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Solitary Wave and Periodic Wave Solutions for a Class of Singular p-Laplacian Systems with Impulsive Effects

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Abstract. This work deals with the existence of periodic wave solutions and nonexistence of solitary wave solutions for a class of second-order singular p-Laplacian systems with impulsive effects. A sufficient criterion for the solutions of the considered system is provided via an innovative method of the mountain pass theorem and an approximation technique. Some corresponding results in the literature can be enriched and extended.

Keywords: periodic wave solution, solitary wave solution, singular p-Laplacian systems, impulsive effects, mountain pass theorem.

AMS Subject Classification: 34B16; 34B37; 34C25; 34C37.

1 Introduction

In this paper, we consider a class of second-order singular p-Laplacian systems with impulsive effects described by

$$\begin{cases} \dfrac{d}{dt}\big(|u'(t)|^{p-2}u'(t)\big) + f(u(t)) = e(t), \quad t \neq t_j,\ t \in \mathbb{R}, \\ -\Delta\big(|u'(t_j)|^{p-2}u'(t_j)\big) = g_j(u(t_j)), \quad j \in \mathbb{Z}, \end{cases} \tag{1.1}$$

where $p \geq 2$, $u = (u_1, u_2, ..., u_N)^\top \in \mathbb{R}^N$, $f \in C(\mathbb{R}^N, \mathbb{R}^N)$ and f may be singular at $u = 0$, $g_j(u) = \text{grad}_u G_j(u)$ for some $G_j \in C^1(\mathbb{R}^N, \mathbb{R})$, $e \in C(\mathbb{R}, \mathbb{R}^N)$, $\Delta(u(t)) = u(t_j^+) - u(t_j^-)$, $u(t_j^\pm) = \lim_{t \to t_j^\pm} u(t)$. It is assumed that there exist an $m \in \mathbb{N}$ and a $T > 0$ such that $0 = t_0 < t_1 < t_2 < \cdots < t_{m-1} < t_m = T$, $t_{j+m} = t_j + T$, $g_{j+m} = g_j$, $j \in \mathbb{Z}$.

During the past years, different types of impulsive differential equations have been studied by many authors. Some classical tools have been widely used to get the solutions of impulsive differential equations, such as fixed point theorems in cones, topological degree theory (including continuation method and coincidence degree theory), the method of lower and upper solutions, and the critical point theory. For the theory and classical results, we refer the readers to the references, [4], [17], [19], [26], [32] and books [2], [22], [31].

Recently, the study on the existence of homoclinic solutions for the impulsive differential equations has attracted many researchers' attention. See, to name a few, [8], [29], [32]. For example, in [32], by applying the variational methods, Zhang and Li established the existence result of homoclinic solutions of the following second order impulsive differential equations

$$\begin{cases} q'' + V_q(t, q) = f(t), & \text{for } t \in (s_{k-1}, s_k), \\ \Delta q'(s_k) = g_k(q(s_k)), \end{cases}$$

where $k \in \mathbb{Z}$, $q \in \mathbb{R}^N$, $\Delta q'(s_k) = q'(s_k^+) - q'(s_k^-)$ with $q'(s_k^\pm) = \lim_{t \to s_k^\pm} q'(t)$, $V_q(t, q) = \text{grad}_q V(t, q)$, $f \in C(\mathbb{R}, \mathbb{R}^N)$, $g_k(q) = \text{grad}_q G_k(q)$, $G_k \in C^1(\mathbb{R}^N, \mathbb{R}^N)$ for each $k \in \mathbb{N}$, and there exist an $m \in \mathbb{N}$ and a $T \in \mathbb{R}^+$ such that $0 = s_0 < s_1 < ... < s_m = T$, $s_{k+m} = s_k + T$ and $g_{k+m} \equiv g_k$ for all $k \in \mathbb{Z}$ (that is, g_k is m-periodic in k).

Singular equations appear in a lot of physical models, see [9], [15], [18], [21], [30] and the references therein. The existence periodic solutions and homoclinic solutions of different kinds of singular equations has been proposed by many authors, see [3], [5], [6], [7], [11], [13], [20], [27], [28], [33] and the references therein. Singular problems with impulsive effects have been scarcely studied, see [1], [12], [23], [24]. For example, in [12], the author and Luo considered the following first-order singular problems:

$$x'(t) + x^{-\alpha(t)} = e(t)$$

under impulsive conditions

$$\Delta(x(t_k)) = x(t_k^+) - x(t_k^-) = I_k(x(t_k)), \quad k = 1, 2, \ldots, q-1,$$

where $\alpha > 0$, $e : \mathbb{R} \to \mathbb{R}$ is continuous and T-periodic, $I_k : \mathbb{R} \to \mathbb{R}(k = 1, 2, ..., q-1)$ are continuous and $I_{k+q} \equiv I_k$. t_k, $k = 1, 2, ...q-1$, are the instants where the impulses occur and $0 = t_0 < t_1 < t_2 < t_3 < \cdots < t_{q-1} < t_q = T$, $t_{k+q} = t_k + T$. By applying the continuation theorem due to Mawhin and Gaines, the authors proved that the positive periodic solution was generated by impulses.

However, to the best of our knowledge, few researchers have studied the existence of periodic wave solutions and the nonexistence of solitary wave solutions for the singular p-Laplacian problems with impulsive effects. Inspired by the works mentioned above, in this paper, by means of the mountain pass theorem and an approximation technique, we establish the existence results of periodic wave solutions and nonexistence results of solitary wave solutions for system (1.1).

