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OPTIMISATION OF A BEAM TRANSFER FODO LINE

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Abstract

With in view the design of the CLIC long transfer lines, we develop a formal approach for the optimisation of a straight FODO line. Optimum phase advance and cell length depending on beam parameters are derived for power consumption, overall cost and sensitivity to quadrupole misalignment.

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Optimisation of a beam transfer FODO line

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INTRODUCTION

In the CLIC project, the drive beam and the main beam will travel from their central production sites towards the head of the main linacs over a straight distance $L > 20$ km [1]. With such long lines, a systematic approach must be used in order to optimize many parameters. This includes optics, beam dynamics, operational issues like reliability, diagnostics and safety with respect to beam losses. Finally, the cost of the equipment and the power consumption are major issues. In this paper, an attempt to optimize a long line is presented. We consider a line made of FODO cells and search for the minimum of a few figure of merits which depend on optics functions. We identified the sensitivity of the beam emittance to quadrupoles misalignment, the number of cells and the electrical power which supply the magnets. We briefly discuss further steps of optimization which require a study of parameters which are beyond pure optics studies. We finally discuss some choices of parameters which are partly specific to the CLIC transfer lines.

FODO BASICS

We consider a symmetric FODO cell of length L_c, which starts and ends in the middle of focussing quadrupoles (QF). A de-focussing quadrupole (QD) is located at the center of the cell. The gradient of the quadrupoles are equal in magnitude, i.e $k = k_{\mathrm{QF}} = -k_{\mathrm{QD}}$. We further consider a thin length approximation, with the focal length $f = 1/kl$. QF and QD are separated by a drift (D) of length $L_c/2$. The cell contains no dipoles, because we consider a straight beam line. A derivation of several formulae can be found in [2] and a summary in [3], with in both cases slightly different notations than here. The length of the line is L.. The transfer matrix of the horizontal plane through the cell is obtained with $M = M_{\mathrm{qf}/2}M_{\mathrm{d}}M_{\mathrm{qd}}M_{\mathrm{d}}M_{\mathrm{qf}/2}$ which expands to

$$M_{\mathrm{FF}} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 1-\frac{L_c^2}{8f^2} & L_c(1+\frac{L_c}{4f}) \\ -\frac{L_c}{4f^2}(1-\frac{L_c}{4f}) & 1-\frac{L_c^2}{8f^2} \end{pmatrix} \tag{1}$$

The right side of Eq. (1) is then identified with the parametrization of Courant-Snyder for a periodic and symmetric structure ($\alpha = 0$)

$$M_{\mathrm{FF}} = \begin{pmatrix} \cos\mu & \beta\sin\mu \\ \frac{\sin\mu}{\beta} & \cos\mu \end{pmatrix} \tag{2}$$

With Eqs. (1) and (2), $\cos\mu = 1 - \frac{L_c^2}{8f^2}$ and

$$\sin\frac{\mu}{2} = \frac{L_c}{4f}\ . \tag{3}$$

The β-functions reaches its maximum at the extremities of the cell. We write $\beta = \hat\beta$ and identify m_{12} in Eqs. (1) and (2) using Eq. (3). We get

$$\hat\beta = L_c\frac{1+\sin\frac{\mu}{2}}{\sin\mu}\ . \tag{4}$$

Considering the transfer matrix from QD to QD,

$$\check\beta = L_c\frac{1-\sin\frac{\mu}{2}}{\sin\mu}\ . \tag{5}$$

At this point, we note that Eq. 2 contains two free parameters, namely β and μ. While other choices might be considered, optimization shall preferably be made with using optic functions proper, here β and μ, instead of say, L_c and f. We will see below that this choice is indeed good. It allows to express all the useful quantities and our figures of merit with functions which allow to separate the variables, i.e. $A(\beta,\mu) = \mathrm{const}\times F(\beta)G(\mu)$.

