

XIV. NEUROPHYSIOLOGY*

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A. A COMMENT ON SHIFT REGISTERS AND NEURAL NETS[†]

Shift registers, linear and nonlinear, can be regarded as a subclass of neural nets,¹ and, therefore, the results of the general theory of neural nets apply to them, not vice versa. A useful technique for investigating neural nets is provided by the state transition matrix. We shall apply it to shift-register networks. The notation will be the same as that previously used.¹

Let an SR network be a neural net of N neurons and M external inputs, defined by a set of N Boolean equations of the form

$$\begin{aligned}
 y_1(t) &= f_1(x_1(t-1), x_2(t-1), \dots, x_M(t-1); y_1(t-1), \dots, y_N(t-1)) \\
 y_2(t) &= y_1(t-1) \\
 &\vdots \\
 &\vdots \\
 y_N(t) &= y_{N-1}(t-1),
 \end{aligned}
 \tag{1}$$

where $f_1(x_1, x_2, \dots, x_M; y_1, y_2, \dots, y_N)$ can be any Boolean function of the external inputs x_1, x_2, \dots, x_M , and of the outputs y_1, y_2, \dots, y_N . The SR network corresponds to a shift register of N delay elements, which is linear or nonlinear, depending upon the nature of the function f_1 .

The state transition matrix of the SR network, obtained by applying Eq. 5 of a previous report,² is

$$\mathcal{M}(X)_{ij} = f_1(X; a, b, \dots, d, g)^P \cdot a^Q \cdot \dots \cdot dt,
 \tag{2}$$

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where (a, b, \dots, d, g) and (p, q, \dots, s, t) are the strings of 0's and 1's defining S_i and S_j , respectively.

Since the term $a^q \cdot \dots \cdot dt$ is 0 unless

$$a = q, \dots, d = t, \tag{3}$$

it follows that the only $\mathcal{M}(X)_{ij}$ terms that may be different from 0 are those for which S_i and S_j satisfy the conditions of Eqs. 3.

For every $S_i = (a, b, \dots, d, g)$ there are only two states S_j that fulfill the conditions above, namely, $S_j = (0, a, b, \dots, d)$ and $S_j = (1, a, b, \dots, d)$. Thus, every state of an SR network can only go to one of the two corresponding states, instead of being able to go to any state, as in a general neural net. Similarly, any state $S_j = (p, q, \dots, s, t)$ may only be reached from either state $(q, \dots, s, t, 0)$ or $(q, \dots, s, t, 1)$.

For example, the state transition matrix of an SR of three delays may have terms that are 1 only in the positions marked with an x in the following matrix

		S_j							
		000	001	010	011	100	101	110	111
S_i	000	x				x			
	001	x				x			
	010		x				x		
	011		x				x		
	100			x				x	
	101			x				x	
	110				x				x
	111				x				x

Conversely, any network of N neurons whose state transition matrix has 0 terms everywhere except for pairs S_i, S_j which satisfy Eqs. 3, is an SR network of N delay elements.

The number of possible modes of oscillations for an SR network of N delays is considerably lower than for a general neural net of N neurons. For example, for an SR network of three delays, there are only two modes of oscillation of length 8, whereas a general neural net can have $7! = 5040$. There is also a limitation in the sequences that can be obtained, under constant input, out of a neuron in an SR network. For example, the sequence 11110000 cannot be realized in an SR network of three delays, whereas it can be obtained out of a neuron in a network of 3 neurons. Actually, for an SR of 3 delays there are only two possible sequences of length 8, which correspond to the two possible modes of oscillation, namely, 11101000 and 11100010. Other sequences of length 8 that cannot be realized by an SR network of 3 delays, but which are obtainable out of a neuron in a network of 3 neurons are 11010100, 00101011, 11011000, 00100111, 00101101, and 11010010.

A neuron in a general net can produce any sequence of length up to 2^N if the number

of 0's and the number of 1's in the sequence are not larger than 2^{N-1} . The rules to synthesize a neural net of N neurons that realizes any of the possible sequences are easily obtained from the state transition matrix, and will be given in a future note.

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References

1. W. S. McCulloch and J. Simões da Fonseca, Quarterly Progress Report No. 85, Research Laboratory of Electronics, M. I. T., April 15, 1967, pp. 325-327.
2. R. Moreno-Diaz, "Stability of Networks with Loops," Quarterly Progress Report No. 83, Research Laboratory of Electronics, M. I. T., October 15, 1966, pp. 165-171.

B. THE COLORS OF COLORED THINGS

Part I. The Inductive Argument

This report appears in two parts. In the first, I present the grouping of phenomena and psychological opinion that I believe to be relevant.

In the second part I show a model of the receptors that yields an output from which the conversion of C_1 to C_2 can be derived.

The first part is necessarily abundant because the material has not been brought together in this manner elsewhere, and I felt that the reader ought to know what is being modelled by the theory.

1. Introduction

Judgment of color (including brightness) seems not to depend on extension. We can see red star points, Mars, and red fields, the sky in a dusty sunset. For a red spot on the gray wall, its redness seems to us most primitive: the redness is like nothing else but itself, it cannot be decomposed or described, but only exhibited; it is a simple. Color is an absolute judgment – the spot is red in itself, not relatively red with respect to something else. Even when we induce a "color illusion" making a "gray" spot turn "red" by "contrast enhancement," that spot is absolutely red, not merely redder than the background or in any other wise contingently red. With our tradition it is natural to suppose that the perception expressed as that red spot on that gray wall is put together out of sensations that come prior, e. g., redness, grayness, etc., in the same way that sentences are composed of words. In this view sensations of color come prior to perceptions of colored things, and a red spot is bounded redness. But what seems to come before and what after in our introspections need not reflect a similar order in the underlying physical causation. Thus, even if sensations are to be taken as psychological primitives, they ought not to be confused with sensory events or supposed to be more reflective of them than perceptions.

Sensations, insofar as they are introspectively abstracted from perceptions, are apperceptive. Sensory events, insofar as one can detect the working of sense organs,

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are not accessible to perception directly but only through channels that pass outside the system. For example, I could imbed a microelectrode in a single nerve cell of my eye, and listen to its pulsing as I look at different things, but I should not be able to identify the sensory events with the sensations I infer from my perceptions. Even if there were a direct causal chain, say, "red" neurons whose discharge rate went pari passu with how much redness I saw – even then there would be the distinction between my seeing a red thing, my sensitivity to whatever in me represents the nervous activity, and my listening to the actual nervous activity that meant "red." The three representations are different, however well they might map each other. It has been a strong hope, however, that such a mapping might be found to give anteriority to sensations over perception – and where this false hope has most intruded has been in color theory. But that is because the criteria for a color theory are most often improper.

A color theory must account for the absolute judgments that are the colors I see. It need not account for how I perceive, nor need it solve the mind-body problem, but it should at least present a model that utters the same judgments as a man whom I examine with arrangements of colored patches. In the past, from Helmholtz on, there has been the feeling that one could pass from a theory of sensory events to a theory of color sensations, provided one could first define a distance element in some representation of the space of sensory events. Experiment shows that such a rigid transformation is not sufficient. The most revealing recent studies have been those of Dr. Land who revived the question of simultaneous contrast.

To understand the work of Dr. Edwin Land, we must first review the elements of the sensory space of color. Dimensional arguments about sensory events are important, for if it can be shown that sensory events, constrained to two degrees of freedom, generate sensations with three degrees of freedom, then we are driven to suppose noncausal relations between our perceptions and what occasions them, and we may as well forget about a physiological psychology. It is almost such a question that Land poses. But to understand the problem more fully we shall have to revert to a common-sense argument about the purpose of a color system, for I think it can be shown that, historically, we have erred in what we take to be the nature and use of color. Such a discussion is necessary, for teleology plays an important part in the analysis of informational systems, and it is only a positivist superstition that gives it a bad name in biology.

I shall present the account inductively.

a. Sensory Mechanism

Our present theory of the sensory events that underlie the seeing of color was developed almost in its final form by Isaac Newton. Indeed, all of the later emendations and exactnesses contribute no new ideas to the basic notion as set forth in Opticks. The relevant sections in that book are cursively written, a delight to read after the slow,

data-weighted approach of a modern textbook. Newton's theory has now been confirmed by the discovery of those pigments embodying the processes conceived by Young to be entailed by Newton's model. This work has been done at several laboratories, but most convincingly by Marks at Johns Hopkins.

After Newton had decomposed sunlight and candle light and analyzed the spectrum, he addressed himself to the question of how color can be a function of the spectrum. He intuited the barycentric model and did the crucial experiments to show its aptness. Slightly modified, the notion is as follows.

Lay the spectrum out along a weightless wire. Bend the wire around into an open plane curve. Let us call this figure the spectral-line boundary. To get the color of a ray of light, we attach weights to the wire at every point, each weight proportionate to the flux at that point in the spectrum. The center of gravity of the figure will then represent the chromaticity (color independent of brightness). There will be a set of points corresponding to different mixtures of spectral extremes, and these will lie along an imaginary straight line connecting the two spectral extrema, the ends of the wire. Those hues that lie along this straight line are called nonspectral hues. The hue boundary is thus closed.

A uniform distribution of flux along the spectrum (as in "white" noise) yields huelessness or white, the "middling color between all colors." Flux concentrated at one point of the spectrum yields a pure spectral hue. Fluxes concentrated at both extrema of the spectrum yield nonspectral hues or purples that vary according to the ratio of the two fluxes. Thus, as one moves along the spectrum from one end to the other and then back through different ratios of extrema, the hue changes continuously. There exists no hue that does not correspond to a value on the hue boundary; huelessness corresponds to the center of the enclosed area.

For any distribution of flux along the spectrum the center of gravity will lie somewhere between the hue boundary and the white center. There is an infinite set of distributions that will yield the same center of gravity. These are called metamers. A unique metamer in each set will be that consisting of the mixture of a uniform distribution of flux along the spectrum, together with flux concentrated at one point on the hue boundary. It is this metamer that corresponds to the color that we attribute to the set — we see the color as having the hue of the point on the boundary and the enhuedness (or saturation) corresponding to the ratio of the flux at that point to the flux uniformly distributed. In other words, white is the most desaturated of all colors, and any color can be represented by a mixture in varying proportion of white with a maximally saturated hue, spectral or nonspectral. Incidentally, no two spectral lines, when mixed in any proportions, can yield a color more saturated than the color of the single spectral line (or combination of extremum spectral lines) that has the same hue as the mixture. Thus the hue boundary is everywhere not concave.

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This wonderful conceit makes vectors of colors, and the rules for composition of colors reduces to the rules for composition of forces in three-space, as Grassman later pointed out. Thomas Young remarked that the closed hue boundary could be bounded by a triangle, whence the transformation of light to color required but three different sensitive processes in the retina. His comment was taken up by Helmholtz and is known as the Young-Helmholtz trichromatic theory. But, indeed, it only says explicitly what is implied by Newton's model.

What remained was to establish the metameric sets, and by matrix inversion to compute the local stretching and condensation of the spectrum along the hue boundary, and the curvature of the boundary in at least one projection of it. This purely algebraic job on psychological measurements was done within the last hundred years, begun by Maxwell and finished by committee.