Definition 1. *Suppose that $u(s)$ is a solution of the system* (1.1) *for $s \in \mathbb{R}$, $u(s)$ is called a solitary wave solution if $\lim_{s\to-\infty} u(s) = \lim_{s\to+\infty} u(s)$. Usually, a solitary wave solution of system* (1.1) *corresponds to a homoclinic solution of system* (1.1)*. Similarly, a periodic wave solution of system* (1.1) *corresponds to a periodic solution of system* (1.1)*.*

Thus, in order to investigate the existence of periodic wave solutions and nonexistence of solitary wave solutions of system (1.1), we only need to prove the existence of periodic solutions and nonexistence of homoclinic solutions of system (1.1).

A function $u \in C(\mathbb{R}, \mathbb{R}^N)$ is a solution of system (1.1) if function u satisfies (1.1). A solution u of system (1.1) is homoclinic to 0 if $u(t) \to 0$ and $u'(t^{\pm}) \to 0$ as $t \to \pm\infty$, and the corresponding orbit is called a homoclinic orbit.

In general, it is very difficult (if not impossible) to construct a suitable functional such that the existence of its critical point implies to that of a homoclinic orbit of system (1.1). First of all, we assume that there exists homoclinic orbits of system (1.1). Then, by applying an approximation technique, we show that the existence of homoclinic orbit is obtained as a limit of $2kT$-periodic solutions of the following sequence of impulsive differential equations

$$\begin{cases} \dfrac{d}{dt}\big(|u'(t)|^{p-2}u'(t)\big) + f(u(t)) = e_k(t), \quad t \neq t_j,\ t \in \mathbb{R}, \\ -\Delta\big(|u'(t_j)|^{p-2}u'(t_j)\big) = g_j(u(t_j)), \quad j \in \mathbb{Z}, \end{cases} \tag{1.2}$$

where $e_k : \mathbb{R} \to \mathbb{R}^N$ is a $2kT$ periodic extension of the restriction of e to the interval $[-kT, kT]$.

Remark 1. Note that the domain under consideration is unbounded, and thus, there is a lack of compactness for the Sobolev embedding. To overcome this difficulty, we show the existence of a homoclinic orbit of system (1.1) by proving that (1.2) has $2kT$ periodic solutions whose limit gives a homoclinic orbit of system (1.1). Another difficulty is that we have to deal with the impulsive perturbations in system (1.2).

For the sake of convenience, we list the following assumptions:

- [H1] There exist constants $a > 0$ and $\gamma \in (1, p]$ such that for all $u \in \mathbb{R}^N$,

$$-F(u) \geq a|u|^{\gamma},$$

where $F \in C^1(\mathbb{R}^N, \mathbb{R})$ and $f(u) = \mathrm{grad}_u F(u)$.

- [H2] There exists a constant $\theta > p$ such that

$$-F(u) \leq -f(u)u \leq -\theta F(u), \text{ for all } u \in \mathbb{R}^N.$$

- [H3] $G_j(0) = 0$, $g_j(u) = o(|u|^{p-1})$ as $|u| \to 0$, $j = 1, 2, ..., m$.

- [H4] There are two constants μ, β with $\mu > \theta$ and $0 < \beta < \mu - \theta$ such that

$$0 < \mu G_j(u) \leq g_j(u)u + \beta a|u|^\gamma, \quad u \in \mathbb{R}^N \backslash \{0\}, \ \ j = 1, 2, ..., m.$$

- [H5] $e \in C(\mathbb{R}, \mathbb{R}^N) \cap L^p(\mathbb{R}, \mathbb{R}^N) \cap L^q(\mathbb{R}, \mathbb{R}^N)$ with

$$\Big(\int_{-\infty}^{+\infty} |e(t)|^q dt\Big)^{1/q} < \frac{1}{C^{p-1}} \min\Big\{\frac{\delta^{p-1}}{p}, \Big[a\Big(1 - \frac{\beta}{\mu - \gamma}\Big)\delta^{\gamma-1} - M\delta^{\mu-1}\Big]\Big\},$$

where $M = \sup\big\{G_j(u) : j = 1, 2, ..., m, |u| = 1\big\}$, $1/p + 1/q = 1$, $\delta \in (0, 1]$ such that

$$a\Big(1 - \frac{\beta}{\mu - \gamma}\Big)\delta^{\gamma-1} - M\delta^{\mu-1} = \max_{x\in[0,1]} \Big[a\Big(1 - \frac{\beta}{\mu - \gamma}\Big)x^{\gamma-1} - Mx^{\mu-1}\Big]$$

and $C > 0$ is a constant.

- [H6] $M_1 > \beta a/(\mu - \gamma)$, where $M_1 = \inf\big\{G_j(u) : j = 1, 2, ..., m, |u| = 1\big\}$.

The remainder part of this paper is organized as follows. Section 2 is devoted to state some necessary definitions, lemmas and the variational structure. Section 3 is devoted to state the main results and an example is given to support the established results. Section 4 is devoted to prove the main results.

2 Preliminary

Throughout this paper, we adopt the convention that $|u| = \sqrt{\sum_{j=1}^N u_j^2}$ and $uv = \sum_{j=1}^N u_j v_j$ for $u = (u_1, ..., u_N)^\top \in \mathbb{R}^N$ and $v = (v_1, ..., v_N)^\top \in \mathbb{R}^N$.