The focal length is expressed as a function of $\hat\beta$ and μ with Eqs. (3) and (4) :

$$f = \frac{\hat\beta}{2}\frac{\cos\frac{\mu}{2}}{1+\sin\frac{\mu}{2}} \tag{6}$$

and the number of cells is

$$N = \frac{L}{L_c} = \frac{L}{\hat\beta}\frac{1+\sin\frac{\mu}{2}}{\sin\mu}\ . \tag{7}$$

The chromaticity of a cell is $C_{\mathrm{cell}} = -\tan\frac{\mu}{2}/\pi$ [2, 3]. The chromaticity of the line is $C = NC_{\mathrm{cell}}$. With Eq. (7), we obtain

$$C = -\frac{L}{2\pi\hat\beta}\frac{1+\sin\frac{\mu}{2}}{\cos^2\frac{\mu}{2}} \tag{8}$$

OPTIMIZATION

In order to preserve the emittance at best, the displacement Δ of the beam at the end of the line must be minimized. The main source of displacement comes from quadrupole displacement ('parasitic kicks'). If the momentum width $\pm\hat\delta_p$ of the beam is not negligible, a parasitic dispersion $\tilde{D}$ is associated to Δ and smears the beam. A large chromaticity aggravates the smearing by filamenting the beam. The effect of parasitic kicks is computed below, while the chromaticity $C(\hat{\beta},\mu)$ is given by Eq. 8. The number of cells N is given by Eq. 7. We do not explore explicitely the cost optimization of the vacuum system, because it is quite case specific. If the need for beam aperture is small, the radius of the vacuum chamber may be fixed to a larger value for adequate conductance. In other cases, the radius is fixed by collective effect issues. We therefore limit our evaluation to different hypothesis for the choice of the radius, see below the section on magnet power.

Parasitic Kicks

The displacement Δ of the beam at the end of the line ($\beta(L) = \hat{\beta}$), which results from random kicks $\delta x' = (kl)\delta x = \delta x/f$ associated to a r.m.s. displacement $\delta x_{\rm rms}$ of the quadrupoles is

$$\Delta = \sum_{\rm i=1}^{\rm 2N} \sqrt{\hat{\beta}\beta_i}\sin\phi_i\,\delta x_i' \;\text{ with }\; \phi_i = i\frac{\mu}{2} \tag{9}$$

The average quadratic sum $\Delta^2_{\rm rms} =< \Delta^2 >$ of Δ is

$$\Delta^2_{\rm rms} = \sum_{\rm i}^{\rm 2N} < \Delta_i^2 > = \frac{1}{f^2}\sum_{\rm i}^{\rm 2N}\hat{\beta}\beta_i\sin^2\phi_i\delta x^2_{\rm rms}\,. \tag{10}$$

With alternating QF ($\beta = \hat{\beta}$) and QD ($\beta = \check{\beta}$) and with $< \sin^2\phi_i > \simeq 1/2$, $\sum_{\rm i}^{\rm 2N}\beta_i < \sin^2\phi_i > \simeq (\hat{\beta}+\check{\beta})N/2$. It follows $\Delta^2_{\rm rms} = \hat{\beta}(\hat{\beta}+\check{\beta})N/2f^2 \times \delta x^2_{\rm rms}$. Using N from Eq. 7, f from Eq. (6), $\hat{\beta}+\check{\beta} = 2L_c/\sin\mu$ from Eqs. 4 and 5 and using the trigonometric relation $\sin\mu = 2\sin\frac{\mu}{2}\cos\frac{\mu}{2}$, we get

$$\Delta_{\rm rms} = \sqrt{\frac{4L}{\hat{\beta}}}\frac{1+\sin\frac{\mu}{2}}{\sqrt{\sin\mu}\cos\frac{\mu}{2}}\delta x_{\rm rms} \tag{11}$$

The figure of merit is $A_k = \Delta\epsilon/(\epsilon\delta x_{\rm rms})$, with $\epsilon = \sigma_\beta\sigma'_\beta = \sigma^2_\beta/\hat{\beta}$, its derivative $\Delta\epsilon = 2\sigma_\beta\Delta\sigma_\beta/\hat{\beta}$ and $\Delta\sigma_\beta = \Delta_{\rm rms}$:

$$A_k = \frac{2\Delta_{\rm rms}}{\sqrt{\epsilon\hat{\beta}}\delta x_{\rm rms}} = \frac{4}{\hat{\beta}}\sqrt{\frac{L}{\epsilon}}\frac{1+\sin\frac{\mu}{2}}{\sqrt{\sin\mu}\cos\frac{\mu}{2}} \tag{12}$$

Parasitic Dispersion

A estimator of the parasitic dispersion is obtained by modifying the phase advance for chromatic error in Eq. 9, i.e. with $\phi = i\mu/2 - i\pi C\delta_p/N$, with C the chromaticity of the line, see Eq. 8. We then write

$$< \Delta(\delta_p)^2 > = \frac{\hat{\beta}(\hat{\beta}+\check{\beta})}{2f^2}\sum_{\rm i}^{\rm 2N} < \sin^2(i\mu/2 - i\pi C\delta_p/N) >$$