You will notice that this is a theory of the transductive or sensory process only and takes indiscernibility to establish identity according to the dicta later expressed so well by Helmholtz. Whatever are the processes that lie behind the sense organs, they have no access to the world save through the sense organs, so that if two stimuli are unaidedly indistinguishable under all circumstances, they must set up identical representational processes. If sensorily indistinguishable events can be distinguished by physical measurements, then one can write a degenerative transformation that carries the measurements into their sensory representation. Implicit in this outlook is the conviction against occasionalism — no information about the world can occur save through the senses.

Thus, Newton's theory states the rules of the sensory processes involved in seeing color. It does not establish what colors we shall see, but only what spectral distributions of flux we cannot tell apart. Only if we insist on a rigid transformation that carries the space of sensory events into the space of sensations, can we say we now have a color theory. But the adequacy of such a transformation can be disproved by experiment.

To reduce confusion in the rest of this paper, I shall call Newton's color space C_1 , the vectors in this space, colors. The space of the colors we see I shall call C_2 , the vectors in this space, COLORS.

In summary, color space, C_1 , can be described as a three-dimensional, uniform and homogeneous vector space. Three arbitrary reference vectors can be calculated (as one of a set of triads) by matrix inversion from lists of indistinguishable action spectra and the intensities at which they cannot be distinguished. The space is usually described in conical coordinates. Hue corresponds to θ , saturation to ϕ , and brightness to r . The chromaticity is defined by θ and ϕ .

Thus chromaticity is well-defined. Brightness is a no more difficult matter in C_1 . On a matching basis, where the test color must be matched both in chromaticity and brightness, r is linear. Brightness, used in this way, is contrary to common usage,

for I distinguish it from BRIGHTNESS in C_2 , which is a far more complex quality to discuss. Brightness, however, is not simply the incident flux of light on the retina, rather it is a function of that flux given by the colorimetric references — the three processes of Young's hypothesis.

I am a little reluctant to try to give a precise, general account of C_2 . Where θ in C_1 can be given in terms of hue, i.e., λ , or, in the case of purples, the ratio of fluxes for the two extremum λ 's, θ' in C_2 is given in terms of HUE as given by HUE names. Where ϕ in C_1 can be given in terms of saturation, the ratio of flux at a hue point to flux at the chromaticity of the action spectrum chosen as white for a sample of light of any spectrum, in C_2 SATURATION is the distance of a COLOR from ACHROMATICITY. Finally, where r in C_1 is brightness, r' in C_2 is BRIGHTNESS, and by this I mean what is called, usually, "lightness." All three parameters in C_2 are subjectively absolute judgments; but insofar as they can be related to C_1 , the sensory space, they are order judgments, i.e., they are relative. From the subjective point of view, furthermore, it is as if the coordinate system of C_1 is not that of C_2 but that some correspondence exists between them. That is to say, the COLORS are definite and differ only by HUE, SATURATION and BRIGHTNESS between a variety of patches that do not differ in texture or other surficial quality, and these three parameters are, subjectively, laid out not in a conical coordinate system, but in a spherical one.

To make reading easier, henceforth I shall use lower case for color names in C_1 , such as "blue," to indicate θ , rather than, as is more proper, some λ ; and I thus set them off from, e.g., "BLUE," the name of a COLOR that I see.

The carrying of C_1 into C_2 is further confounded by the existence of COLOR names that do not correspond to colors. For example, BROWN is a color that we distinguish from YELLOW rather more strongly than we distinguish a dark RED from a bright RED. But BROWN is the name for a YELLOW that is much less bright than other COLORS around it. It is a contextually judged COLOR, as you can determine for yourself by Helmholtz's test of looking only at a BROWN patch through a tube coated black on the inside. It looks yellow then.

Newtonian color space, C_1 , does not require that white be given as equal flux through the spectrum. Any flux distribution can be called white, or the achromatic axis of the conical coordinate system. All of the spectral points along the boundary can be given their λ numbers rather than color names, and the points along the nonspectral line can be given "complementary" numbers, obtained by the intersection of that line with another line produced from a particular λ through the white point (described as a particular flux distribution). Thus all identities could be established without using COLOR names.

Indeed, the method used for constructing C_1 must not use COLOR names. This method, called colorimetry, is substantially as follows: A subject is confronted by a large GRAY wall through which two holes are cut. Behind each

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hole at a short distance is a diffusely reflecting white surface (i. e., it has uniform reflectance across the spectrum), a field. One field is illuminated by a particular color of light. The subject is then given intensity control of three non-coplanar chromaticities of light (i. e., no combination of any two will look like the third). He may add the three together on the other field, or he may take any one of the three and add it in varying intensity to the field illuminated by the test light. The object of the manipulation is to make the two fields indistinguishable in respect to COLOR. This can always be accomplished by a unique combination of colors (a unique combination of the intensities of the given chromaticities). If only two chromaticities are used for matching a third, most matches cannot be done. If four chromaticities are used for matching a fifth, the combination for matching is not unique. Hence, C_1 is three-dimensional.

What is interesting, subjectively, is that when a color match is done, the COLOR of the match may be ambiguous. The uniform field of COLOR if yellow, may appear YELLOW or ORANGE or YELLOW-GREEN. It fluctuates, but the matching of the two halves is not destroyed. The identity of the match is not affected by the absolute COLOR that it evokes. All of what I have said thus far follows from Grassman's laws.

1. Any mixed color, no matter how it is composed, must have the same appearance as the mixture of a certain saturated color with white.
2. When one of two kinds of light that are to be mixed together changes continuously, the appearance of the mixture changes continuously also.
3. Colors that look alike produce a mixture that looks like them. (I add: When the mixture is reduced to the same brightness as the component colors.)

The reason that HUE names are used for hues is, of course, obvious. Spectral lines at the longest λ 's most often appear RED, etc., so that we can ordinarily talk of the red end of the spectrum, or the blue end, and not be misunderstood. But, e. g., BLUENESS, as will be shown, does not depend on blue light being present.

Defects in color vision are of several types. The simplest ones are dichromacies wherein it appears that one of Young's processes is absent. Dichromats can accomplish most color matches with two spectral lines as the chromaticities that are varied in brightness.

Another type is the anomalous color-seer who may have the three processes, but one or more of the processes does not have the action spectrum that is most common, or else the processes are not combined in the normal way. The matches made by such a person will differ from the normal: his C_1 will have a changed boundary. Such people can, for example, distinguish colors that are metameric for most others – as happened in the recent Arab-Israel war when the protective coloration of Egyptian forces was penetrated by an anomalous Israeli bombardier. (At the same time we can distinguish many of the spectral distributions that they cannot distinguish.)

b. Purpose of Color Vision

The second comment that I must make before going on to Land's experiments concerns the visual advantage that shapes color vision. In a world of generally opaque objects seen by changeable illumination, the invariant by which we could track a surface from one place to another, from the yellow light of noonday sun through the blue light of the sky when only the sun is overcast, from the red of setting sun through the green shadows of leaves, is not the light that is reflected from the surface, but rather the reflectance itself. That is, if we could know both the spectral composition of light incident on a thing, as well as that reflected from that thing, we could make a judgment that would be largely independent of the accident of illumination.

It is well known that our perceptions of the COLORS of objects in a welter of objects is remarkably stable under change of lighting. The COLORS of sweaters, frocks and other artefacts, subtle skin tones, facial makeup, etc., do not change markedly as one goes from outdoors into a tungsten-lit or even a fluorescent-lit room, although the colors change. If, in a tungsten-lit room we change the chromaticity of the light by filters that only bias the distribution of flux but preserve the wide band, we are fairly insensitive to the resulting change in chromaticity of the surfaces seen by that light. In Helmholtz' words it is as if we "subtract the illuminant." More exactly, it is as if we treat the illuminant as WHITE, as the middling COLOR between all the COLORS seen. This phenomenon is called "color constancy."

It is quite true that when we go from broadband light to monochromatic light, or light of very few spectral lines, noticeable changes begin to occur when the action spectrum gets very rough, but over an extraordinarily wide range of action spectra with a smooth contour, COLOR constancy does hold.

Subjectively, then, we see things as were we sensitive more to their reflectances than to the light reflected from them, and, at first guess, we might suppose that we are also sampling the illuminant so as to compute reflectance automatically. This cannot be true, however, for we neither look at the sun nor at light bulbs to achieve color constancy.

Referring chromaticities (colors independent of brightness) to the color triangle we do note some alternatives. If the reflectances and illuminants are relatively smooth functions of the spectrum, then, for a given set of uniformly lit reflectances the reflected lights from which shown as a cluster of points inside the chromaticity triangle, a change of common illuminant moves the cluster about inside the chromaticity triangle, condensing it in one direction or another, but always preserving "mutual order." Our COLOR constancy reflects either that we are sensitive to the "mutual order" of the points or else that we do a barycentric operation on the point cluster, call the center of gravity white, and see colors according to some rule that takes θ into θ' . But a quality more related

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to reflectance than to reflected light is what is embodied in the "mutual order."

(Disruptions of the order can, obviously, be done by the use of "notch" filters, ones that exclude a band of light. A very interesting étude is to get a sheet of cellophane that excludes from yellow-green to green or green-blue, and walk outside looking at grass and yellow-painted trucks, etc. Grass looks red, some yellow objects look red, others white. It is possible, if you use metamers with rough-action spectra for protective coloration, to break that protection with such a notch filter, or even a simple "highpass" or "lowpass" filter with a steep step gradient, e. g., such as exists with the oil droplets in bird eyes.)

Suppose that we take it as an engineering problem, that we are confronted by an array of patches of colored paper lying helter-skelter overlapping each other. Let this array be illuminated in such a way that we have no assurance that incident light is the same either in intensity or action spectrum from one broad region to another. About the only thing that we can say confidently is that the distribution of lighting is not correlated with the distribution of patches. Finally, we are not to be permitted absolute references of any sort. Of these stipulations, the last arises from the physiological and psychological studies over the last century. It is a most restrictive constraint. Question: How reliably can we track a patch as the array is shuffled and the illumination changes? It is a most restrictive constraint. In this problem we cannot use shape or size of a patch because in the shuffling different overlaps will occur. But we shall assume that all papers have the same texture.

Our initial attempt will, of course, be photometric. We will have restricted the engineer to an array of photocells imitating an arrangement of rods and cones, but put no stipulations on the cells. They can have a flat spectral sensitivity curve. Now the image of the patch array is cast on this surface. We can pass this image through filters each of which passes only one λ . There are at least two strategies that come to mind instantly.

1. We can divide the total flux at λ_1 coming from each patch by the area of the patch, or else average the response of the photocells inside the image of the patch if the cells are linear transducers. Then we can order the patches with respect to this intensity function. The trouble with this method is that the chromatic order of the patches will not be invariant under spatial variations in the action spectrum and intensity of the illuminant.

2. We can take the ratios of flux at λ_1 across every boundary using only the photocells close to a boundary. Since the spatial variations in illuminant are not correlated with patch boundaries, we take the local order of patches given by ratios of the patch to its bounding patches, and, having done this for all patches, set up the global order of all patches with respect to reflectance of λ_1 . This method yields an invariance of the order under any spatial distribution of intensity of λ_1 providing some flux of λ_1 is available from each patch. Background counts as a patch, and an isolated patch on the

background, of course, has a boundary. But if we have the ratios across the whole boundary of a patch reported point by point along the boundary, we must take all points reporting identical ratios and divide by the number of those points and do this for all different ratios across the boundary. Alternatively, we can report only the ratios of these ratios at those singular points, the vertices along the boundary, where the ratios change. This vertical operation is the easier, requires less "data processing."