Define the space

$$H_{2kT} = \big\{u : \mathbb{R} \to \mathbb{R}^N \,|\, u, u' \in L^p([-kT, kT], \mathbb{R}^N),\ u(t) = u(t + 2kT),\ t \in \mathbb{R}\big\}.$$

Then H_{2kT} is a separable and reflexive Banach space with the norm defined by

$$\|u\|_{H_{2kT}} = \Big(\int_{-kT}^{kT} |u'(t)|^p dt + \int_{-kT}^{kT} |u(t)|^p dt\Big)^{1/p}.$$

Denote $L^\infty_{2kT}(\mathbb{R}, \mathbb{R})$by the space of $2kT$ periodic essentially bounded measurable functions from $\mathbb{R}$ into $\mathbb{R}$ with norm given by

$$\|u\|_{L^\infty_{2kT}} = ess \sup_{t\in[-kT,kT]} |u(t)|.$$

Let $\Omega_k = \{-km+1, -km+2, ..., 0, 1, 2, ..., km-1, km\}$ and define a functional φ_k as

$$\varphi_k(u) := \frac{1}{p}\eta_k^p(u) + \int_{-kT}^{kT} e_k u dt - \sum_{j\in\Omega_k} G_j(u(t_j)), \quad u \in H_{2kT}, \tag{2.1}$$

where

$$\eta_k(u) := \left(\int_{-kT}^{kT} |u'(t)|^p dt - \int_{-kT}^{kT} pF(u(t))dt \right)^{1/p}. \tag{2.2}$$

Then φ_k is Fréchet differentiable at any $u \in H_{2kT}$. For any $v \in H_{2kT}$, by a simple calculation, we have

$$\begin{aligned}\varphi_k'(u)v = &\int_{-kT}^{kT} |u'(t)|^{p-2}u'(t)v'(t)dt - \int_{-kT}^{kT} f(u(t))v(t)dt \\ &+ \int_{-kT}^{kT} e_k(t)v(t)dt - \sum_{j\in\Omega_k} g_j(u(t_j))v(t_j).\end{aligned}$$

Thus, by [H2], we get

$$\begin{aligned}\varphi_k'(u)u \le &\int_{-kT}^{kT} |u'(t)|^p dt - \int_{-kT}^{kT} \theta F(u(t))dt \\ &+ \int_{-kT}^{kT} e_k(t)u(t)dt - \sum_{j\in\Omega_k} g_j(u(t_j))u(t_j).\end{aligned} \tag{2.3}$$

It is evident that critical points of the functional φ_k are classical $2kT$ periodic solutions of system (1.2).

Lemma 1. *[10] There is a positive constant C such that for each $k \in \mathbb{N}$ and $u \in H_{2kT}$ the following inequality holds:*

$$\|u\|_{L^\infty_{2kT}} \le C\, \|u\|_{H_{2kT}}.$$

Lemma 2. *[14] There exists $r_p > 0$, for any $x, y \in \mathbb{R}^N$ such that*

$$(|x|^{p-2}x - |y|^{p-2}y)(x-y) \ge r_p|x-y|^p, \quad p \ge 2.$$

Lemma 3. *Suppose $u : \mathbb{R} \to \mathbb{R}^N$ is a continuous mapping such that*

$$\begin{aligned}&u' \in L^p_{loc}(\mathbb{R}, \mathbb{R}^N) \\ &\quad := \Big\{u : \mathbb{R} \to \mathbb{R} | \text{for any finite interval } [a,b], u|_{[a,b]} \in L^p([a,b], \mathbb{R}^N)\Big\}.\end{aligned}$$

Then for $a, b \ge 0$ with $a + b > 0$, the following inequality holds:

$$|u(t)| \le 2^{\frac{p-1}{p}} (a+b)^{-\frac{1}{p}} \max\{1, (a+b)\} \left(\int_{t-a}^{t+b} (|u'(s)|^p + |u(s)|^p)ds \right)^{1/p}.$$

In particular,

$$|u(t)| \leq 2^{\frac{p-2}{p}} a^{-\frac{1}{p}} \max\{1, 2a\} \Big(\int_{t-a}^{t+a} (|u'(s)|^p + |u(s)|^p) ds \Big)^{1/p},$$

$$|u(t)| \leq 2^{\frac{2p-1}{p}} \Big(\int_{t-1}^{t+1} (|u'(s)|^p + |u(s)|^p) ds \Big)^{1/p}.$$

Proof. Fix $t \in \mathbb{R}$, for any given $\delta \in \mathbb{R}$, we can have

$$|u(t)| \leq |u(\delta)| + \Big| \int_{\delta}^{t} u'(s) ds \Big|. \tag{2.4}$$

Integrating (2.4) on the interval $[t-a, t+b]$ with respect to δ, then it follows from the Jensen and Hölder inequalities that

$$\begin{aligned}
(a+b)|u(t)| &\leq \int_{t-a}^{t+b} \Big(|u(\delta)| + \Big| \int_{\delta}^{t} u'(s) ds \Big| \Big) d\delta \\
&\leq (a+b)^{\frac{p-1}{p}} \Big(\int_{t-a}^{t+b} \Big(|u(\delta)| + \Big| \int_{\delta}^{t} u'(s) ds \Big| \Big)^p d\delta \Big)^{\frac{1}{p}} \\
&\leq (2a+2b)^{\frac{p-1}{p}} \Big(\int_{t-a}^{t+b} \Big(|u(\delta)|^p + \Big| \int_{\delta}^{t} u'(s) ds \Big|^p \Big) d\delta \Big)^{\frac{1}{p}} \\
&\leq (2a+2b)^{\frac{p-1}{p}} \Big(\int_{t-a}^{t+b} |u(\delta)|^p d\delta + (a+b)^p \int_{t-a}^{t+b} |u'(s)|^p ds \Big)^{\frac{1}{p}} \\
&\leq (2a+2b)^{\frac{p-1}{p}} \max\{1, (a+b)\} \Big(\int_{t-a}^{t+b} |u(\delta)|^p d\delta + \int_{t-a}^{t+b} |u'(s)|^p ds \Big)^{\frac{1}{p}},
\end{aligned}$$

then, we obtain

$$\begin{aligned}
|u(t)| \leq & 2^{\frac{p-1}{p}} (a+b)^{-\frac{1}{p}} \cdot \max\{1, (a+b)\} \\
& \times \Big(\int_{t-a}^{t+b} |u(\delta)|^p d\delta + \int_{t-a}^{t+b} |u'(s)|^p ds \Big)^{\frac{1}{p}}.
\end{aligned}$$