With the condition $\pi C\delta_p \ll 1$,

$$\sum_{\rm i}^{\rm 2N} < ... > = N + \frac{\pi^2C^2\delta_p^2}{2N^2}\sum_{\rm i}^{\rm 2N} i^2 = N + \frac{4\pi^2C^2\delta_p^2 N}{3} \tag{13}$$

Then by definition of $\tilde{D}$, we get from the second term

$$\tilde{D}^2\delta_p^2 = \frac{\hat{\beta}(\hat{\beta}+\check{\beta})}{2f^2}\times\frac{4\pi^2C^2N}{3}\delta_p^2 \tag{14}$$

With the relations used to get Eq. 12 and C from Eq. 8, we finally obtain in the limit of validity $\pi C\delta_p \ll 1$

$$\tilde{D}_{\rm rms} \simeq \left(\frac{L}{\hat{\beta}}\right)^{3/2}\frac{(1+\sin\frac{\mu}{2})^2}{\sqrt{3\sin\mu}\cos^3\frac{\mu}{2}} < \delta x^2 >^{1/2} \tag{15}$$

Its relative dependence with μ (Fig. 1) fits rather well with a MadX code simulation, but it underestimates it by a factor $\sim 25\%$. This requires further refinement. Following the similar case of parasitic kicks, the figure of merit is here

$$A_D = \frac{2\tilde{D}_{\rm rms}}{\sqrt{\epsilon\hat{\beta}}} \sim \frac{1}{\hat{\beta}^2} \tag{16}$$

Overall Magnet Power

The current I of a resistive quadrupole scales with $I \sim (kl)a^2 = a^2/f$, where a is the coil radius. With $2N$ quadrupoles, the total power P of the line is proportional to $P \sim 2Na^4/f^2$. With Eq. 7 for N, Eq. 6 for f and disregarding constant factors, we get the figure of merit

$$A_p(\hat{\beta},\mu) = \frac{La^4}{\hat{\beta}^3}\frac{(1+\sin\frac{\mu}{2})^3}{4\sin\mu\cos^2\frac{\mu}{2}}. \tag{17}$$

As for the radius a and writing $A_p = F(\hat{\beta},a)G(\mu)$, we may consider different cases, namely

- Radius fixed by beam size, or $a \sim \hat{\sigma}_\beta \Rightarrow a^4 \sim \hat{\beta}^2$. Then, $F_1(\hat{\beta},a) = F_1(\hat{\beta}) \sim 1/\hat{\beta}$.
- Radius fixed by external criterion , e.g. vacuum conductance. Here, $a = \text{const}$, such that $F_2(\hat{\beta},a) = F_2(\hat{\beta}) \sim 1/\hat{\beta}^3$.
- A CLIC case : multi-bunch resistive wall instability. This case is discussed in [4]. The trajectory error grows across the line with $\delta x_n \sim \hat{\beta}^{1/2}/a^3$. Keeping $\delta x = \text{const}$, we get $a \sim \hat{\beta}^{1/6}$ and $F_3(\hat{\beta},a) = F_3(\hat{\beta}) \sim 1/\hat{\beta}^{7/3}$.

Results

The figures of merit A_k, $\tilde{D}$, C, A_p and N are given in Figs. 1, 2, 3 and 4 as a function of the phase advance. A summary of the F and G functions is found in Table 1. As for parasitic kicks (Fig. 1), a minimum is reached near $\mu = \pi/4$, while the parasitic dispersion $\tilde{D}$ is minimized near $\mu = \pi/8$. The power is also minimized near $\mu = \pi/4$, see Fig. 3. The number of cells is not minimized at $\mu = \pi/4$, but $N(\pi/4)/N_{\rm min} = 1.18$ is acceptable. On the other hand $N(\pi/8)/N_{\rm min} = 1.86$, somewhat conflicting with a minimization of $\tilde{D}$. It seems therefore quite obvious to choose a phase advance of $\mu = \pi/4$, which is also useful for being a half of the often mandatory $\mu = \pi/2$ for some correction systems. Then quite obviously, with all the functions varying with $1/\hat{\beta}^n$, with $1 < n < 3$, the largest possible value of $\hat{\beta}$ must be preferred.