The second strategy using boundaries is certainly preferable to the first using areas, and, for a minimal system, the vertical measure is preferable to averaging measures along boundary segments.

Having done this operation λ by λ for n wavelengths, we can now set up an n -dimensional ordering of all patches in the same way as they would be ordered if we knew the reflectance of each patch λ by λ . The more λ 's used, the more patches in the array and the more different reflectances they exhibit, the closer does the order derived from mutual comparisons of reflected light approach the order these patches would have in a "reflectance space."

Finally, if we now applied to this exhaustive measure the degenerative transformation that carries the spectrum into color space, we would have a chromatical ordering of the patches that would be relatively invariant under smooth local fluctuations of intensity and action spectrum of broadband illumination. Alternatively, and more simply, if we took the separate ratios of each of the fundamental retinal color processes across boundaries, measured the changes of these ratios at vertices around the boundary of each patch, and ordered each of them globally, we would have a tendency to chromatical invariance under changes in the color of the light and the shuffling of the patches. (This invariance would obviously be most marked if we use but three λ 's widely spaced in the spectrum so as to make ourselves insensitive to local roughness in the action spectra of the reflectances.) Such a global ordering process is what Land means by his "retinex," and such a profound invariance or CHROMATICITY constancy is what he shows with restricted illumination. It is important to remark here that COLOR constancy holds over a much narrower range than CHROMATICITY constancy.

You will note, however, that the preferred strategy, whether using λ by λ or using the three reference chromaticities of color space, violates the last condition that we gave at the beginning of the discussion (the use of an absolute spectral reference, we know which λ is which, or which reference chromaticity is which over the whole plane of the image) unless we import extensional information into "getting" C_1 to C_2 , given that C_2 is devised for determining invariances related to reflectances.

This extensional information involves, depending on the method you choose: 1.) averaging the color over an area (that must be defined) so that this, the color of that area can be compared with the colors of other areas; 2.) averaging the ratio of colors along a boundary (whose length must thereby be known) so that this ratio can be compared

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with other ratios; or 3.) taking the change of ratios at vertices along a boundary (whereupon a vertex must be defined as the intersections of boundaries, thus requiring boundary recognition). I only mention these obvious matters to point out that the sensation of REDNESS in a real RED SPOT (that stays stubbornly RED as we change the light by which we see it), if extensionless in quality, does not imply that it is generated without use of extension.

c. Land Experiments

I shall give Land's first studies in their simplest form initially. Suppose you have three bright light sources, say tungsten bulbs. And suppose you choose three filters, say Wratten Nos. 25, 95 and 47, passing red, green and blue respectively with but little overlap. Each light is covered by one of the filters, and there are no other sources of illumination. Now provide yourself with a set of highly colored papers which you arrange on a table so that they overlap each other every which way. If you now restrict yourself to viewing the array under each of the lights separately, you will discover that there is practically no variation of HUE. If you view only by light through filter #25, the array will be predominantly RED. Areas that reflect little if any light will appear a GREENISH-BLACK. Slightly lighter areas will appear GRAY, but most will be a variety of RED. This had been already remarked by Helmholtz. Similar effects will occur with light coming only through the #95 filter or the #47. Now, when you turn on light through all three filters, you will see a wide variety of HUES at different SATURATIONS and BRIGHTNESSES. These may not be the COLORS of the papers that you would see under white light, but then you have restricted the conditions of viewing greatly. What is important is that your COLOR judgments will have three degrees of freedom – to be quite exact, you will see various GRAYS, and WHITE and a BLACK as well as differently saturated HUES, if you have included achromatic papers in the array. Now you can assure yourself of the phenomenon of COLOR constancy – for if you have further provided a stop, or occluder (such as a sheet of cardboard) by which you can reduce the light from any of the three sources, or any two of them, you will see that over a wide range, the CHROMATICITIES are not dependent on the relative amounts of the different lights striking the array.

But now a miracle! Instead of partially occluding the light through filter #47 (blue), you turn off that light entirely. To your astonishment you will see very little change in COLOR. There will still be BLUES, PURPLES, WHITE, GRAYS, BLACK as well as REDS and GREENS. In other words, from a coplanar set of points in Newtonian color space you are getting sensations that are trivariantly distributed in C_2 . For surely BLACK and WHITE are not coplanar with all three, RED, GREEN and BLUE, in perceptual color space.

This one experiment, done cheaply, for the filters are inexpensively got from

camera stores, instantly wrecks the hopes of finding a rigid transformation that carries colorimetric color space (Newtonian) into perceptual COLOR space.

Land's next experiment is more difficult to perform. It requires somewhat more expensive equipment. Take two step-wise gray wedges (where every stripe as you move from left to right is some multiple as dark as the preceding stripe). These can be made photographically from ordinary black and white film. Take two slide projectors and, having registered their projections on a screen, place one wedge in the first projector so that the stripes run horizontally, and the other wedge in the other projector so that the stripes run vertically. The resulting image of an array of squares will have a bright corner and, diagonally opposite, a dark corner. Now place a red filter, say #25, in front of one projector, and a neutral gray filter, as compensation, in front of the other. The resulting image is not remarkable. The bright corner is PINK, diagonally opposite it the dark corner is BLACK, one of the other corners is WHITE, and one is deep RED. The squares vary in REDNESS through different tints and shades to WHITE and BLACK. The shades may even look a bit PURPLISH.

You have prepared, however, two other pairs of slides. In one pair you use the same gray stripes as in the first and unremarkable set, except that you have randomized the stripes in each. They do not go from light to dark in an orderly way. When you insert this pair in the projectors, crossed in the same way as the first pair, you are suddenly confronted by COLORS – REDS, YELLOWS, GREENS and BLUES. They are not vivacious, but they are there.

In the final pair of slides you have randomized the squares in the registered image. This is difficult to do. It means cutting up each stripe into squares on each slide and distributing the squares randomly in a square array. But the result is worth it. The registered images from the two projectors yield squares in a riot of COLORS.

You will note that in this experiment every combination of relative intensities of red and white light that occurs with one pair of slides occurs with the others. All that differs between the slide pairs is the degree of spatial order applied to the combinations. Why is it that so little COLOR is provided by the first pair, so much by the last? To answer that this is a case of "contrast enhancement" or "contrast induction" or "simultaneous contrast" is to substitute one mystery for another.

To continue on Land's studies, it is an obvious application of the first experiment mentioned to photograph a scene on one piece of black and white film through a red filter, and then photograph it on another piece through a green filter. The projection of the resultant first photograph through a red filter and the second through a green filter with registration of the projected images yields a fully-colored picture. As a result of the second experiment one can say that colors ought be equally present if we leave off either the red or the green filter in the projection. Land then shows that the HUES are invariant whether one doubles (uses two identical copies in series) either of the two

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slides projected. Thus, what he establishes is that under these circumstances HUE is given by rank orders rather than by exact ratios. In a last experiment Land uses in his projectors two monochromatic yellows, 1 μ apart, the shorter wavelength in place of the green light, the longer in place of red, and one can see dimly all the HUES – in pastel or washed-out water-color SATURATION, but definite.

The display of HUE constancy under restricted illumination is one of Land's most spectacular shows. Consider the colored patch array under red, blue and green light. We saw that turning off the blue light did not much alter the HUES seen. Now, too, we can vary the ratio of red illuminant to green illuminant over a wide range and still the HUES do not change. Thus, if patches A and B are different COLORS under one illumination ratio, and the illumination ratio of red light to green light is then changed so that the light that comes from B now is what came from A before, or vice versa, patches A and B are unaffected in HUE by either change. Thus HUE in this case is quite independent of the absolute action spectrum and utterly dependent on the relations between the action spectra of contiguous areas.

The vividness of the COLORS seen in this case where the absolute action spectra do not independently determine color leads us to ask why COLORS are so predominantly connected to colors in ordinary experience.

The major paradox brought out by these études has been mentioned. Furthermore, it is lawful in this sense, that everybody with normal color vision confronted by these pictures names the seen COLORS in the same way. The COLORS are not ambiguous.

There are many changes that can be rung on Land's theme. One can seek minimal configurations. Such experiments are tedious because temporal boundaries (blinking the eyes) can be exchanged for spatial boundaries vis-a-vis COLOR, and so, too, can afterimage boundaries, as we know from comparing simultaneous and successive contrast; nevertheless, these configurations can be made; four COLORS that are distinctly not coplanar can be assigned to a configuration of only four areas, the background serving as one of them.

d. Some Interpretations Made by Land and by Yilmaz

Land's demonstration controverts any theory, such as that of Yilmaz, that tries to account for the production of all HUES from the use of two chromaticities by means of a rigid transformation of C_1 to yield C_2 . Yilmaz supposed that the coplanar set of all colors in the field of vision was tilted (a translation and rotation of the coordinate system of C_1 by a Lorentz transformation). In his theory, however, one of the three variables, BRIGHTNESS, HUE, or SATURATION, will be dependent on the other two globally. Furthermore, in Yilmaz' model the rotation of the coordinate system depends on color at every point in the image. That is, the COLORS become area-dependent, for he explicitly remarks that the COLORS are weighted by area.

A theory of "retinexes" has been put forth by Land. It has some good features and, at first sight, seems to save some of the phenomena. Briefly, it is this: Suppose, using only black and white film, you photograph a scene with a filter of chromaticity A, say red, and then do the same with a filter of chromaticity B, say green. If you now examine the first photograph, you will, on inspection, be able to order the areas in subjective BRIGHTNESS, called by Land, lightness. So, for example, a white bit of paper in the shade of a tree will have a lightness (BRIGHTNESS) greater than a gray bit of paper in the sunlight, although the amount of light coming from the first may be much less than what comes from the second. Lightness, done as a purely subjective judgment in full context, can be written down as the order of areas from light to dark for A, and then can also be written for B. Land, then, says that if there are three processes in the eye, red, blue and green sensitive, each process acts as an independent sheet, like a photograph, and you, behind the three sheets, take the red lightness, the green lightness and the blue lightness, and from this get a three-dimensional order. But you will note that while this seems to explain the results of presenting an array of squares randomly colored with two chromaticities as opposed to the same array spatially ordered with respect to both chromaticities, it does so by a deus ex machina, a judger of "lightness" sitting behind everything, who takes contexts intelligently to issue his judgments. But, what is more to the point, it does not get us around the problem of how one actually goes to a trivariant sensation from a divariant stimulus except by arbitrary laws — such as, the rules for taking lightness differ between the green retinex and the blue retinex. Recently Land has been looking at the effects of boundaries in ordering "lightness."

e. Contrast Phenomena

The experiments just described have been taken as only a special case of what has been known as simultaneous contrast for over a century, and, in general, treated most extensively by Helmholtz. Even when he is not explicitly mentioned, it is his argument that is used, and rightly so, for there has never been a more astute and critical psychologist. Helmholtz has magnificent chapters on Contrast (Chapters 23 and 24) in Vol. II of Physiological Optics. I know of no better handling of the phenomenology anywhere, and, unless one is driven to consult for other reasons the equally admirable essays of Purkinje, Chevreul, Fechner, Plateau and other natural philosophers of the 19th Century, Helmholtz' essay suffices for the argument. The experiments he describes are simple and revealing. In this one section he discusses and explains in part what, much later, were called Mach bands. He notes and explains in part the fading of stabilized images on the retina, and in general gives so rich and original an account that psychologists have still not mined it to the limit.