In particular, if $a = b$ and $a = b = 1$, respectively, we have

$$|u(t)| \leq 2^{\frac{p-2}{p}} a^{-\frac{1}{p}} \max\{1, 2a\} \Big(\int_{t-a}^{t+a} (|u'(s)|^p + |u(s)|^p) ds \Big)^{1/p},$$

$$|u(t)| \leq 2^{\frac{2p-1}{p}} \Big(\int_{t-1}^{t+1} (|u'(s)|^p + |u(s)|^p) ds \Big)^{1/p}.$$

Therefore, the proof is completed. □

3 Main results

Theorem 1. *Assume that [H1]–[H6] hold, then the system* (1.1) *possesses at least one* $2kT$*-periodic wave solution.*

Theorem 2. *Assume that [H1]–[H6] hold, then the system* (1.1) *possesses no solitary wave solution.*

We conclude this section considering the following example.

Example 1. Consider system (1.1) with $p = 4$, $t_j = \frac{2j\pi}{m}$, $j \in \mathbb{Z}$, $T = 2\pi$, $m = 5$, $F(u) = -\frac{1}{\gamma}u^{\gamma} + \frac{1}{1-\gamma}u^{1-\gamma}$, $e(t) = \sin t/4$, $G_j(u(t_j)) = (1 + |\sin(t_j/2)|)|u(t_j)|^6$. Then, it is easy to verify that F, f, G_j, g_j, e satisfy the assumptions of Theorem 1-2 with $a = 1/4$, $\gamma = 4$, $\mu = 6$, $\theta = 5$ and $\beta = 1/2$. Therefore, system (1.1) possesses at least one periodic wave solution and no solitary wave solution, which are induced by impulses.

4 Proofs of main results

Now, we give the proof of Theorem 1 by using the mountain-pass theorem [16].

4.1 Proof of Theorem 1

Proof. For any given sequence $\{u_n\} \in H_{2kT}$ such that $\{\varphi_k(u_n)\}$ is bounded and $\lim_{n\to\infty} \varphi'_k(u_n) = 0$, there exists a constant $C_1 > 0$ such that

$$|\varphi_k(u_n)| \le C_1, \ \ \|\varphi'_k(u_n)\|_{H^*_{2kT}} \le C_1, \ \forall n \in \mathbb{N},$$

where H^*_{2kT} is the dual space of H_{2kT}. The rest of the proof is divided into three steps.

Step 1. We show that $\{u_n\}$ is bounded. In fact, by (2.1) and [H4]

$$\begin{aligned}\eta_k^p(u_n) &= p\varphi_k(u_n) + p\sum_{j\in\Omega_k} G_j(u_n(t_j)) - p\int_{-kT}^{kT} e_k(t)u_n(t)dt \le p\varphi_k(u_n) \\ &+ \frac{p}{\mu}\sum_{j\in\Omega_k}\left[g_j(u_n(t_j))u_n(t_j) + \beta a|u_n(t_j)|^{\gamma}\right] - p\int_{-kT}^{kT} e_k(t)u_n(t)dt. \quad (4.1)\end{aligned}$$

By (2.3) and $\mu > \theta > p$, we have

$$\begin{aligned}\frac{p}{\mu}\varphi'_k(u_n)u_n &\le \int_{-kT}^{kT}\left[\frac{p}{\mu}|u'_n(t)|^p - \frac{p}{\mu}\theta F(u_n(t)) + \frac{p}{\mu}e_k(t)u_n(t)\right]dt \\ &- \frac{p}{\mu}\sum_{j\in\Omega_k} g_j(u_n(t_j))u_n(t_j) \le \int_{-kT}^{kT}\left[\frac{\theta}{\mu}|u'_n(t)|^p - \frac{\theta}{\mu}pF(u_n(t)) \right. \\ &\left. + \frac{p}{\mu}e_k(t)u_n(t)\right]dt - \frac{p}{\mu}\sum_{j\in\Omega_k} g_j(u_n(t_j))u_n(t_j) \\ &= \frac{\theta}{\mu}\eta_k^p(u_n) + \frac{p}{\mu}\int_{-kT}^{kT} e_k(t)u_n(t)dt - \frac{p}{\mu}\sum_{j\in\Omega_k} g_j(u_n(t_j))u_n(t_j). \quad (4.2)\end{aligned}$$

From (4.1) and (4.2), we can obtain

$$\left(1 - \frac{\theta}{\mu}\right)\eta_k^p(u_n) \le p\varphi_k(u) - \frac{p}{\mu}\varphi'_k(u_n)u_n - \left(p - \frac{p}{\mu}\right)\int_{-kT}^{kT} e_k(t)u_n(t)dt$$

$$+\frac{p\beta a}{\mu}\sum_{j\in\Omega_k}|u_n(t_j)|^\gamma \le pC_1+\frac{p\beta a}{\mu}\int_{-kT}^{kT}|u_n(t)|^\gamma dt$$
$$+\left[\frac{pC_1}{\mu}+\left(p-\frac{p}{\mu}\right)\left(\int_{-kT}^{kT}|e_k(t)|^q dt\right)^{1/q}\right]\|u_n\|_{H_{2kT}}, \tag{4.3}$$

where $q>1$ satisfying $1/p+1/q=1$.

