - 3\beta - 2\beta^2 + 2\beta^3)R + 2(1 - \beta)\tau}{2 - 8\beta + 4\beta^2}. \tag{33}$$

Then innovation difference and total innovation are:

$$\Delta x^{c*} = \left| \frac{2(1 - \beta)\tau}{1 - 4\beta + 2\beta^2} \right|, \tag{34}$$

$$\Sigma x^{c*} = (1 + \beta)R. \tag{35}$$

Combining (15), (16), (32) and (33), we obtain equilibrium inputs and input difference:

$$r_1^{c*} = \frac{(1 - 4\beta + 2\beta^2)R - \tau}{2(1 - 4\beta + 2\beta^2)}, \tag{36}$$

$$r_2^{c*} = \frac{(1 - 4\beta + 2\beta^2)R + \tau}{2(1 - 4\beta + 2\beta^2)}, \tag{37}$$

$$\Delta r^{c*} = \left| \frac{\tau}{1 - 4\beta + 2\beta^2} \right|. \tag{38}$$

All the conclusions under non-cooperative innovation are still valid under cooperative innovation. Moreover, comparing the results of cooperative innovation with those of non-cooperative innovation, we have the following conclusions.

Proposition 7. $\Delta x^c = \Delta x^{c*}$ and $\Delta r^c = \Delta r^{c*}$.

Proof: From equation (27), (31), (34) and (38), we get $\Delta x^{c*} = \left| \frac{2(1-\beta)\tau}{1-4\beta+2\beta^2} \right| = \Delta x^c$,
 $\Delta r^{c*} = \left| \frac{\tau}{1-4\beta+2\beta^2} \right| = \Delta r^c$.

Conclusions are therefore achieved and the proof is complete.

Remarks. Proposition 7 shows that if firms make cooperative innovation decisions, no matter input constraint is tight or not, the differences of innovation and input are the same.

Proposition 8. Innovation and input difference and total innovation of non-cooperative and cooperative meet the following relationship: $\Delta x^c < \Delta x$, $\Delta x^{c*} < \Delta x^*$, $\Delta r^c < \Delta r$,

$$\Delta r^{c*} < \Delta r, \begin{cases} \Sigma x^c \leq \Sigma x, & 0 < \beta < \frac{1}{2} \\ \Sigma x^c \geq \Sigma x, & \frac{1}{2} < \beta \leq 1 \end{cases} \text{ and } \begin{cases} \Sigma x^{c*} \leq \Sigma x^*, & 0 \leq \beta \leq \bar{\beta} \\ \Sigma x^{c*} \geq \Sigma x^*, & \bar{\beta} < \beta \leq 1 \end{cases}.$$

Proof: See in appendix.

Remarks. Cooperative innovation always reduces the difference of inputs (or firm size), but cooperative innovation increases total innovation if spillover is large. If spillover is small, comparing with non-cooperative innovation, cooperative innovation inhibits total innovation. The results above illustrate that the disadvantaged firm always benefits from cooperative innovation, but things for the advantaged firm and consumers are ambiguous. Our research partly explains the phenomenon that non-cooperative innovation and cooperative innovation coexist. Because spillovers of innovation are different, firms take different innovation strategies (cooperative innovation or non-cooperative innovation) under which they carry out different innovation. The breakpoint β in Proposition 8 is dependent on the relationship of R and $(A - c_0 - \frac{1}{2}\tau)$.

Discussion and conclusions

In this study, we investigate how innovation spillover affects input competition and how cost difference, spillover and capacity constraints affect innovation with input constraints under Cournot competition. Different from earlier study, our study shows that even when cooperative innovation strategies are adopted by firms, both input and innovation of different firms are not the same due to the existence of cost difference. Prior studies illustrated that capacity constraints lead to symmetric firms, which means capacity constraints reduce the high cost firm’s disadvantage. Our study shows that the high cost firm can even acquire competition advantage or plunder more input than the low cost firm by cost-reducing innovation under low spillover condition. But high innovation spillover enlarges the input difference between firms and high spillover inhibits innovation investment of high cost firm. Increasing total

input change nothings because any unit increase input will be divided equally by the two firms. Interestingly, the increase of constraints bound raises innovation.

The conclusion reached by early studies that cooperative innovation results in symmetric innovation is not a general conclusion if firms compete with asymmetric costs, but cooperative innovation reduces input difference and innovation difference. Although cooperative innovation reduces innovation and input difference, it increases total innovation under high innovation spillover and consumers do not always benefit from cooperative innovation. Furthermore, this study reveals that the high cost firm prefers low innovation spillover, while the low cost firm prefers high spillover.