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It is tempting to launch directly into making a model for C_2 on the assumption that everyone has read the Physiological Optics. But, unfortunately, this is not the case, even among contemporary color theorists, since many have accepted, in the main, his conclusion that pure simultaneous contrast, the essence of Land's demonstrations, is attributable to a "change, not of sensation, but of judgment." Helmholtz felt that certain contrast COLORS arose from a Kantian judgment, an unconscious decision on evidence rather than from an obligatory color-COLOR transformation in perception. Thus, some psychologists even now talk of "memory colors," etc., as if thereby to explain Land's findings. But this is nonsense, since some of the demonstrations use only polygons, and there is no intrinsic reason why everyone ought "remember" that an L-shaped patch ought to be, say, RED as an apple is. What seems to be remembered are Helmholtz' conclusions but not the arguments and evidence. Therefore we must review them. They contain an oversight that led to error.

The experiments that I will now mention occur on pp. 271-293 in the English version, Southall's translation. All can be done out of hand with minimal equipment.

2. Colored Shadows

1. Take two ordinary lamps, play them on a white surface from different angles. Put a colored filter over one of the lights. Introduce a pencil upright on the white surface. Of the two shadows cast, one has the COLOR of the filter, the other has the complementary COLOR. So, for example, if you use a red filter, the shadow it casts is a bright, vivid GREEN. If you turn off the unfiltered light, the GREEN shadow goes BLACK. As soon as the unfiltered light is turned on again, that shadow goes GREEN, and no movement of your eyes, no application of your will to see it otherwise, makes it aught but GREEN. (If you introduce a green filter over the unfiltered light, the shadow becomes a deeper GREEN.)

2. Now replace the red filter by a blue one. The colored shadow is YELLOW – very YELLOW. (If you add a red filter to the unfiltered light, the shadow turns ORANGE.) And so it is for all colored filters on one light – the addition of white light from the other lamp to the field turns the shadows the complementary color. Nor need the filters be brightly saturated. The effects are quite as strong with quite low saturations. And the COLORS of the shadows do not vary much as the relative intensity of the illuminant is changed.

3. Put a tube to your eye with, say, the induced GREEN shadow first mentioned and look only at the boundary of shadow and field. The GREENNESS against the PINK background is still clear. Now after looking at the boundary for a while, bring the tube to look but at the GREEN shadow and turn off the light behind the red filter. That place – now not shadowed, now only a white surface under white illuminant – persists as GREEN for a while, or until you remove the tube from your eye. What Helmholtz did not say

is that it does not matter if you move the tube over the white surface after you have turned off the red light – it looks equally GREEN everywhere and fades equally rapidly over where the shadow wasn't as over where it was. These effects last a few seconds, long enough for you to turn switches on and off comfortably. It was the persistence of the uniform GREEN over an area illuminated only by white light (the shadow) where the contrast-inducing color had been screened out by the tube a few seconds earlier (moving from the shadow-field boundary to the shadow alone) that led Helmholtz to feel that the GREEN was remembered—therefore was not "sensed" but "judged." Thus he illegitimately ruled out the GREEN as an afterimage of simultaneous contrast.

3. Paper Patches behind Diffusers

1. Against a bright red sheet of paper seen by reflected light, lay in the center a patch of gray paper of about the same BRIGHTNESS as near as you can judge. Overlay the array with another white sheet of paper so that the red paper gives a pinkish tinge to the overlying sheet. Then the gray patch is seen as GREEN.

2. Outline the patch delicately with a fine pencil line. The GREEN vanishes, the patch turns GRAY.

3. By isolating the patch seen through paper, using a narrow black tube, pick out from a series of gray papers one that matches the patch seen through the diffuser. Cut it out in the same shape as the patch and overlay it on the image of the patch through the diffuser. It is not GREEN.

4. Bring any patch of any sort of gray to bound both pink background and induced GREEN. The GREEN vanishes or desaturates.

From these findings Helmholtz concluded that if we supposed the superjacent paper to be uniformly pink, we judge that the underlying gray patch had to be GREEN to appear GRAY on the surface, and therefore, judging the patch as GREEN, we see it as GREEN.

4. Paper Patches Alone – Not Explicitly in Helmholtz

Suppose in the first experiment of A, leaving the white light and red light fixed, you match the red light-white light mixture on the field to a pink sheet of paper under the white light alone. Similarly, match the GRAY in the shadow with a gray patch of paper under the white light alone. In other words, match the field and shadow separately as close as you can to a pair that provides the same colors under white light as the shadow-background gives. Now cut out the gray paper in the shape of the shadow, lay it against the pink paper and illuminate only with the white light. The gray looks GRAY, not GREEN, no matter how closely you have matched the values unless the gray patch is relatively small and the pink paper very large. But when the areas are constrained by the black tube, then we can always tell one from the other. Indeed, we knew this before doing the experiment. For if, no matter what filter we used, the shadow was colored, then certainly some patches of gray against some colored papers ought to appear

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complementarily colored under white light. But they don't (except in the diffuser case)!

Thus, given two pairs of identical colors and two identical spatial configurations, that the COLORS should be different made Helmholtz think, quite justifiably it would seem on first glance, that the difference occurred because of a judgment not related to color at all – namely, our guess as to the nature of the objects – one being a shadow and the other a patch on the background. Even when I put the tube to my eye so as to see only a segment of shadow or patch and a segment of the bounding background, and the COLORS are different though the colors are the same, then, most of all, Helmholtz would seem to be right. But what if someone else makes the arrangement and still I see a difference?

Here, where we are faced with the same paradox as arises from Land's use of simultaneous contrast, it almost seems that we must now reluctantly opt for the "judger" using information not supplied by the image vis-à-vis color.

Let us revert back to the early pages of Chapter 24. We must read these paragraphs of Helmholtz in the light of a prior discussion of afterimages. When you look fixedly for a while at, say, a bright red spot against a white background and then close your eyes sharply, the instantaneous feeling is that the image persists though the eyes are closed – the spot thereafter persists a short while as RED (longer if the red spot has been very bright) then fades and at the same time turns GREEN. Very soon you are not aware of any sensation. But the spot is still there latently. For, if on your closed eyes you play a light suddenly, the spot stands out as GREEN, then fades; if you turn off the light, the spot suddenly appears as RED, then fades, and this can be repeated. Alternatively you can open and close your eyes while facing a featureless white surface. These afterimages slowly fade (unless, like Ritter or Plateau, you gaze fixedly at the sun and burn a hole in your retina). The afterimage then can be induced for many days. (A reflected laser flash from an absorbing piece of black velvet once gave me an afterimage evokable for over half an hour; I was quite worried.)

These are also contrast phenomena and play a part in the induction of color across boundaries. Helmholtz' description is vivacious, and I shall quote a long passage from it:

"The phenomena of successive contrast, which will be considered first, are easily comprehended from what has been stated in the previous chapter. After looking at a field of colour A and medium brightness, suppose the eye turns to look at another field of colour B. Then as a rule, the residual stimulation of the impression A will not be strong enough for a positive after-image to be projected on a second field of medium brightness; and so there will be a negative after-image of A upon the field B. Thus those parts of the colour B that are like A will be diluted. If B is of the same hue as A, it becomes whiter by contrast; if it is complementary, it becomes more saturated. If it lies on one side or the other of the colour circle between A and its complementary colour, it changes into an adjacent hue farther from A and nearer the complementary

colour. Incidentally, the brighter A was, the darker B looks. Accordingly, this would be the general law of successive contrast, on the supposition that the luminosities of the two fields were such that only negative after-images could occur.

Even in comparing coloured areas with each other that lie side by side in the visual field, successive contrast, that is, contrast caused by after-images, is a very important factor, as any one can easily verify. It has generally been supposed that in these cases it was simply a matter of simultaneous contrast, because hitherto in the theory of contrast little account has been taken of a certain characteristic of human vision. Under ordinary circumstances, we are accustomed to let our eyes roam slowly about over the visual field continuously, so that the point of fixation glides from one part of the observed object to another. This wandering of the eye occurs involuntarily, and we are so used to it that it requires extraordinary effort and attention to focus the gaze perfectly sharply on a definite point of the visual field even for 10 or 20 seconds. The moment we do it, unusual phenomena immediately take place. Sharply defined negative after-images of the objects develop, which coincide with the objects as long as the gaze is held steady, and hence cause the objects soon to get indistinct. The result is a feeling of not seeing and of having to strain the eyes, if we persist in trying to look at the fixed place; and the impulse to move the eye becomes more and more irresistible. The little deviations of its position are scarcely noticeable in the strain, but they are revealed by parts of the negative after-images flashing up on the edges of the objects, first on one side and then on the other. This wandering of the gaze serves to keep up on all parts of the retina a continual alternation between stronger and weaker stimulation, and between different colours, and is evidently of great significance for the normality and efficiency of the visual mechanism. For nothing affects the eye so much as frequent development of negative after-images caused by staring a long time at surfaces even only moderately illuminated. Strong negative after-images are, indeed, always an indication of a high degree of retinal fatigue.

Now let us consider what happens when the eye wanders in this way over a field where there are different colours or areas of different luminosity. If we observe a limited coloured field with the eye accurately focused on some point of it, a sharply defined after-image will be developed, which is therefore easily recognized. If two different points of the object in the same line of sight have been observed for a long time, two well defined after-images will be formed partly overlapping each other; but without special attention they are not now easily recognized as being copies of the object. But if the gaze has moved slowly over the object, without being held on any point, naturally the after-image will be simply a faded spot, and it is no longer so easy to recognize, although it is actually there for the attentive observer. Now if the look is transferred to an adjacent field of another colour, this colour of course will be altered by the influence of the after-image, exactly as if we had had these different colours one after the other in the field of vision. Accordingly, in a case like this, we do not have simultaneous contrast, at least not by itself; but we have here also successive contrast, and the phenomena are entirely, or in large part, identical with those described in the preceding chapter. In order to have simultaneous contrast alone, special pains must be taken to keep the fixation of the eye absolutely steady during the experiment.

Later we shall examine more carefully the phenomena of pure simultaneous contrast which continue during steady fixation of the eye. Now the phenomena will be described that belong partly to simultaneous, but

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mainly to successive contrast, as they are manifested under ordinary natural conditions of vision. The colour changes that occur in these circumstances are exactly the same as those already described for pure successive contrast. In general they are much more distinct and striking than those of pure simultaneous contrast; and when the two might cause different results, those of successive contrast invariably predominate in the natural use of the eye; and when both evoke the same effects, the alterations of colour always become much more considerable when the gaze ceases to be steady and begins to wander.

In general, contrast effects are promoted when the inducing colour is more intense than the reacting one, because then the after-images of the former are more vivid and more lasting. For example, if a small wafer of white paper is laid on a coloured sheet, this white will have the complementary colour. The colouring is more impressive, however, when grey is used instead of white; or even black, since in these subjective experiments all black is to be considered as a dark grey. However, as a rule, a medium grey is more satisfactory for the experiment than black. In such cases the contrast action may go so far that a tolerably vivid colour is reversed into the complementary. For example, if a small piece of orange-red paper (coloured with red lead) is laid on a red glass disc and held up against the bright sky, the reddish paper looks a vivid green-blue, that is, complementary to the colour of the red glass, being almost its own complementary colour too.