On the other hand, from [H1], (2.2) and Lemma 1, we have

$$\left(1-\frac{\theta}{\mu}\right)\eta_k^p(u)=\left(1-\frac{\theta}{\mu}\right)\int_{-kT}^{kT}\Big(|u'(t)|^p-pF(u(t))\Big)dt$$
$$+\frac{p\beta a}{\mu}\int_{-kT}^{kT}|u_n(t)|^\gamma dt-\frac{p\beta a}{\mu}\int_{-kT}^{kT}|u_n(t)|^\gamma dt$$
$$\ge\left(1-\frac{\theta}{\mu}\right)\int_{-kT}^{kT}|u'(t)|^p dt+pa\left(1-\frac{\theta}{\mu}-\frac{\beta}{\mu}\right)\int_{-kT}^{kT}|u_n(t)|^\gamma dt$$
$$+\frac{p\beta a}{\mu}\int_{-kT}^{kT}|u_n(t)|^\gamma dt\ge\min\Big\{\left(1-\frac{\theta}{\mu}\right)\|u_n\|^p_{H_{2kT}},$$
$$paC^{\gamma-p}\left(1-\frac{\theta}{\mu}-\frac{\beta}{\mu}\right)\|u_n\|^\gamma_{H_{2kT}}\Big\}+\frac{p\beta a}{\mu}\int_{-kT}^{kT}|u_n(t)|^\gamma dt. \tag{4.4}$$

It follows from (4.3) and (4.4) that

$$\min\Big\{\left(1-\frac{\theta}{\mu}\right)\|u_n\|^p_{H_{2kT}},paC^{\gamma-p}\left(1-\frac{\theta}{\mu}-\frac{\beta}{\mu}\right)\|u_n\|^\gamma_{H_{2kT}}\Big\}$$
$$\le pC_1+\left[\frac{pC_1}{\mu}+\left(p-\frac{p}{\mu}\right)\left(\int_{-kT}^{kT}|e_k(t)|^q dt\right)^{1/q}\right]\|u_n\|_{H_{2kT}}.$$

Since $p\ge\gamma>1$ and $0\le\beta<\mu-\theta$, then we can see that $\|u_n\|_{H_{2kT}}$ is bounded.

Because H_{2kT} is a reflexive Banach space, we can pick $\{u_n\}$ be a weakly convergent sequence to u in H_{2kT}, and $\{u_n\}$ converges uniformly to u in $C[-kT,kT]$. So, we have

$$u_n(t)-u(t)\to 0 \text{ as } n\to\infty,\ t\in[-kT,kT],\quad \sum_{j\in\Omega_k}[g_j(u_n(t_j))-g_j(u(t_j))]$$
$$\times[u_n(t_j)-u(t_j)]\to 0,\quad \int_{-kT}^{kT}[f(u_n(t))-f(u(t))][u_n(t)-u(t)]dt\to 0. \tag{4.5}$$

Then

$$\int_{-kT}^{kT}[|u_n'(t)|^{p-2}u_n'(t)-|u'(t)|^{p-2}u'(t)]\big[u_n'(t)-u'(t)\big]dt=\big[\varphi_k'(u_n(t))-\varphi_k'(u(t))\big]$$
$$\times\big[u_n(t)-u(t)\big]+\int_{-kT}^{kT}\big[f(u_n(t))-f(u(t))\big]\big[u_n(t)-u(t)\big]dt$$
$$+\sum_{j\in\Omega_k}\big[g_j(u_n(t_j))-g_j(u(t_j))\big]\big[u_n(t_j)-u(t_j)\big]. \tag{4.6}$$

Thus, it follows from (4.5), (4.6) and Lemma 2 that $\|u_n - u\|_{H_{2kT}} \to 0$. Therefore, the functional φ_k satisfies the Palais-Smale condition.

Step 2. Define

$$\varphi(s) = s^{\mu} G_j\left(\frac{u}{s}\right) - \frac{\beta a s^{\mu-\gamma}}{\mu-\gamma}|u|^{\gamma}, \quad j = 1,2,...,m, \ \ s > 0.$$

From [H4], we can have

$$\begin{aligned}\varphi'(s) =& \frac{d}{ds}\left[s^{\mu} G_j\left(\frac{u}{s}\right) - \frac{\beta a s^{\mu-\gamma}}{\mu-\gamma}|u|^{\gamma}\right] \\ =& s^{\mu-1}\left[\mu G_j\left(\frac{u}{s}\right) - g_j\left(\frac{u}{s}\right)\frac{u}{s} - \beta a\left(\frac{|u|}{s}\right)^{\gamma}\right] \le 0,\end{aligned}$$

which implies that $\varphi(s)$ is non-increasing for $s > 0$. Thus,

$$\begin{aligned}G_j(u(t_j)) - \frac{\beta a}{\mu-\gamma}|u(t_j)|^{\gamma} &\le G_j\left(\frac{u(t_j)}{|u(t_j)|}\right)|u(t_j)|^{\mu} - \frac{\beta a}{\mu-\gamma}|u(t_j)|^{\mu} \\ &\le \left(M - \frac{\beta a}{\mu-\gamma}\right)|u(t_j)|^{\mu} \le M|u(t_j)|^{\mu},\end{aligned} \tag{4.7}$$

for $0 < |u(t_j)| \le 1$, $j \in \Omega_k$, and

$$\begin{aligned}G_j(u(t_j)) - \frac{\beta a}{\mu-\gamma}|u(t_j)|^{\gamma} &\ge G_j\left(\frac{u(t_j)}{|u(t_j)|}\right)|u(t_j)|^{\mu} - \frac{\beta a}{\mu-\gamma}|u(t_j)|^{\mu} \\ &\ge \left(M_1 - \frac{\beta a}{\mu-\gamma}\right)|u(t_j)|^{\mu},\end{aligned} \tag{4.8}$$

for $|u(t_j)| \ge 1$, $j \in \Omega_k$. Note that [H1] implies that $p \ge \gamma > 0$, thus, we have

$$\int_{-kT}^{kT} |u(t)|^p dt \le \|u\|^{p-\gamma}_{L^{\infty}_{2kT}} \int_{-kT}^{kT} |u(t)|^{\gamma} dt.$$