This paper combines capacity constraints and innovation investment originality. In other words, we offer a valuable discussion about cost reducing innovation under capacity constraints, which expands the application of innovation theory. The major research implications are that disadvantage firms could take innovation spillover and capacity constraints as a competition strategy to obtain competition advantage and regulators should stimulate cooperative innovation to higher social welfare. Besides, people should pay more attention to constrained input capacity because it has significant impact on competition and industrial development.

Comparing with some of the earlier studies, the model of this study is more general. This study only considers quantity-sensitive capacity constraints, while quality-sensitive capacity constraints in a more interesting issue. Another important market factor, product substitution also has major effect on output and innovation competition, so further study will include product substitutability and quality-sensitive capacity constraints. If products offered by different firms are not perfect substitutes, our study can be extended to Stackelberg competition.

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APPENDIX

Table 1. The relationships between variables and parameters

	∂t		$\partial \beta$		$\partial \beta \partial \tau$			∂R	∂t		$\partial \beta$		$\partial \beta \partial \tau$	
	L^a	S^a	L^a	S^a	L^a	S^a			L^b	S^b	L^b	S^b	L^b	S^b
x_1	–	+	–	–	–	–	x_1^*	+	–	+			–	–
x_2	+	–			+	+	x_2^*	+	+	–			+	+
r_1	–	+			–	–	r_1^*	1/2	–	+	–	–	–	–
r_2	+	–			+	+	r_2^*	1/2	+	–	+	+	+	+
Δx	+	+	+	–	+	–	Δx^*	0	+	+	+	–	+	+
Σx	–	–	–	–	+	+	Σx^*	+	–	–			+	+
Δr	–	+	–	–	–	–	Δr^*	0	–	+	–	–	–	–

Notes: L^a and S^a means $\beta < \frac{3-\sqrt{7}}{2}$ and $\beta > \frac{3-\sqrt{7}}{2}$, while L^b and S^b means $\beta < 2-\sqrt{3}$ and $\beta > 2-\sqrt{3}$.

All the results are obtained by differentiated variables by parameters.

Proof of Proposition 1

From (8), we have $\frac{\partial x_1}{\partial \tau} = \begin{cases} -\frac{(4-2\beta)(3-4\beta+2\beta^2)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} < 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ -\frac{(4-2\beta)(3-4\beta+2\beta^2)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} > 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}$ and

from (9) we have $\frac{\partial x_2}{\partial \tau} = \begin{cases} \frac{4(2-\beta)(1+\beta)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} > 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ \frac{4(2-\beta)(1+\beta)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} < 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}$.

Conclusions are therefore achieved and the proof is complete.

Proof of Proposition 2

From (8) easy to get $(x_1 - x_2) = \begin{cases} \frac{(4-2\beta)\tau}{6\beta-1-2\beta^2}, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ \frac{(4-2\beta)\tau}{6\beta-1-2\beta^2}, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}$,

$\frac{\partial \Delta x}{\partial \beta} = \begin{cases} \frac{2(11-8\beta+2\beta^2)\tau}{(6\beta-1-2\beta^2)^2} > 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ -\frac{2(11-8\beta+2\beta^2)\tau}{(6\beta-1-2\beta^2)^2} < 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}$ and

$\frac{\partial^2 \Delta x}{\partial \beta \partial \tau} = \begin{cases} \frac{2(11-8\beta+2\beta^2)}{(6\beta-1-2\beta^2)^2} > 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ -\frac{2(11-8\beta+2\beta^2)}{(6\beta-1-2\beta^2)^2} < 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}$. From equation (9), we achieve

$\frac{\partial(x_1 + x_2)}{\partial \tau} = -\frac{2(2-\beta)}{5-2\beta+2\beta^2} < 0,$

$\frac{\partial(x_1 + x_2)}{\partial \beta} = -\frac{2(1+8\beta-2\beta^2)(2A-\tau-2c_0)}{(5-2\beta+2\beta^2)^2} < 0$ and $\frac{\partial^2(x_1 + x_2)}{\partial \beta \partial \tau} = \frac{2(1+8\beta-2\beta^2)}{(5-2\beta+2\beta^2)^2} > 0.$

Conclusions are therefore achieved and the proof is complete.