Moreover, it is conducive to have the inducing colour occupy a large part of the visual field, because then the various regions of the retina will be frequently and continuously stimulated by this colour and fatigued by it. The result is that the contrast colours are particularly vivid when the reacting colour occupies a small field surrounded by an extensive ground filled with the inducing colour. In this case, it is chiefly simply the colour of the small field that is altered, not that of the large field. But the contrast effects are not absent even when the two fields are of the same size; the influence then being a mutual one, and the colour of each being changed by that of the other.

Finally, the nearer together the inducing and reacting areas are in the visual field, the greater will be the contrast effect; because when the eye glides from one space over to the other, the after-image will be more strongly developed the sooner the gaze encounters the other field. This is shown very strikingly in the arrangement which Chevreul has selected for his experiments. From each of two colours, say yellow and red, he cuts out two similar bands and places them side by side close to each other. Let us call them \underline{Y}_1 and \underline{R}_1 . Then next the yellow band \underline{Y}_1 he lays a second yellow band \underline{Y}_2 at a little interval, and in the same way next the red band \underline{R}_1 another one \underline{R}_2 . In this case the contrast action is not manifested anywhere except at the two middle bands \underline{Y}_1 and \underline{R}_1 . The yellow of \underline{Y}_1 becomes greenish by approaching blue-green that is complementary to \underline{R}_1 , and \underline{R}_1 looks purple by being admixed with some indigo-blue that is complementary to \underline{Y}_1 . On the other hand, the two outside bands \underline{Y}_2 and \underline{R}_2 are not altered in appearance, so that there is a good opportunity of recognizing the contrast action. When the fields in contact are somewhat wider, this is also precisely why the contrast colouring is manifested particularly at the margins. Every time the eye sweeps from one field over \underline{A} into the other field \underline{B} , those parts of the retina that have just left the field \underline{A} will be most fatigued by

the colour A; and these are the places where the image of the edge of B falls now. Those parts of the retina which left A a little sooner and have already moved farther into the field B will be less fatigued; and hence for them the induced colour is not so strong. Consequently, every time the eye passes over the field B, the marginal parts of B are most altered by contrast, and the parts farther from the edge less and less in proportion to their distance away. Thus, for instance, when a green and a blue field are in contact, the edge of the green looks a little more yellowish than the middle, and the edge of the blue a little more violet than its middle; because in the first case there is an admixture of yellow that is complementary to blue, and in the latter case an admixture of purple-red that is complementary to green. The play of after-images at the border of such surfaces can be watched very nicely by marking several points of fixation, and jerking the eye from one to the next, after holding it at each place for a brief time. It is easy to see then the well-defined after-images moving over on the other field. The earlier images, being shifted on ahead, will be paler, while the latest, lingering next the border, will be more intense.

If the question involves not difference of colour, but difference of luminosity, the reacting field will appear to be less bright when it is adjacent to an inducing field that is brighter than it; whereas next to a darker field, the luminosity of the reacting field will seem to be increased.

Incidentally, as compared with the methods of seeing negative images which were described in the preceding chapter, there are also other factors in these experiments that are conducive to eliciting the complementary colour. In general, a coloured object has to be deliberately focused for several seconds in order to obtain afterwards a distinct after-image that will persist for some time on a uniformly coloured ground. But in the experiments on contrast it appears that a tolerably cursory observation of one colour is sufficient to induce the complementary colour on the other field, and that this complementary colour is afterwards much more lasting than an after-image would be which was obtained under the same circumstances. In order to recognize an after-image on a uniformly coloured ground, it must be well developed and clearly outlined. It moves about as the eye moves, and so has to be perceived as any other subjective phenomenon. Ordinarily, we pay attention only to objective visual phenomena. But if a faded after-image covers a smaller coloured field, which has its own objective limitation and always appears under the influence of the after-image, this influence cannot be immediately separated in the perception from the other objective phenomena of the visual field, and hence it becomes much more easily an object of our attention."

There are many games that one can play with the after-COLORS of successive contrast. But the most revealing are those that imply simultaneous contrast too, and here are two major simple experiments:

1. Take a white disc or stripe against any extended uniformly-enhued surround and use a fixation point in the center of the white. The white is little, if at all, COLORED by the surround — there is almost no simultaneous contrast. Now, after looking at the fixation point for a while, close your eyes. The negative afterimage of the white is not black, but almost exactly the color of the inducing surround. Once when the sun was occluded by a translucent cloud and the rest of the sky had a white cloud cover, I tried

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to induce the sequence of afterimages of the sun that Helmholtz discussed. I stood at a window where my wife had put up translucent and highly saturated red-purple curtains. These curtains had been drawn aside but not beyond my peripheral vision. The negative afterimage, to my astonishment, contained the sun, as expected, but as though I were seeing it on the background of the curtain material.

2. Burckhardt did an even more revealing experiment. If one uses a large colored disc concentric with a small white disc, the afterimage of the white disc is the COLOR of the larger disc. If one splits the large disc equally between two colors, the afterimage of the white disc is the COLOR of the equal mixture of those colors. Thus the afterimage of the white area is the COLOR of its bounding COLORS taken aliquantly.

We are now faced with a further puzzle. Reverting back to the case where we compared the images of the simultaneous contrast and the contrast of papers under white light, we now arrange two annular displays. In one case we play blue light through a projector with a slide in it containing a black spot. On the shadow of this on the screen we can play a spot of white light varying the intensity. In the other case we mount a large disc of blue paper. On it we place a small gray disc and illuminate with white light. We can now vary the lights at will and come to what is almost a match between the two annuli. Nevertheless, at arm's length, the two situations differ. The center of the annulus of blue light is certainly vivid YELLOW. The center gray patch on blue paper under white light shows YELLOW edges, a la Helmholtz, but the center never turns the beautiful uniform clear YELLOW of the other display. Yet in my experience the afterimages set up by the two displays are very similar. Both centers are BLUE, then, both annuli are YELLOW.

Thus the two situations are similar in this sense, that both displays are similar in chromaticity and similar in the after-CHROMATICITIES induced. But they differ as to CHROMATICITY on direct viewing.

More and more we begin to tend toward Helmholtz' dictum:

"In these last experiments the contrast action no longer depends simply on a definite distribution of colours in the field of vision. We have seen that this effect can be exactly the same with two different simple modifications of the experiment, and yet in the one case the contrast effect appears, in the other it does not. The moment the contrasting field was recognized as an independent body laid over the coloured ground, or was even divided off enough by something to indicate that it was a separate field, the contrast was absent. Accordingly, since the judgment of the position in space, i. e., of the corporeal independence of the object in question, is the decisive factor in the determination of the colour, the consequence is that the contrast colour here is not due to an act of sensation but to an act of judgment. The nature of this act of judgment by which we reach the perception of objects with definite characteristics will be more accurately described in Part III (VOL. III). As the acts of judgment here spoken of are always executed unconsciously and involuntarily, naturally it is often hard to determine what chain of impressions is responsible for the final results, and in

the nature of the case very different circumstances may affect it."

In general, as he pointed out, these simultaneous contrast effects are most striking if the two areas are quite close in luminosity (brightness) and differ only in chromaticity.

There is, however, a tertium quid.

a. Simultaneous Contrast at Vertices

Let us now do a few more easy experiments, variations on the theme set above. Make yourself a projector of any sort that produces a sharply bounded uniform disc of white light. Now set up a wide field with another colored light, say with a blue filter over it: let it be collimated by a lens so as to cast a sharp shadow, and play this colored light on a white piece of paper, using a dull or black object to cast the shadow. This shadow will appear quite DARK. If it has a CHROMATICITY, this can't be seen. Then play your disc of white light, suitably stepped down in brightness, into that shadow but away from the boundary. The disc has no distinguishable COLOR, despite you are now convinced after playing all day with simultaneous contrast, that it ought be YELLOW. Then bring the disc so that it is almost, but just distinguishably not quite, tangent to the inner boundary of the shadow. Still it is not definitely COLORED. By quickly switching your eyes from field to shadow you may transiently induce the possibility that the disc may have a YELLOWISH cast, but it is not very definite. Now bring the disc so that the boundary crosses it diametrically. Mirabile dictu, the half in the shadow is distinctly YELLOW, that half on the field is distinctly PURPLISH-PINK, and the background is split between BLACK and BLUE. Then move the disc of white light off the boundary in either direction. Back totally inside the boundary it is possibly just faintly YELLOW, totally outside the boundary it is definitely PURPLISH-PINK. The ambiguity of the COLOR totally within the boundary is much lessened, not only by moving the eyes from field to shadow but by switching the field light, the BLUE, on and off. However, what is more astonishing is that if you bring the disc from almost tangent to the inner margin of the shadow to where the shadow forms a just perceptible chord, the definiteness of the YELLOW instantly appears and stays.

You will note that in this experiment we have, by shadows, evoked a non-coplanar set of COLORS in C_2 , BLACK, BLUE, YELLOW and PURPLISH-PINK. The latter three are not colinear, therefore determine a CHROMATICITY PLANE, for not one is the result of mixing the other two, and BLACK is more DARK than the DARKEST of any of the three other COLORS.

If instead of an opaque object to make a shadow, you use a sheet of gray film that only cuts down the light through it by about 10X or less, then the shadow background is

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not BLACK but rather GRAY or GRAY-BLUE, the YELLOW turns YELLOW-ORANGE, the PURPLISH-PINK becomes PINKER and the BLUE stays BLUE.

You can play variations on this theme with different filters in chromaticity and grayness.

It seems quite a complex matter when we construct vertices rather than simple boundaries using colored shadows. Let us consider one such vertex made by intersecting two shadows. The rule is relatively simple and we shall see later how it derives from the facts of adaptation. There are four phases in the experiment given, the colored light in the shadow of the white source, the white in the shadow of the colored source, the shadows of both sources, the colored light and white light added together. Ordinarily in simultaneous contrast, using shadows, only the second and fourth combinations are taken. The former is then the COMPLEMENTARY COLOR to the latter.

Let us call the chromaticity of the colored light A , and of the white light W . The full light from both on the white sheet is $A_1 + W_1$, in the shadow for A , the light is $A_0 + W_1$, in the shadow for W , it is $A_1 + W_0$ and in the mutual shadow it is $A_0 + W_0$. The shadows need not come from complete opacities so that A_0 and W_0 may both be set up by simple attenuating neutral filters. If A is blue, then $A_1 + W_0$ is BLUE, $A_0 + W_1$ is YELLOW, $A_1 + W_1$ is PURPLISH-PINK (very DESATURATED) and $A_0 + W_0$ is DARK GRAY or BLACK. Of course, if W_1 instead of being white has a hue, other and even more vivid COLORS appear. It is worth playing in this way with COLORED shadows to get an insight into mechanism for the simplest case. For it is instantly apparent that the COLORS, whatever HUE for A , are not coplanar in C_2 ; the line in C_2 that joins $A_0 + W_0$ with $A_1 + W_1$ cannot but be skew to the line that joins $A_1 + W_0$ with $A_0 + W_1$, e. g., both DARK GRAY and BRIGHT, DESATURATED PURPLISH-PINK cannot be in the same plane as BLUE and YELLOW.