If $\|u\|_{L^{\infty}_{2kT}} \le \delta \le 1$, then from (2.1), [H1], [H2], (4.6) and (4.7), we have

$$\begin{aligned}\varphi_k(u) =& \frac{1}{p}\eta_k^p(u) + \int_{-kT}^{kT} e_k(t)u(t)dt - \sum_{j\in\Omega_k} G_j(u(t_j)) = \frac{1}{p}\left[\int_{-kT}^{kT}|u'(t)|^p dt\right. \\ &\left. - \int_{-kT}^{kT} pF(u(t))dt\right] + \int_{-kT}^{kT} e_k(t)u(t)dt - \sum_{j\in\Omega_k} G_j(u(t_j)) \\ \ge& \frac{1}{p}\int_{-kT}^{kT}|u'(t)|^p dt + a\int_{-kT}^{kT}|u(t)|^{\gamma}dt - \sum_{j\in\Omega_k}\left(M|u(t_j)|^{\mu} + \frac{\beta a}{\mu-\gamma}|u(t_j)|^{\gamma}\right) \\ &- \left(\int_{-kT}^{kT}|e_k(t)|^q dt\right)^{1/q}\left(\int_{-kT}^{kT}|u(t)|^p dt\right)^{1/p} \\ \ge& \frac{1}{p}\int_{-kT}^{kT}|u'(t)|^p d + a\int_{-kT}^{kT}|u(t)|^{\gamma}dt - \int_{-kT}^{kT}\left(M|u(t_j)|^{\mu} + \frac{\beta a}{\mu-\gamma}|u(t_j)|^{\gamma}\right)dt \\ &- \left(\int_{-kT}^{kT}|e_k(t)|^q dt\right)^{1/q}\left(\int_{-kT}^{kT}|u(t)|^p dt\right)^{1/p}.\end{aligned}$$

Further, we have

$$\varphi_k(u) \geq \frac{1}{p}\int_{-kT}^{kT}|u'(t)|^p dt + \left[a\Big(1-\frac{\beta}{\mu-\gamma}\Big) - M\,\|u\|_{L^\infty_{2kT}}^{\mu-\gamma}\right]\int_{-kT}^{kT}|u(t)|^\gamma dt$$
$$-\left(\int_{\mathbb{R}}|e_k(t)|^q dt\right)^{1/q}\left(\int_{-kT}^{kT}|u(t)|^p dt\right)^{1/p} \geq \frac{1}{p}\int_{-kT}^{kT}|u'(t)|^p dt$$
$$+\left[a\Big(1-\frac{\beta}{\mu-\gamma}\Big)\|u\|_{L^\infty_{2kT}}^{\gamma-p} - M\,\|u\|_{L^\infty_{2kT}}^{\mu-p}\right]\int_{-kT}^{kT}|u(t)|^p dt$$
$$-\left(\int_{\mathbb{R}}|e_k(t)|^q dt\right)^{1/q}\left(\int_{-kT}^{kT}|u(t)|^p dt\right)^{1/p} \geq \min\left\{\frac{1}{p}, a\Big(1-\frac{\beta}{\mu-\gamma}\Big)\|u\|_{L^\infty_{2kT}}^{\gamma-p}\right.$$
$$\left.-M\,\|u\|_{L^\infty_{2kT}}^{\mu-p}\right\}\|u\|_{H_{2kT}}^p - \left(\int_{\mathbb{R}}|e_k(t)|^q dt\right)^{1/q}\|u\|_{H_{2kT}}.$$

Set $\rho = \delta/C$, and

$$\alpha = \frac{1}{C^p}\min\left\{\frac{\delta^p}{p}, a\Big(1-\frac{\beta}{\mu-\gamma}\Big)\delta^\gamma - M\delta^\mu\right\} - \left(\int_{\mathbb{R}}|e_k(t)|^q dt\right)^{1/q}\frac{\delta}{C} > 0,$$

where C is defined in Lemma 1. Let $\|u\|_{H_{2kT}} = \rho$, then $\|u\|_{L^\infty_{2kT}} \leq \delta \leq 1$. Therefore, $\varphi_k(u) \geq \alpha > 0$.

Step 3. We choose $\zeta \in \mathbb{R}$, $\omega = \frac{\pi}{T}$, $Q(t) = (\sin(\omega t), 0, ..., 0) \in H_{2T}\backslash\{0\}$. Then, we can see that $Q(\pm T) = 0$. Let

$$m_1 = \min\{F(u) : |u(t)| \leq 1, t \in [0,T]\},\ m_2 = \min\{F(u) : |u(t)| = 1, t \in [0,T]\},$$

then $0 > m_2 \geq m_1 > -\infty$.

Let $h(s) = s^{-\theta}F(su)$, $s > 0$. It follows from [H2] that

$$h'(s) = \big(f(su)su - \theta F(su)\big)/s^{\theta+1} \geq 0.$$

Then, we can get

$$F(u) \geq |u|^\theta F(u/|u|), \quad |u| \geq 1, \tag{4.9}$$

which implies that

$$F(u) \geq m_2|u|^\theta + m_1, \quad u \in \mathbb{R}.$$

Set $|\zeta u(t_j)| \geq 1$, $j \in \Omega_k$. From (4.8), we can have

$$G_j(\zeta u(t_j)) - \frac{\beta a}{\mu-\gamma}|\zeta u(t_j)|^\gamma \geq \Big(M_1 - \frac{\beta a}{\mu-\gamma}\Big)|\zeta u(t_j)|^\mu,\ t_j \in \Omega_k.$$

Define

$$\widetilde{Q}(t) = \begin{cases} Q(t), & t \in [-T,T], \\ 0, & t \in [-kT,kT]\backslash[-T,T]. \end{cases} \tag{4.10}$$