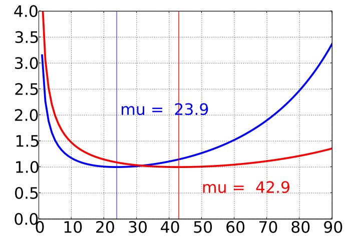

Figure 1: The beam displacement Δ (red) and the parasitic dispersion $\tilde{D}$ (blue) as a function of the phase advance of the cell μ [°] and at the end of the line, resulting from trajectory errors in the quadrupoles. Both functions are normalized to their respective minmum value r.m.s. quadrupole displacement.

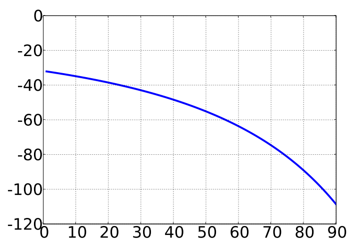

Figure 2: The Chromaticity C in a.u. as a function of μ.

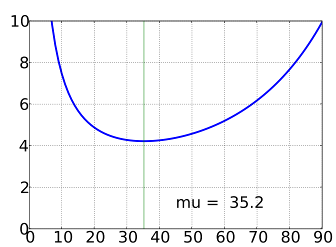

Figure 3: The overall magnet power (a.u.) as a function of μ.

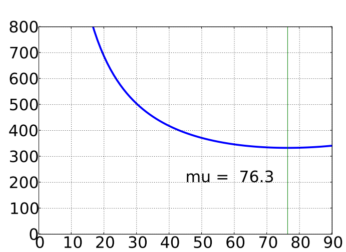

Figure 4: The number of cells (a.u.) as a function of μ.

Table 1: The figures of merit split in their dependence with $\hat{\beta}$ and μ, and disregarding constant factors.

Case	$F(\hat{\beta})$	$G(\mu)$
N cells	$\frac{1}{\hat{\beta}^{7/3}}$	$\frac{1+\sin\frac{\mu}{2}}{\sin\mu}$
Paras. kicks	$\frac{1}{\hat{\beta}}$	$\frac{1+\sin\frac{\mu}{2}}{\sqrt{\sin\mu\cos\frac{\mu}{2}}}$
$\tilde{D}/\sqrt{\hat{\beta}}$	$\frac{1}{\hat{\beta}^2}$	$\frac{(1+\sin\frac{\mu}{2})^2}{\sqrt{3\sin\mu\cos^3\frac{\mu}{2}}}$
Power :		
$a \sim \hat{\sigma}_\beta$	$\frac{1}{\hat{\beta}}$	$\frac{(1+\sin\frac{\mu}{2})^3}{4\sin\mu\cos^2\frac{\mu}{2}}$
$a =$ cst.	$\frac{1}{\hat{\beta}^3}$	”
n-bunch res.	$\frac{1}{\hat{\beta}^{7/3}}$	”

THE CASE OF THE CLIC MAIN BEAM TRANSFER LINE

The transfer line of CLIC Main Beam is constrained only at the extremities of the line, where $\hat{\beta}$ must be matched to synchronous curved lines. With $L = 21$ km, it is therefore economical to foresee a matching section at each extremity. The radius of the vacuum chamber will be fixed to a large value in order to limit multi-bunch resistive wall instabilities[4]. Therefore a large $\hat{\beta} = 860$ m is considered. With $\mu = \pi/4$ the cell length is $L_c = 438$ m which is half of the sectorization length of the tunnel. But with so long cells, some parasitic effects must be further worked out, like the impact of the earth magnetic field. With these value, the chromaticity is $C = -5.3$. The phase error at the edge of the momentum band $\hat{\delta}_p = 0.01$ is $\delta\phi = 2\pi\hat{\delta}_p C = -18°$ and the chromatic error $\delta\beta/\beta < 1\%$. The vertical emmitance of the main beam is $\epsilon_n = 10$ nm [1]. At 9 GeV, the beam size is $\hat{\sigma}_\beta = 15$ μm. With $\mu = \pi/4$, $\tilde{D} \simeq 0.02$ m, such that the chromatic smear is $\tilde{D}\hat{\delta} = 200$ μm with $\delta x_{\mathrm{Q}} = 100$ μm. With a ratio $\tilde{D}\hat{\delta}_p/\hat{\sigma}_\beta = 13.5$, a beam based alignment remains necessary even with the optimized line.

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