When we look more generally for the action of vertices in seeing COLORS, we come across cases fairly readily. Whoever has gone to an "Op Art" show knows that the contemporary colorist has gone in for doubly-bounded areas, concentric annular arrays of color or long, long parallel stripes of it. And indeed, that quality that they feel makes the painting "alive" is a kind of ambiguity or uncertainty that one feels on viewing such fields. The COLORS are not settled, they fluctuate. Most of this effect can be instantly abolished by putting a boundary, as with a piece of cardboard, diametrically across the annular arrays or stripes and attending to that boundary. If you try to produce the Land effects with annular arrays, the results are poor, and it will be that the two borders of each annulus will be differently COLORED so that the area does not have a single COLOR as when there are singly-bounded areas bounding each other.

5. Some Facts Leading to Explanation

There are two mechanisms (among a great many others that we have no room to discuss here) that enter into an account of simultaneous contrast; adaptation and

Mach bands. Adaptation is a variety of smoothing operation applied to the image. Mach bands are evidence for a kind of sharpening operation similar to, but not exactly, a second spatial derivative of some function of the distribution of flux.

a. Adaptation

Your ability to see a change in lighting, spatially or temporally, depends on the level of lighting itself. Thus, subjectively, objects that bound each other maintain their relative BRIGHTNESS as you change the level of lighting – just barely discriminable patches of GRAY at low lighting become slowly better discriminable as the lighting increases – but in general the lightness values are better described as were you sensitive to ratios rather than to differences in flux. This is reflected in our language. One thing is several times as BRIGHT as another – not two lumens (imagine a vernacular equivalent!) BRIGHTER.

When you are exposed to a uniformly bright field, and then plunged into a dark milieu, your ability to discriminate BRIGHTNESSES immediately thereafter is low, as if that light were still present, and, as it were, added to the light coming from objects. Your discrimination then improves gradually as if that light equivalent were fading. You have been light-adapted. Initially, however, your discrimination is as poor as if, while you were at an outdoor movie someone added his headlights to the image on the screen. This is why Helmholtz used the term "intrinsic light" in discussing adaptation. When you come to the best resolution you can in the crepusculation, you are then dark-adapted. Now if the lighting is suddenly increased, you are dazzled, everything is too bright to discriminate, as if your visual system were saturated. Very soon your discrimination returns to an optimum, and then you are light-adapted again. Your level of adaptation is defined in terms of the equivalent background light necessary to account for your ability to just see a test light.

There are two species of adaptation – one for the cones, and one for the rods. The cones dark-adapt much faster than the rods. Our color mechanism is predominantly cone-dependent. There are three varieties of cone, each containing one of the three pigments used in color vision.

Very little is known about cone adaptation as compared to rod adaptation. For the latter the best work in humans has been done by Rushton and his collaborators. Rods have a special pigment, rhodopsin, that bleaches with light. Rushton found that the absolute threshold for light in the rods was related simply to how much bleached rhodopsin there was per rod in an area of rods. He expresses the relation as

$$\frac{I}{I_D} = e^{ay/y_0},$$

where a is a constant, y is the amount of bleached rhodopsin, y_0 is the amount of rhodopsin, I is the absolute threshold for light under the bleached amount y , and I_D is

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the absolute threshold, i. e., the sensitivity to a step of light against a background field acted as if the incremental threshold could be given as a fixed ratio of $\Delta L/L$ for a wide range. Adaptation, thus, is not to be expressed simply in terms of how much pigment is bleached. Clearly other factors enter, nervous and chemical.

Rushton and Westheimer, working only peripheral to the fovea, then showed that if they produced a stabilized grating of light in sharp focus on the retina, thereafter the same level of adaptation, set up by the light where it hit, spread $1/2^\circ$ of visual angle away without significant attenuation. The angle of spread increased as they moved away from the fovea. Thus there is a spread of light adaptation for rod mechanism.

Despite the great dissimilarities of rods and cones, certain crude features are held in common between their effects. Cones also light-adapt and dark-adapt and there is also a spread of adaptation, although it is not a simple one. The time constants are subjectively easily appreciated. In the colored shadow intersection experiment just substitute suddenly another chromaticity B for A and watch the change of events.

Thus at a boundary I will take it that the level of adaptation, whether because of spread or eye-jiggle, corresponds to some operation like adding the colors that meet at the boundary to yield a reference color of adaptation there at the boundary and to either side. At a vertex it is all the colors that meet there that yield this reference color.

It was Helmholtz who pointed out that, in the absence of certainty (whatever that can mean) about the illuminant, it was this level of adaptation, called by him "fatigue," that we could take as the ACHROMATIC reference for the COLORS seen in the area fatigued.

Now, before we consider Mach bands we must look at some of the implications of what we already know about after-images.

Let us suppose you have fixedly looked at a spot of bright yellow light, a pure spectral line, in a darkened room. You have suddenly closed your eyes. For the first instant the YELLOW spot persists as had you not closed your eyes (the positive after-image), then, after a short indescribable period, it turns BLUE (the negative after-image). Then it fades. While your eyes are closed, the yellow spot has been expanded to a large and much dimmer area on the screen. It remains monochromatic. You open your eyes and suddenly see the afterimage spot as BLUE against the background. The spot fades – you close your eyes and it appears as YELLOW. This can be repeated several times if the inducing spot was very bright.

Now the blue process is very little excited by this light if it is excited at all. The dichromat who misses the blue process will not see BLUE as a negative afterimage. We are faced with the fact that to sense BLUE, the blue process must be present even if unaffected by the light. Thus the unbleached blue process must signal its presence and the state of its dark-adaptation relative to the two other processes. The same, of course, must be true for the other two processes. Thus our receptors must each have two degrees of freedom in their action on the subsequent retinal stages.

The afterimage BLUE adds with other COLORS. If you produce it by the yellow spot and look at a red screen, the negative afterimage looks PURPLE against a RED background. Thus, given a reference adaptation level, the impression of BLUER than it arises from the measure of less yellow than it, and so for the other processes. This, in every sense, is an opponent mechanism, and one does not need to recast the Young-Helmholtz trichromatic theory into the Hering account. Such a notion is, however, only a development of Helmholtz' idea of "intrinsic light."

Simultaneous contrast produced by shadows is not much affected by slightly defocusing the image as with a diffuser or with poorly-marked shadows, and this is true also for Land's pictures. But both it and successive contrast are sensitive to diffusion in the case of "real" boundaries. Thus we are brought to consider the arguments from Mach bands.

(Example — a gray patch of paper against a not-too-wide red background looks gray until you defocus, looking at it, or diffuse the image. The afterimage shows the patch as red, the background as green. If you use a red light on a white sheet of paper and have a patch shadow and throw some white light on the paper from a different angle, the patch is vivid GREEN. The afterimage is the same as for the "real" gray patch against a "real" red background.)

So far we have considered only the light coming from surfaces, and while boundaries have crept in, they have not been handled explicitly.

b. Mach Bands

Helmholtz remarked, and, of course, it had been known earlier, that if two slightly different BRIGHTNESSES of GRAY bound each other, the LIGHTER looks BRIGHTER at the boundary than over the rest of the area, and the DARKER is even MORE DARK at the boundary than elsewhere.

"Incidentally, it comes out plainly in the capricious results of these experiments, how hard it is for us to make accurate comparisons of luminosity and colour of two surfaces that are not directly in contact with each other and have no border between them. In the case of photometric methods we saw that the only certain and exact way of making the comparison was when there was nothing to distinguish the border between the two fields except difference of colour or illumination. The farther they are apart, the more inexact the comparison becomes; so that in such a case there is distinctly a wider latitude for the influence of accessory circumstances on our judgment of luminosity or colour. In the experiments which have been described the difference between the induced and inducing surfaces is brought out under the most favourable conditions; but the induced surface has to be compared with other surfaces lying off to the side in the visual field, so that this comparison can only be very imperfect.

This is shown still more plainly in the experiments now to be described, where the induced surface is in contact with two different colours on opposite sides. Then it will have the complementary colour on the corresponding edges. Or when the induced surface touches a darker surface on one edge and a brighter one on the other, the first edge will look brighter and the second edge darker. However, these contrast phenomena are likewise not distinct unless the only distinction

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between the inducing and induced fields is simply the difference of colour or luminosity, with no other border of any kind.

The experiments can readily be performed with transparent paper covers. Pieces of green and pink-red paper are fastened together so as to make a single sheet, half one colour and half the other. On the border line between the two colours a little strip of grey paper is attached; and over it all is laid a sheet of thin letter paper just large enough to cover it. The grey strip, where it touches the green, will now look pink-red, and where it touches pink-red it will look green. In the middle of it the two colours fuse into each other through an indefinite hue which perhaps is really grey, although it cannot be definitely recognized by us as such. The phenomenon is much more vivid when the length of the grey strip is oblique to the line of separation of the colours. Then the part of the grey that projects into the green may look just as vividly pink-red as the pink-red ground of the other side. The contrast colour is fainter, yet distinctly perceptible, when the middle longitudinal line of the grey strip is directly over the line of separation of the colours. Then the lateral edges of the grey appear coloured with a narrow border of complementary colour faded out towards the middle."

It was Mach who systematically showed that these effects were due to our sensitivity to the Laplacian of some function of brightness in the brightness distribution over an image. The effect is as if we take the value of brightness at every point in the image and compare it with the brightness of its surround weighted inversely by radial distance away from the point. In a word, a sharpening mechanism. Such a mechanism, given high resolution of the image on the retina and high sensitivity, should distinguish not only boundaries but the character of a boundary extremely well. Ratliff has translated Mach's papers into English and written a physiological, psychological and philosophical exegesis thereon. It is well worth the reading.

Various workers have studied the subtlety of boundary detectable by this mechanism. The most penetrating among them has been Békésy. O'Brien and Cornsweet separately have devised illusions based on Mach's work. In general, we are very sensitive to the actual curve describing the change of intensity with distance across a boundary. (This curve, you must understand, ought to be considered in view of the image-smearing due to the dioptrics of the eye. Westheimer has a stimulus star point imaged into a blur falling off to half intensity in several cones diameter under optimum focus and pupil diameter.) If we propose to construct a GRAY that varies smoothly from one place to another, we must keep the derivative of the local change of intensity everywhere minimal. Two grays of different BRIGHTNESS may be made to seem the same BRIGHTNESS by the use of "false" Mach bands. The BRIGHTNESSES can even be reversed over the brightnesses thereby. You must remember only that large values of the Laplacian are somewhat more weighted than small values.

What is most astonishing about O'Brien's and Cornsweet's illusions is that the value of gray for a patch is mostly determined by the Laplacian at its boundary if there is not too great difference of brightness across the boundary. Cornsweet's illusion is,

possibly, the easiest to produce. Against a uniform gray field a sharp difference between a darker and a lighter gray is established along a line bisecting the field and then shaded off with constant, or close to constant, second derivative of intensity to the gray of the field within about a centimeter or so.

