

Large Hadron Collider Project

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# NON-LINEAR CORRECTION SCHEMES FOR THE PHASE 1 LHC IR INSERTION REGION UPGRADE AND DYNAMIC APERTURE STUDIES

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### Abstract

The Phase 1 LHC Interaction Region (IR) upgrade aims at increasing the machine luminosity essentially by reducing the beam size at the Interaction Point (IP). This requires a total redesign of the full IR. A large set of options have been proposed with conceptually different designs. This paper reports on a general approach for the compensation of the multipolar errors of the IR magnets in the design phase. The goal is to use the same correction approach for the different designs. The correction algorithm is based on the minimization of the differences between the IR transfer map with errors and the design IR transfer map. Its performance is tested using the dynamic aperture as figure of merit. The relation between map coefficients and resonance terms is also given as a way to target particular resonances by selecting the right map coefficients. The dynamic aperture is studied versus magnet aperture using recently established relations between magnetic errors and magnet aperture.

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#### Abstract

The Phase 1 LHC Interaction Region (IR) upgrade aims at increasing the machine luminosity essentially by reducing the beam size at the Interaction Point (IP). This requires a total redesign of the full IR. A large set of options have been proposed with conceptually different designs. This paper reports on a general approach for the compensation of the multipolar errors of the IR magnets in the design phase. The goal is to use the same correction approach for the different designs. The correction algorithm is based on the minimization of the differences between the IR transfer map with errors and the design IR transfer map. Its performance is tested using the dynamic aperture as figure of merit. The relation between map coefficients and resonance terms is also given as a way to target particular resonances by selecting the right map coefficients.

The dynamic aperture is studied versus magnet aperture using recently established relations between magnetic errors and magnet aperture.

#### MATHEMATICAL BACKGROUND

A general introduction to this subject is given in [1]. The transfer map between two locations of a beam line is expressed in the form

$$\vec{x}_f = \sum_{jklmn} \vec{X}_{jklmn} \ x_0^j \ p_{x0}^k \ y_0^l \ p_{y0}^m \ \delta_0^n, \tag{1}$$

where  $\vec{x}_f$  represents the vector of final coordinates  $(x_f, p_{xf}, y_f, p_{yf}, \delta_f)$ , the initial coordinates being represented with the zero subindex, and  $\vec{X}_{jklmn}$  is the vector containing the map coefficients for the four phase-space coordinates and the momentum deviation  $\delta$ , considered as a parameter. The MAD-X [2] program together with the Polymorphic Tracking Code (PTC) [3] provide the computation of the quantities  $\vec{X}_{jklmn}$  up to any desired order.

To assess how much two maps, X and X' deviate from each other, the following quantity is defined:

$$\chi^2 = \sum_{jklmn} ||\vec{X}_{jklmn} - \vec{X}'_{jklmn}||$$
(2)

where  $|| \cdot ||$  stands for the quadratic norm of the vector. To disentangle the contribution of the various orders to the global quantity  $\chi^2$ , the partial sum  $\chi^2_q$  over the map coefficients of order q is defined, namely

$$\chi_q^2 = \sum_{j+k+l+m+n=q} ||\vec{X}_{jklmn} - \vec{X}'_{jklmn}||$$
(3)

so that

$$\chi^2 = \sum_q \chi_q^2. \tag{4}$$

In principle, this definition could be used to introduce a weighting of the various orders, using a well-defined amplitude in phase space. This option is not considered in the applications described in this paper.

Furthermore,  $\chi^2_q$  is split into a chromatic  $\chi^2_{q,c}$  and achromatic  $\chi^2_{a,a}$  contribution, corresponding to

$$\chi_{q,a}^2 = \sum_{j+k+l+m=q} ||\vec{X}_{jklm0} - \vec{X}'_{jklm0}||.$$
(5)

It is immediate to verify that  $\chi_q^2 = \chi_{q,c}^2 + \chi_{q,a}^2$ . As shown in [1] the relation between map coefficients and resonance terms follows,

$$X_{p0000}^{x} = -ie^{-i\Delta\phi_{x}} \sqrt{\frac{\beta_{xf}}{\beta_{x}^{p}}} \sum_{q=1}^{p+1} qh_{q(p-q+1)00} + c.c.$$
(6)

This expression already captures the most important features of the relation between map coefficients and resonance terms. For example the sextupolar map coefficient  $X_{20000}^x$  depends linearly on  $h_{3000}$  and  $h_{1200}$ , or the (3,0) and (1,0) resonances respectively. It can be proved that the coefficient  $X_{pq000}^x$  depends on the same terms as  $X_{(p+q)0000}^x$ . The number of resonances involved in the relation increases linearly with the order of the map coefficient. Therefore minimizing local map coefficients implies a minimization of a collection of resonances. Therefore this approach might be useful when the knowledge of the full accelerator is limited.

## **CORRECTION OF MULTIPOLAR** ERRORS

#### Algorithm

The basic assumption is that the multipolar field errors of the IR magnets are available as the results of magnetic measurements. The ideal IR map X without errors is computed using MAD-X and PTC to the desired order and stored for later computations. Including the magnetic errors to the IR elements perturbs the ideal map. To cancel or compensate this perturbation, distributed multipolar correctors need to be located in the IR. The map including both the errors and the effect of the correctors will be indicated with X'. The corrector strength is determined by simply minimising  $\chi_a^2$ for these two maps. For efficiency, the minimisation is accomplished order-by-order (see, e.g., Ref. [4] for a description of the dependence of the various orders of the nonlinear transfer map on the non-linear multipoles). In such an approach the sextupolar correctors are used to act on  $\chi^2_2$ , the octupolar ones on  $\chi^2_3$ , and so on.