Proof of Proposition 3

Subtract (12) by (13), we have $(r_1 - r_2) = \begin{cases} \frac{3\tau}{6\beta-1-2\beta^2} < 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ \frac{3\tau}{6\beta-1-2\beta^2} > 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}$. From (12)

to (14), we get

$$\frac{\partial r_1}{\partial \tau} = \begin{cases} -\frac{3(3-4\beta+2\beta^2)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} < 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ -\frac{3(3-4\beta+2\beta^2)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} > 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases},$$

$$\frac{\partial^2 r_1}{\partial \beta \partial \tau} = -\frac{2(19-31\beta+44\beta^2-44\beta^3+20\beta^4-4\beta^5)}{(1-6\beta+2\beta^2)^2(5-2\beta+2\beta^2)^2} < 0,$$

$$\frac{\partial r_2}{\partial \tau} = \begin{cases} \frac{6(1+\beta)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} > 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ \frac{6(1+\beta)}{(1-6\beta+2\beta^2)(5-2\beta+2\beta^2)} < 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases},$$

$$\frac{\partial^2 r_2}{\partial \beta \partial \tau} = \frac{6(37-48\beta+24\beta^2+16\beta^3-12\beta^4)}{(1-6\beta+2\beta^2)^2(5-2\beta+2\beta^2)^2} > 0,$$

$$\frac{\partial \Delta r}{\partial \tau} = \begin{cases} \frac{3}{6\beta-1-2\beta^2} < 0, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ \frac{3}{6\beta-1-2\beta^2} > 0, & \frac{3-\sqrt{7}}{2} < \beta \leq 1 \end{cases}, \frac{\partial \Delta r}{\partial \beta} = -\frac{3(6-4\beta)\tau}{(6\beta-1-2\beta^2)^2} < 0 \text{ and}$$

$$\frac{\partial^2 \Delta r}{\partial \beta \partial \tau} = -\frac{3(6-4\beta)}{6\beta-1-2\beta^2} < 0.$$

Conclusions are therefore achieved and the proof is complete.

Proof of Proposition 6

It is easy to check that $\Sigma x^* - \Sigma x = \frac{(10\beta - 4\beta^2 + 4\beta^3)R + 2(1 - 5\beta)(A - c_0 - 0.5\tau)}{(1 + \beta^2)(5 - 2\beta + 2\beta^2)} > 0$

$$\text{and } (\Delta x^* - \Delta x) = \begin{cases} -\frac{(1+\beta)\tau}{(1-4\beta+\beta^2)(1-6\beta+2\beta^2)}, & 0 \leq \beta < \frac{3-\sqrt{7}}{2} \\ -\frac{(7-37\beta+24\beta^2-4\beta^3)\tau}{(1-4\beta+\beta^2)(1-6\beta+2\beta^2)}, & \frac{3-\sqrt{7}}{2} < \beta < 2-\sqrt{3}, \text{ so} \\ \frac{(1+\beta)\tau}{(1-4\beta+\beta^2)(1-6\beta+2\beta^2)}, & 2-\sqrt{3} < \beta \leq 1 \end{cases}$$

$$\begin{cases} \Delta x^* < \Delta x, & 0 \leq \beta < \frac{5-\sqrt{7}}{4} \\ \Delta x^* > \Delta x, & \frac{5-\sqrt{7}}{4} < \beta \leq 1 \end{cases}. \text{ For the same reason we achieved}$$

$$\begin{cases} \Delta r^* < \Delta r, & 0 \leq \beta < \frac{12 - \sqrt{109}}{7} \\ \Delta r^* > \Delta r, & \frac{12 - \sqrt{109}}{7} < \beta \leq 1 \end{cases}.$$

Conclusions are therefore achieved and the proof is complete.

Proof of Proposition 8

We have $\Delta x^c = \left| \frac{2(1-\beta)\tau}{1-4\beta+2\beta^2} \right|$ and $\Delta x = \left| \frac{2(2-\beta)\tau}{1-6\beta+2\beta^2} \right|$. Because $2(\beta-1)\tau < 2(2-\beta)\tau$ and $|1-4\beta+2\beta^2| > |1-6\beta+2\beta^2|$, so easy to get $\Delta x^c < \Delta x$. For the same reason, we have $\Delta x^{c*} < \Delta x^*$, $\Delta r^c < \Delta r$ and $\Delta r^{c*} < \Delta r^*$. Subtracted equation (28) by (11), we have $\Sigma x^c - \Sigma x = \frac{18(2\beta-1)(2A-2c_0-\tau)}{(5-2\beta+2\beta^2)(7-4\beta-2\beta^2)}$ and subtracted equation (35) by (20), we get $\Sigma x^{c*} - \Sigma x^* = \frac{(1-\beta+\beta^2+\beta^3)R-2(1-\beta)(A-c_0-0.5\tau)}{1+\beta^2}$. So there exist the following re-

lationships: $\begin{cases} \Sigma x^c \leq \Sigma x, & 0 < \beta < \frac{1}{2} \\ \Sigma x^c \geq \Sigma x, & \frac{1}{2} < \beta \leq 1 \end{cases}$ and $\begin{cases} \Sigma x^{c*} \leq \Sigma x^*, & 0 \leq \beta \leq \bar{\beta} \\ \Sigma x^{c*} \geq \Sigma x^*, & \bar{\beta} < \beta \leq 1 \end{cases}$.

Conclusions are therefore achieved and the proof is complete.

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