If the two half-fields are several centimeters wide over the great majority of their expanse, they are the same in brightness save close to the boundary. Nevertheless they seem fairly different in BRIGHTNESS and uniformly so. Cornsweet used a whirling disc with a black sector to yield the uniform gray. He then picked a point midway up the radius on one side of the sector, and drew a small arc concentric to the disc part way across the sector, part way across the white. He then blacked in a short smooth curve on the outer annulus flaring out the black sector to the arc and whited in under a similar short curve on the central part, flaring out the white background into the black sector. When this figure is whirled, the uniform gray is interrupted by a sharp boundary between darker and lighter grays than the background, and both shade into the background. The result is startling. The inner disc is uniformly LIGHT GRAY, the outer is uniformly DARK GRAY away from the sharp boundary. This effect can be got, however, much more easily. Sharply crease a clean piece of white paper with your fingernail so that the crease remains after you open the paper up. Tack the paper, crease upright, against the wall so that the creased part projects outward and so that at about 1-inch to either side of the crease the paper is a flat surface. From there the paper curves smoothly to the crease. Illuminate it from a distant diffuse source at about a 45° angle to the wall. The crease toward the light is brighter than the background; away from the light it varies from deep shadow at the crease to background illumination. Stand back far enough, about 5-6 feet, so that local markings on the paper do not intrude, and you get the two GRAYS.

One additional experiment is worth doing. Intersect two creases, one completely across the paper, the other at right angles and only part way, extending from one edge of the paper to the crease. You can now induce three shades of GRAY with a proper rotational position of the sheet on the wall. You find that the sequence of BRIGHTNESS induced by these "false" Mach bands is independent of the color of light by which you view. Thus, probably all three processes in the retina, certainly at least two, exhibit this effect.

The false Mach bands can be used to give colors as well. The method of the crease, given opaque enough white paper (so no reflections come from the wall in back) and with both leaves coming to the same plane on each side of the crease, allows you to use two illuminants, say red and green lights, coming one from one side of the crease, the other from the other side, both playing on the paper. Then you see a PINK field to one side of the crease and a GREEN field to the other, and the effect is quite marked if the wall is of dark material. Now, too, you can use the three fields set up by the intersecting creases to give you three distinguishable COLORS with proper arrangement

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of the crease angles vis-a-vis the lights. This experiment shows that COLOR depends not only on what light comes from the surfaces, but what bounds the surfaces. This is a case of simultaneous contrast between identical areas identically illuminated save at the boundaries. True, the COLORS are not vivid, but they are there.

The two functions that have been adduced psychologically have their physiological counterparts. These two functions are a smoothing operation inferred from the nature of adaptation and a sharpening function inferred from the Mach bands. The former corresponds to a long spatial period operation, a lowpass filter across the retina sensitive to uniformities in brightness, relatively indifferent to sharp spatial changes in brightness except as they are accompanied by longer period changes (as if the image were defocused, and the more defocused when the higher the brightness). The latter corresponds to a short spatial period operation, a highpass filter, indifferent to uniformities and sensitive to spatial changes in brightness.

It is pleasant, then, to find in the optic nerve of vertebrates that the majority of fibers by far report as were they taking a Laplacian of some function of brightness. Some vertebrates have fibers that report a smearing function, or adaptation measure, as well, notably the frog. (Paper by Shin-Ho Chung and myself in preparation.) In mammals this adaptation can be detected as a modification of the action of the first sort of fiber either in average frequency and/or extent of surround against which the Laplacian-like operation is done. In this sense psychology has informed physiology. Ratliff's book goes into some detail on the earlier work of this sort. An extensive discussion of these matters is beyond the scope of this paper, unfortunately.

6. Notes Toward the Resolution of the Paradoxes – Part I

We are now in a position to consider the two main paradoxes that slowly emerged from the discussion. The first paradox is this: Two fields are seen through a dull black tube. Each field is divided into two equal halves. In one case we look at the boundary of a COLORED shadow ($A_1 + W_1$ vs $A_0 + W_1$). In the other case we look, under white light, at the boundary between a gray patch of the same color as $A_0 + W_1$ and a colored background of the same color as $A_1 + W_1$. In the former case we see $A_0 + W_1$ as the complementary CHROMATICITY to A. In the latter case the gray patch looks stubbornly GRAY. But the afterimages of both are identical and are as if $A_1 + W_1$ and $A_0 + W_1$ in the colored boundary case are reversed.

Let us consider this first paradox. The description, of course, is incomplete. In the case of the COLORED shadow the light at the boundary varies from $A_1 + W_1$ to $A_0 + W_1$ continuously, however sharp the shadow. There is no light value in crossing the boundary that does not lie between $A_1 + W_1$ and $A_0 + W_1$ in C_1 . The level of adaptation at such a boundary is combinative of $A_1 + W_1$ and $A_0 + W_1$ and represents an

ACHROMATIC point through which the ACHROMATIC axis goes. $A_0 + W_1$ is less A-ish than both the level of adaptation and $A_1 + W_1$, etc. Given a θ determined by $A_1 + W_1$ and $A_0 + W_1$ in C_1 , this is the θ in C_2 and the ACHROMATIC axis moves between $A_1 + W_1$ and $A_0 + W_1$, whereupon $A_0 + W_1$ has the COLOR in C_2 corresponding to the color of $\theta + 180^\circ$ in C_1 . Very little except SATURATION change can be contributed by the Laplacian mechanism. But these are not trivial. You will remember the case of a spot of $A_0 + W_1$ added in the shadow A_0 . Its COLOR could be determined in part by moving the eyes around, but it fluctuated and tended to stay desaturated until $A_0 + W_1$ actually bounded $A_1 + W_1$. Then the COLOR became more definite and stable.

In the second case what was left out in the description is, of course, the color of the boundary. A "real" boundary due to a piece of paper lying on another, or two pigments brought to exact proximity so that there is, if not an actual mixing, still a subtraction process from adjacency (see Helmholtz on pigment-mixing), a "real" boundary, I say, has a reflected color in white light different from the colors of the two bounding phases and not that of any mixture of those phases. If you doubt that you are sensitive to the color of a fine line, buy some legal paper with red or green margin indicators and try. You can always distinguish a boundary between real objects from a boundary of a shadow because of the different nature of the two with respect to sharpness, gradient, etc. I do not mean to go into the physical optics of the difference, however. But try the following experiment which comes, again somewhat modified, from Helmholtz. Take a thick pad of clean white paper. Fold the top sheet back on itself so that the edge of the sheet lies over the sheet itself, say the bottom third folded back. Now press it down with a piece of clean glass, or, more simply, use your hand, and look at it with the step down away from you. See if you can find an illumination such that you cannot tell where the edge lies on the paper at any angle of view. You will find to your astonishment, as Helmholtz did, that most illumination produces the false Mach band effect. One or the other phase at and away from the boundary will look darker depending on the angle at which the uniform lighting falls on the average. One angle of lighting, quite sharply determined, yields no false Mach band effect, and then you are uncertain. Turn the step down toward you. Now see if there is an angle of lighting whereat the step becomes invisible. There is always a shadow or reflection at the edge, and I assure you that at arm's length the color of a line that size can be told. Now if the shadow and/or reflections at the edge of one colored sheet lying on a sheet of another color are produced by white light, the detailed color of that transition (use an ordinary magnifying glass) is not simply combinatory of the two different colors of the two sheets but moves lighter and darker than either and different in hue. Thus, not only is the real boundary in this case provenant of information outside the plane determined in C_1 by both colored phases, but it is appreciable by the mechanism in our eyes. That is why, in fact, Helmholtz, in order to get simultaneous contrast between a field and a patch,

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had to mount the patch not on the field, but closer to himself, thereby not only abolishing the mutual boundary of contact but blurring the boundary to get a reasonable approximation of the boundary of a COLORED shadow.

It is also why the induced color in the patch-background arrangement overlaid with a piece of translucent paper was affected by the faintest pencil line outlining the patch.

Of course this tells you how to make the equivalent of a COLORED shadow out of gray and pink paper. Arrange the angle of light for least perception of the boundary (using the folded sheet of white paper to determine the best angular position of the boundary in the field). Then blur the boundary either by defocusing your eye or using a lens at the end of the tube.

But we have learned from this solution an important new fact. In white light such a boundary is virtually a vertex, three phases are compresent, and not one is colinear in C_1 with the other two. The exiguosness of the boundary as a phase does not diminish its effect (up to vanishingly small dimensions).

7. Further Observations Needed before Observing the Second Paradox

a. The nature of the blue process in color vision

A normal man has a luminosity curve that describes his threshold for just seeing a light as a function of λ . In a dichromat who lacks either the red or the green process the luminosity curve is changed and the absolute thresholds are raised. In a dichromat who lacks the blue process the luminosity curve is unchanged, the thresholds are normal.

The set of primary color processes that can be computed from the data of color matching has a subset such that the blue process has no luminosity or brightness contribution. The construction of C_1 does not require the blue process to have a luminosity function.

A monochromat who lacks any pair of processes except red and green can make brightness discriminations above the threshold of saturation of the rods. A man who has only the blue process cannot so discriminate brightnesses.

If you exhaust the red process in your eye with red light, immediately after the exhausting light is turned off you can discriminate brightness. If you exhaust the blue process in your eye with blue light, you discriminate brightnesses well. If you exhaust both red and green processes with yellow light, you affect the blue process but little, but thereafter you are almost blind foveally, you cannot discriminate brightnesses at all.

Finally, if you restrict blue light to the fovea, not only is the threshold high, but the subjective assessment of BRIGHTNESS change with brightness change is much lower than for green or red light.

These facts suggest that while all three primary color processes contribute to determine θ and ϕ , only two are involved in determining r , the red and green processes. We see a blue light only to the extent that the red and green processes provide information.

Almost in Hering's manner we can say that the seeing of BLUE arises from the judgment "This light is less YELLOW than it should be for its brightness." And it is commonly given that blue light affects CHROMATICITY much more than BRIGHTNESS.

b. Relations between BRIGHTNESS and CHROMATICITY

While brightness and chromaticity are independent of each other, this is not true for BRIGHTNESS and CHROMATICITY. This you have already seen by doing Helmholtz' experiments looking through narrow band filters. Not only is there a SATURATION difference between highlights and shadows, but even a HUE difference depending on the contrast. When, on looking through a red filter you see DARK objects as GREEN, this tells you that DARK, having less red than the adaptation level locally, is GREENER than it, for the local adaptation level for all pigments is the comparison GRAY.

Now the chromaticity in C_1 is a line passing through the black point intersecting the achromatic axis there (the origin of the coordinates used). CHROMATICITY in C_2 , this finding suggests, is a skewed line that passes through the ACHROMATIC axis at some r' away from the origin and intersecting the GREEN axis. This would imply, however, if the transformation were rigid from C_1 to C_2 (a straight line going into a straight line) that not only would REDNESS become more saturated as BRIGHTNESS increased, but very BRIGHT REDS ought to be supersaturated unless the line were curved. The impression one has is that a quite different rule applies. For if one superimposes a slow gradient of brightness on the field seen through the red filter and uses a repeated pattern of grays along the gradient, the same reflectance is identified as gray in the repeated pattern. Thus it is not an absolute value of gray that determines the ACHROMATIC axis in the monochromatic view, but a brightness relative to the surround brightness. Such an operation carries the chromaticity in C_1 not into a line in C_2 but a surface unless we insist that the coordinates of C_2 are set locally with respect to the colors as a rotation and a translation of C_1 as in Yilmaz' theory for the global field.

In effect this operation occurs because the lightnesses or BRIGHTNESSES are determined in part by the Mach-band mechanism specifying the difference across the boundary, in part by the adaptation mechanism specifying a reference BRIGHTNESS level. At this point we must refer back to Steven's work on "scaling." You will remember that you cannot discriminate objects well if you come from dark into bright light or the reverse. You will also remember that dazzle, or light adaptation, spreads across the retina from bright points as a smoothed function. You will also recall that you generally move your eyes about.