The code MAPCLASS [5] already used in [6] has been

extended to compute  $\chi_q^2$  from MAD-X output. The correction is achieved by the numerical minimisation of  $\chi_q^2$  using any of the existing algorithms in MAD-X for this purpose.

#### Performance evaluation

The evaluation of the performance of the method previously described is carried out using two of the three layouts proposed for the upgrade of the LHC insertions (see, e.g., Ref. [7]). The field quality of the low-beta triplets is considered to follow the assumption reported in Ref. [8], which implies that large-bore quadrupoles feature a better field quality than smaller aperture ones. An example of the order-by-order correction is shown in Fig. 1 for the so-called low  $\beta_{\text{max}}$  configuration [7]. A total of sixty realisations of the LHC lattice are used in the computations. Non-zero systematic errors are included in the simulations.



Figure 1: Evaluation of the various orders of  $\chi_q^2$  (upper plot) before (blue markers) and after (red markers) correction. Sixty realisations of the random magnetic errors are used. The layout is the low  $\beta_{\text{max}}$ , whose optics is also reported (lower plot).

One corrector per IR side and per type (normal or skew component) are used. Different locations of the non-linear correctors can be used for the minimisation of  $\chi_q^2$ . The configuration having the lowest  $\chi_q^2$  after correction is selected for additional studies (see next section). The difference between a non-optimised positioning and the best

possible one is illustrated in Fig. 2. There, the results of the proposed correction scheme in the case of a symmetric configuration (see Ref. [7]) are shown. The configuration corresponding to the grey dots achieves slightly better corrections over the ensemble of realisations and therefore is selected for further studies.



Figure 2: Evaluation of the various orders of  $\chi_q^2$  (upper plot) before (blue markers) and after (grey and red markers) correction. The red markers represent a non-optimised (in terms of correctors location) compensation scheme. Sixty realisations of the random magnetic errors are used. The layout is the symmetric one, whose optics is also reported (lower plot).

#### DYNAMIC APERTURE COMPUTATION

## Assessment of the non-linear correction algorithm

The main goal of the error compensation is to increase the Dynamic Aperture (DA), defined as the minimum initial transverse amplitude becoming unstable after N turns. Details of the DA computation are given in [1]. In Fig. 3 the DA for the two LHC upgrade options, low  $\beta_{max}$  and symmetric, as a function of phase space angle is plotted with and without non-linear corrections schemes.

The correction algorithm proved to be particularly successful in the case of the symmetric layout. Indeed, for this configuration about 2.5  $\sigma$  are recovered thanks to the cor-



Figure 3: Comparison of the dynamic aperture for the socalled LHC upgrade layouts low  $\beta_{max}$  and symmetric with and without correction of the non-linear magnetic errors in the low-beta quadrupoles.

rection of the non-linear  $b_3$  and  $b_6$  errors.

The compensation in the case of the low  $\beta_{max}$  layout is less dramatic, allowing to recover 2.5  $\sigma$  for small angles, only. It is also important to stress that the baseline DA is not the same for the two layouts, as the low  $\beta_{max}$ is already well above 14.5  $\sigma$  without any correction. Furthermore, not only the optics is different for the options, but also the triplets' aperture. The first implies a different enhancement of the harmful effects of the triplets field quality, while the latter has a direct impact on the actual field quality because of the scaling law [8]. It is clear that the DA for the low  $\beta_{max}$  is already well beyond the targets used for the design of the nominal LHC even without nonlinear correctors. The situation for the symmetric option is slightly worse and a correction scheme might be envisaged.

# Digression: Dynamic aperture vs. low-beta triplet aperture

A third layout proposed as a candidate for the LHC IR upgrade is the so-called compact [7]. It features very large aperture triplet quadrupoles (150 mm diameter for  $Q_1$  and 220 mm for  $Q_2$  and  $Q_3$ ). Thanks to the proposed scaling law, the field quality is excellent and the result DA is beyond 16  $\sigma$  and hence does not require any correction scheme. Nevertheless, a detailed study of the dependence of the dynamic aperture on the magnets aperture is carried out. The DA is computed versus the aperture of  $Q_1$  and simultaneously over the apertures of  $Q_2$  and  $Q_3$ . The optics is assumed to be constant. The results are shown in Fig. 4. The minimum, average, and maximum (over the realisations) DA are shown for the two type of scans. The horizontal lines represent the asymptotic value. The dependence on the aperture of  $Q_1$  is rather mild, because of the not too high value of the beta-function, and there exists a rather wide range of apertures for which the DA is almost constant. In particular for  $\phi > 110 \text{ mm}$  the asymptotic value of the DA is reached. A constant drop of DA is ob-



Figure 4: DA as a function of the low-beta quadrupoles aperture. The scan over the aperture of  $Q_1$  is shown in the upper plot (nominal aperture 150 mm), while  $Q_2$  and  $Q_3$  are considered in the lower plot (nominal aperture 220 mm). The layout is the so-called compact one.

served for  $\phi < 100 \text{ mm}$  and, in general, the three curves behave the same.

The dependence of DA on the  $Q_2$  and  $Q_3$  aperture is somewhat different. The asymptotic value is hardly reached for apertures larger than 250 mm and the DA drop with aperture is monotonic and smooth. The spread between the asymptotic values for minimum, average, and maximum DA is smaller than for the case of the scan over the aperture of  $Q_1$ . As mentioned above the DA scales with a power law of the magnet aperture.

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