It follows from (2.1), (2.2) and (4.9)–(4.10) that

$$\varphi_k(\zeta\widetilde{Q}) = \frac{1}{p}\int_{-T}^{T}|\zeta Q'(t)|^p dt - \int_{-T}^{T} F(\zeta Q(t))dt - \sum_{j\in\Omega_1} G_j(\zeta Q(t_j))$$
$$+ \int_{-T}^{T} e_1(t)\zeta Q(t)dt \le \frac{T|\zeta|^p\omega^p}{p} - m_2|\zeta|^\theta \int_{-T}^{T}|Q(t)|^\theta dt$$
$$- |\zeta|^\gamma \sum_{j\in\Omega_1} \frac{\beta a}{\mu-\gamma}|Q(t_j)|^\gamma - |\zeta|^\mu \sum_{j\in\Omega_1} \Big(M_1 - \frac{\beta a}{\mu-\gamma}\Big)|Q(t_j)|^\mu$$
$$+ |\zeta|\Big(\int_{\mathbb{R}}|e_1(t)|^q dt\Big)^{1/q}\Big(\int_{-T}^{T}|Q(t)|^p\Big)^{1/p} - 2m_1T.$$

Clearly,

$$\varphi_k(\zeta\widetilde{Q}) \to -\infty \ \text{ as } |\zeta| \to +\infty.$$

Consequently, φ_k possesses a critical value $c_k \ge \alpha > 0$. Let u_k denote the corresponding critical point of φ_k on H_{2kT}, that is,

$$\varphi_k(u_k) = c_k, \ \varphi_k'(u_k) = 0. \tag{4.11}$$

Hence, system (1.2) possesses a $2kT$-periodic solution u_k. Therefore, the system (1.1) possesses at least one $2kT$-periodic wave solution. □

Theorem 3. *Let $\{u_k\}$ be the sequence defined in (4.11). Then there exist a subsequence $\{u_{k,k}\}$ of $\{u_k\}$ and a function $u_0 \in W^{1,p}_{loc} \cap L^\infty_{loc}(\mathbb{R},\mathbb{R}^N)$ such that $\{u_{k,k}\}$ converges to u_0 weakly in $W^{1,p}_{loc}$ and strongly in $L^\infty_{loc}(\mathbb{R},\mathbb{R}^N)$.*

Proof. We claim that there is a constant $M_3 > 0$ independent of k such that $\|u_k\|_{H_{2kT}} \le M_3$. Let $e_1 \in H_{2T}\setminus\{0\}$ such that $e_1(\pm T) = 0$, $e_1(t_k) \ne 0$ for some $t_k \in (-T,T)$ and $\varphi_1(e_1) \le 0$. Define

$$e_k(t) = \begin{cases} e_1(t), & |t| < T,\\ 0, & T \le |t| \le kT, k = 2,3,\dots. \end{cases}$$

We then extend $e_k (k = 1,2,...)$ to be $2kT$ periodic, which, for convenience, we denote also by e_k. It is clear that $e_k \in H_{2kT}$ and $\varphi_k(e_k) = \varphi_1(e_1) \le 0$.

Define $g_k : [0,1] \to H_{2kT}$ by $g_k(s) = se_k$ for $s \in [0,1]$. Then, we can have

$$c_k \le \max_{s\in[0,1]} \varphi_k(g_k(s)) = \max_{s\in[0,1]} \varphi_1(g_1(s)) \equiv c_0$$

independently of k, where c_k is a constant of (4.11).

As in Step 1 in the proof of Theorem 1, we can prove that $\{u_k\}$ is a bounded sequence in $W^{1,p}((-T,T),\mathbb{R}^N)$. Hence, we can choose a subsequence $\{u_{1,k}\}$ such that $\{u_{1,k}\}$ converges weakly in $W^{1,p}((-T,T),\mathbb{R}^N)$ and strongly in $L^\infty((-T,T),\mathbb{R}^N)$. Note that $\{u_{1,k}\}$ is a bounded sequence in $W^{1,p}((-2T,2T), R^N)$, we can choose a subsequence $\{u_{2,k}\}$ such that $\{u_{2,k}\}$ converges weakly in $W^{1,p}((-2T,2T),R^N)$ and strongly in $L^\infty((-2T,2T),R^N)$. Repeating this process, we obtain, for any positive integer n, a sequence $\{u_{n,k}\}$ that converges weakly in $W^{1,p}((-nT, nT), \mathbb{R}^N)$ and strongly in $L^{\infty}((-nT,\ nT), \mathbb{R}^N)$ satisfying

$$\{u_k\} \supset \{u_{1,k}\} \supset \{u_{2,k}\} \supset ...\{u_{n,k}\} \supset ....$$

Therefore, for any positive integer n, the sequence $\{u_{j,j}\}$ converges weakly in $W^{1,p}((-nT, nT), \mathbb{R}^N)$ and the sequence $\{u_{j,j}\}$ converges strongly in $L^{\infty}((-nT, nT), \mathbb{R}^N)$. Therefore, there exists a function $u_0 \in W^{1,p}_{loc}(\mathbb{R}, \mathbb{R}^N) \cap L^{\infty}_{loc}(\mathbb{R}, \mathbb{R}^N)$ such that the sequence $\{u_{j,j}\}$ converges weakly u_0 in $W^{1,p}_{loc}(\mathbb{R}, \mathbb{R}^N)$ and strongly in $L^{\infty}_{loc}(\mathbb{R}, \mathbb{R}^N)$. □

4.2 Proof of Theorem 2

Proof. We divide the proof into three steps.