These observations suggest that the ability to tell a difference across a boundary by the Mach band mechanism, the Laplacian, varies inversely with distance along the brightness line away from the adaptation brightness level. From this one can conclude that the system acts as did it saturate away from the adaptation level, seeing uniform

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gray ratios along the brightness scale as compressed to an indistinguishable WHITENESS or BLACKNESS at moderate distances away from the adaptation level. Stevens determined that our ordering of brightness in an array of grays falls off according to a one-third power law in both directions away from the adaptation level. This can be derived from the considerations given above. You must remember that both Mach bands and adaptation are to be considered on a scale of brightness closer to logarithmic than to any other function. Equal ratios across boundaries generate equal impressions of BRIGHTNESS difference around the level of adaptation. I cannot go into the Fechner law, the Barlow observations and Steven's observations any further now. All I wanted to show was the fall-off in sensitivity of the Mach band mechanism away from the level of adaptation. But now it is not surprising that a photograph, with a dynamic range of 8:1, can be taken as an adequate representation of a scene with a dynamic range of 300:1.

c. BRIGHTNESS as a Function Solely of Red and Green Mechanisms

A simple and informative experiment is to use a large field of any saturated hue and to add a patch of white light of varying brightness to it. One automatically supposes that COLOR follows color and all that would happen is that where the white spot occurs, the HUE will simply become DESATURATED as the intensity of the white light increases. But, instead, a more astonishing thing happens. If the light is blue, then, as white light increases, the BLUE becomes a faint PINKISH-PURPLE, then very desaturated PINK-ORANGE before it becomes WHITE. In the same fashion GREEN becomes YELLOW-GREEN, then very desaturated YELLOW before turning WHITE. RED turns PINK-ORANGE, then very desaturated YELLOW-ORANGE. These phenomena are called the Abney effect.

When the white spot starts at low intensity on a pure yellow field, the spot becomes more saturated YELLOW than the field, then DESATURATES to WHITE as intensity increases.

Thus the trajectories in C_2 of the COLOR, under these simultaneous contrast conditions, is as if as SATURATION decreases so, too, does the absolute value of $|\theta' - \theta'_y|$ where θ' is that of YELLOW. When the HUE is YELLOW, then over a certain range the greater SATURATION lies with the greater BRIGHTNESS, as if ϕ varied in the opposite direction to SATURATION.

The adding of white light to a background-colored field acts as if, up to a certain point, you are adding yellow light alone in C_1 and seeing the correlated COLOR in C_2 . If you use two tungsten lamps, you could argue that one may be yellower than the other, but the changes seen do not change when the lamps are interchanged.

All of Land's demonstrations depend on brightness differences of two chromaticities mixed. It is important to remember that his curve, plotting the differences between spectral hues such that COLORS can be seen (i. e. , a plot of $\Delta\lambda$ as a function of λ) is a

unimodal curve with its nadir at yellow. That is, despite that the blue process is so sensitive, the production of COLORS from two chromaticities acts as if we are only sensitive to relative fluctuations of the red as opposed to the green process, and yellow is where there is the greatest difference of the slopes of the action spectra of the red and green processes.

d. Binocular Land Effects

Finally, providing illumination is kept low so as to get least fluctuations of adaptation as the eyes are moved around, the presentation of two chromaticities, one to each eye, yields COLORS distributed three-dimensionally in C_2 . Here, however, we come for the first time to appreciate the interaction of the Mach bands and adaptation.

Retinal rivalry occurs when the Mach bands do not correspond; retinal cooperation occurs when they do. (Incidentally, in man most of the information in the optic nerve is the Laplacian on some function of brightness in the image.)

If you use an ordinary stereoscope and present a red bar horizontally to one eye, a green bar vertically to the other, you see one bar as if it not only lies over the other, but abolishes any perception of the other for a significant visual angle away from the overlying boundary. You can determine which shall overlie which by moving your eyes (cf. the long quotation from Helmholtz earlier). If you impart a predominantly horizontal movement to your eyes, the vertical bar overlies; with predominantly vertical movements, the horizontal bar overlies. This is because the Laplacian is also a time-dependent function – we are sensitive most to its derivative with respect to time at any place on the retina.

The angular width over which one Mach band from one eye effectively masks an orthogonal one from the other varies with the contrast across each boundary, the greater contrast taking precedence and also having the widest masking width. Thus the rivalry fluctuates as the eyes move and as adaptation changes. (Even in the single eye such effects can be seen. In the Neckar cube illusion where an outlined cube seems to shift back to front, you can govern whether it be one way or the other by concentrating on one of the two intersections and looking frequently just to the right or left of it, or just above and below it. The cube takes the sense given by the line that is made salient by the movement, in that the face that is bounded by that line is that which looks nearer to you.)

If we can get two images with identical boundaries falling on both eyes and register the two binocularly, then the Mach band information is combined. The COLORS seen have the peculiar property of not being in the plane of the surface to which they are attributed, and this is particularly true of RED – but the COLOR names come out right. Also the colors are "lustrous," for "lustre" is a retinal rivalry in respect to surface quality, e. g., fine texture, because of the different fine details between the two images. The reason these effects fail binocularly at high brightness is that the reference adaptation levels change too rapidly as your eyes move about – and the colors fade if you fixate.

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The binocular experiments are the best indication that we operate less on surficial properties in the sensory mechanism than on boundary information, whatever we infer about our sensations from naive perceptions.

8. Notes Toward the Resolution of the Paradoxes – Part II

We are now in a position to consider the Land pictures. The paradox here is as follows: Two chromaticities are used, A and B (one of them can be white). All combinations of A and B form a coplanar set in C_1 . If spatial variations of A and B mutually are very high, the COLORS seen imply a trivariant perception – i. e., one sees BLACK, GRAY and WHITE, and also RED, YELLOW, BLUE, GREEN and PURPLE. That is, the various A and B combinations are three-dimensionally distributed in C_2 . These are the Land demonstrations. They are in their simplest form when we intersect two COLORED shadows.

If we had only an adaptation mechanism and a Mach band mechanism, and the consequence of their interaction, the Stevens BRIGHTNESS compression, and if this applied to all the processes in the retina symmetrically so that BRIGHTNESS were governed by all three, then we could say that in C_2 , on the intersection of two COLORED shadows:

1. The boundaries as one goes round the vertex are $(A_0 + W_0, A_0 + W_1)$, $(A_0 + W_1, A_1 + W_1)$, $(A_1 + W_1, A_1 + W_0)$ and $(A_1 + W_0, A_0 + W_0)$. The level of adaptation is given by W_1 and A_1 . $(A_0 + W_0)$ is less A than L, the adaptation level, so it is COMPLEMENTARY to A. $(A_0 + W_1)$ is more like L than the other areas but BRIGHTER; it should be WHITE. $(A_1 + W_1)$ is more like A than L, so it should have the A COLOR. $(A_1 + W_0)$ is more like L in chromaticity but DIMMER, so should be GRAY or BLACK. This gives us C_2 as a simple projection of C_1 . This is the simple expectation.

2. But now suppose we took the information from the differences across the boundaries and applied both Stevens' BRIGHTNESS compression law and the observation that BRIGHTER than L is equivalent to the color corresponding to YELLOWER than L, and DIMMER than L is equivalent to BLUER than L, always locally. Now we consider what happens at vertices.

Observation: Take two rectangular pieces of cardboard yielding each two shadows under A and W. If the cardboards are moved about, one can get long boundaries of one phase with respect to each of two other phases. When you do this, you discover that the COLOR of that phase is different at each of the long boundaries. At the vertex of the two the COLOR is that which you would predict from knowing the two boundary COLORS separately. When you stand back, the COLOR you see over a great part of the phase is the COLOR of the vertex. If you produce two vertices along a single boundary by using either two shadows through two neutral density filters of different grayness, then, unless the vertices are much separated, the COLOR of the phase combines the COLORS of the vertices. If you produce two vertices along a single boundary by using two shadows

through two neutral density filters of different grayness, then, unless the vertices are much separated, the COLOR of the phase combines the COLORS of the vertices. If a long stretch of boundary (in visual angle) separates the two vertices, the COLOR changes insensibly from that due to one vertex into that due to another.

The notion of "yellowier than" means this: as white is added to a spectral hue, the COLOR of the mixture looks as if spectral yellow had been added to it over a wide range of added white intensity and one took the names usually put on a C_1 chromaticity diagram as the names for the COLORS seen.

Let us now refer to the simplest case, a real world imaged but illuminated with but two spectral lines. These chromaticities form a plane in C_1 . All vertices are assumed to be trihedral. Every vertex can be figured by a triangle connecting the three colors at the vertex represented as points in C_1 . The farther apart are two of the points along r , the more a couple is applied to the line joining them, rotating the point with the higher value of r toward YELLOW, the RED-GREEN plane in the local C_2 , and the lower value away from the RED-GREEN, the line pivoting around L , the adaptation COLOR. If the third point is intermediate in its value of r between the two extrema, the lines joining it with the extrema form a triangle rotated out of the C_1 plane.

Consider one of the areas represented by one of the points in the triangle. Any other vertices involving that area will be other triangles in C_1 of different rotation, if the extremum r 's are different.

By the constraint of a unitary COLOR to a patch area (in the simple model of colored paper patches that introduced Land's experiments) we see that a polygon in C_1 , formed of triangles in C_1 , goes into a polyhedron in C_2 , and uniquely, because of the fact that increased BRIGHTNESS interchanges with yellowness and the blue process does not contribute to BRIGHTNESS.

You will now notice that to preserve the mutual order among COLORS by an internally generated reference system with three degrees of freedom, and to have this order map an external order (of reflectances), it is most useful if the three primary processes differ among themselves differently so as to preserve an internal compass and handedness. In this sense Yilmaz was thoroughly right in looking on the primary color processes as eigenfunctions of the visible spectrum; and, in essence, his theory works if used locally rather than globally, providing one adds the internally generated compass with a YELLOW as the reference axis for θ' . If the blue process contributed to brightness, the going from a polygon of triangles in C_1 to a polyhedron in C_2 would be ambiguous, for there would be no preferred direction for the skewing of any triangle out of the plane. That BLUE is exchangeable with DARK relative to YELLOW provides a unique direction. Furthermore, as the chromaticity diagram of C_1 suggests, hues from about $480 \text{ m}\mu$ on to shorter wave lengths ought to act more like red light than green light in transforming the polygon in Land's system. I believe Land has remarked this.

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Furthermore I suspect from the poor resolution when only the blue process is sensitive, that the blue process does not have a Mach-band mechanism, or not as dense a one, as do the red and green processes. This may arise either because the blue cones are more rare or because they have an intrinsically different action from the other two. Being a conservative, I favor the former view. It is the less salient Mach-band mechanism that allows increased brightness to act as an increase in yellow light. The mechanism applies as much to normal lighting as to the restricted conditions of Land's experiments.

It is, therefore, not an accident or imperfection that the blue process is not used in BRIGHTNESS measure. Had it not been so, restriction of light would have yielded ambiguity in COLOR, or else no COLOR constancy under restricted lighting.

Such a design suggests that the happy coincidence of trihedral vertices predominating in a plane is matched by a three-process system of the sort described. A fourth color destroys COLOR constancy unless absolute references can be used. I suspect that such an argument shows a three-process system to be optimal for the purpose of color vision as set out earlier.

J. Y. Lettvin

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