Step 1. We show that u_0 is a solution to system (1.1). Here, for simplicity, we denote $\{u_{k,k}\}$ by $\{u_k\}$. For any given interval $(a, b) \subset (-kT, kT)$ and any $v \in W_0^{1,p}((a, b), \mathbb{R}^N)$, define

$$v_1 = \begin{cases} v(t), & t \in (a, b), \\ 0, & t \in (-kT, kT) \backslash (a, b). \end{cases}$$

For any $v \in W_0^{1,p}((a, b), \mathbb{R}^N)$, we get

$$\begin{aligned} 0 = \varphi_k'(u_k)(v_1) &= \int_a^b |u_k'|^{p-2} u_k' v' dt - \int_a^b f(u_k) v dt \\ &\quad + \int_a^b e_k v dt - \sum_{t_j \in (a,b)} g_j(u_k(t_j)) v(t_j), \end{aligned}$$

then, it follows that

$$\begin{aligned} &\int_a^b |u_0'|^{p-2} u_0' v' dt - \int_a^b f(u_0) v dt + \int_a^b e v dt - \sum_{t_j \in (a,b)} g_j(u_0(t_j)) v(t_j) \\ &= \lim_{k \to +\infty} \bigg(\int_a^b |u_k'|^{p-2} u_k' v' dt - \int_a^b f(u_k) v dt + \int_a^b e_k v dt \\ &\quad - \sum_{t_j \in (a,b)} g_j(u_k(t_j)) v(t_j) \bigg) = 0. \end{aligned}$$

By using a similar argument as the proof of Lemma 2.5 in [25], we can show that u_0 is a solution to system (1.1).

Step 2. We prove that $u_0(t) \to 0$ as $t \to \pm\infty$. Since $\{u_k\}$ is weakly convergent in $W^{1,p}_{loc}$, it follows from Step 1 of the proof of Theorem 1 that there exists a constant $M_3 > 0$ such that

$$\begin{aligned} \int_{-\infty}^{+\infty} (|u_0'|^p + |u_0|^p) dt &= \lim_{k \to +\infty} \int_{-kT}^{kT} (|u_0'|^p + |u_0|^p) dt \\ &\le \lim_{k \to +\infty} \lim_{j \to +\infty} \int_{-kT}^{kT} (|u_j'|^p + |u_j|^p) dt \le M_3^p. \end{aligned}$$

So, we can have

$$\int_{|t|\geq r}(|u_0'|^p+|u_0|^p)dt\to 0, \text{as } r\to +\infty, \tag{4.12}$$

which together with Lemma 3 yields that $u_0(t)\to 0$ as $t\to\pm\infty$.

Step 3. We prove that $u_0'(t^{\pm})\not\to 0$ as $t\to\pm\infty$.

Note that $0=t_0<t_1<t_2<...<t_m=T$, $t_{j+m}=t_j+T$, $j\in\mathbb{Z}$, $\underline{\delta}=\min_{j\in\mathbb{Z}}\{t_j-t_{j-1}\}>0$, so $\overline{\delta}=\max_{j\in\mathbb{Z}}\{t_j-t_{j-1}\}\geq\delta>0$. By means of the Hölder inequality, we have

$$\begin{aligned}\underline{\delta}|u_0'(t)|^{p-1}&\leq\int_{t_{j-1}}^{t_j}\left||u_0'(\tau)|^{p-2}u_0'(\tau)+\int_\tau^t\frac{d}{ds}\big(|u_0'(s)|^{p-2}u_0'(s)\big)ds\right|d\tau\\&\leq\overline{\delta}^{\frac{1}{p}}\left(\int_{t_{j-1}}^{t_j}|u_0'(s)|^pds\right)^{\frac{p-1}{p}}+\overline{\delta}^{\frac{2p-1}{p}}\left(\int_{t_{j-1}}^{t_j}\left|\frac{d}{ds}\big(|u_0'(s)|^{p-2}u_0'(s)\big)\right|^pds\right)^{\frac{1}{p}}.\end{aligned} \tag{4.13}$$

From (4.12), we can see that

$$\int_{t_{j-1}}^{t_j}|u_0'|^pdt\leq\int_{t_{j-1}}^{t_j}(|u_0'|^p+|u_0|^p)dt\to 0, \text{as } j\to\pm\infty. \tag{4.14}$$

It follows from $e\in L^p(\mathbb{R},\mathbb{R}^N)$ that

$$\int_{t_{j-1}}^{t_j}|e(s)|^pds\to 0, \text{as } j\to\pm\infty. \tag{4.15}$$

In Step 1, we have prove that u_0 is a solution to system (1.1). Then,

$$\begin{aligned}\int_{t_{j-1}}^{t_j}\left|\frac{d}{ds}\big(|u_0'(s)|^{p-2}u_0'(s)\big)\right|^pds&=\int_{t_{j-1}}^{t_j}\big|-f(u_0(s))+e(s)\big|^pds\\&\leq 2^{p-1}\int_{t_{j-1}}^{t_j}\big(|f(u_0(s))|^p+|e(s)|^p\big)ds.\end{aligned} \tag{4.16}$$

However, $f(0)\neq 0$. So that, from $\lim_{t\to\pm\infty}|u_0(t^{\pm})|=0$, (4.15) and (4.16), we can see that

$$\int_{t_{j-1}}^{t_j}\left|\frac{d}{ds}\big(|u_0'(s)|^{p-2}u_0'(s)\big)\right|^pds\not\to 0 \text{ as } j\to\pm\infty. \tag{4.17}$$

Substituting (4.17) and (4.14) into (4.13), we can obtain

$$|u_0'(t^{\pm})|\not\to 0 \text{ as } j\to\pm\infty.$$

Hence, by the definition of homoclinic solutions, we can see that there are no existence of homoclinic solutions for system (1.1). Therefore, the system (1.1) possesses no solitary wave solution. □

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