# DRIVER CHANNELING FOR LASER WAKEFIELD ACCELERATOR

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Guiding the laser pulse in a metal waveguide filled with a rare plasma is proposed as a tool for overcoming driver diffraction and controlling the driver trajectory in the laser wakefield accelerator. The wakefield wavelength in this case is determined by the density of the inner plasma and can be made large enough to provide effective wave generation. For the circular waveguide of the radius about the plasma skin-depth, most of the driver energy is contained in $TE_{11}$ mode, and the waveguide does not drastically diminish the dephasing length.

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Laser-driven plasma-based accelerators[1,2] have already demonstrated tremendous accelerating gradients[3] ($\sim 200$ GV/m) that are several orders of magnitude greater than those obtained in conventional RF linacs. The net energy gain of accelerated particles[4] ($\sim 100$ MeV) is not so impressive since acceleration distance is rather short for typical laser and plasma parameters. The main obstacle on the way to plasma-based high energy accelerators is laser diffraction.

To overcome the diffraction, several mechanisms of optical guiding were proposed.[1,5] Each of them has some imperfections. Relativistic optical guiding, plasma wave guiding, and ponderomotive self-channeling are unable to prevent diffraction of short laser pulses. Tailored

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pulse[6,7] is a halfway solution to the problem since it can only increase the diffraction length several times. Preformed plasma channels can perfectly guide the laser pulse, but, for effective wakefield excitation, the density of channel walls should be much less than the density of a condensed medium. Channel formation in gas or in plasma is a separate, not simple, problem. It is clear that, with present-day plasma channels, it is not possible to achieve a high precision (of precise metallic surface level) field structure necessary for high energy physics applications of laser-driven accelerators.

We propose to use a metal tube filled with a room-temperature gas for preventing laser diffraction. At electromagnetic field intensities typical for laser wakefield acceleration, both the metal waveguide and the inner gas turn to an ordinary plasma. At the moment of particle acceleration this plasma, due to ion inertia, has a very precise density profile perfectly suited for controlling the driver and wake excitation. The constituent elements of the proposed scheme are not new: the ability of solid hollow tubes to guide terawatt laser pulses[8,9] and the efficient wakefield excitation if the plasma is rare and its period is longer than the driver length[4,10,11] were experimentally demonstrated by several groups. Thus, the main idea of the scheme is to combine two well known techniques: the precise metal waveguide which can guide, but in which the wakefield cannot be excited (because of too short plasma wavelength in the walls), and the rare plasma which itself cannot guide the driver, but in which the wakefield can be easily generated.

After the acceleration cycle, the residual plasma energy partly ionizes or vaporizes the walls. When the system cools down, the vapored atoms condense back to the walls. As a result, the quality of the walls will degrade with each shot. The rate of waveguide erosion is now under study, and it is not clear yet how many acceleration cycles solid walls can stand. Perhaps the ideal solution would be a rotating waveguide with liquid walls,[12] which is inherently straight, perfectly cylindrical, and free from erosion.

Let us consider the circular waveguide of radius $R$. For the laser strength parameter $a_0 \ll 1$, the plasma behaves as a linear medium with the dielectric constant $\varepsilon = 1 - \omega_{\mathrm{p}}^2/\omega^2 \approx 1$, where $\omega_{\mathrm{p}} = \sqrt{4\pi n_0 e^2/m}$ is the plasma electron frequency and $\omega$ is the laser frequency ($\omega \gg \omega_{\mathrm{p}}$). Fields inside the waveguide are a superposition of TM and TE modes

which are well known:[13,14]

TM:

$$\vec{E}_{mn} = \nabla \operatorname{div} \vec{\Pi}_{mn} + \frac{\omega^2 \varepsilon}{c^2} \vec{\Pi}_{mn}, \tag{1}$$

$$\vec{H}_{mn} = -\frac{i\omega\varepsilon}{c} \operatorname{rot} \vec{\Pi}_{mn}; \tag{2}$$

TE:

$$\vec{E}_{mn} = \frac{i\omega\sqrt{\varepsilon}}{c} \operatorname{rot} \vec{\Pi}_{mn}, \tag{3}$$

$$\vec{H}_{mn} = \sqrt{\varepsilon} \left( \nabla \operatorname{div} \vec{\Pi}_{mn} + \frac{\omega^2 \varepsilon}{c^2} \vec{\Pi}_{mn} \right); \tag{4}$$

$$\vec{\Pi}_{mn} = A_{mn} J_n\left(\frac{\mu_{mn} r}{R}\right) \cdot \begin{pmatrix} \cos n\varphi \\ \sin n\varphi \end{pmatrix} e^{ik_z z - i\omega t} \vec{e}_z. \tag{5}$$

Here $J_n$ is Bessel function of the order $n$; $\mu_{mn}$ is $m$th zero of $J_n$ for TM modes, or $m$th zero of the derivative of $J_n$ with respect to its argument for TE modes; $A_{mn}$ is a normalization factor;

$$k_z^2 = \frac{\omega^2 \varepsilon}{c^2} - \frac{\mu_{mn}^2}{R^2}; \tag{6}$$

$c$ is the speed of light, and cylindrical coordinates $(r, \varphi, z)$ with $\vec{e}_z$ being the waveguide axis are used.

When the laser beam enters the waveguide, it excites many modes at once. For the case of a wide laser beam entering a wide waveguide $(R \gg \lambda = 2\pi c/\omega)$, the energy falling into dominant modes can be found as follows. Consider, for example, a linearly polarized beam with the Gaussian electric field distribution:

$$E_x = E_0 e^{-r^2/a^2} e^{ikz - i\omega t}, \quad E_y = 0, \tag{7}$$

$$k = \frac{\omega}{c}, \quad x = r \cos\varphi, \quad y = r \sin\varphi. \tag{8}$$

For $a \gg \lambda$, we can neglect the longitudinal component of the laser electric field both before the waveguide and inside it, neglect the difference between $k_z$ and $k$, and put $H_y = E_x$ and $H_x = -E_y$ everywhere. In this approximation, the amplitude of the transverse electric field inside the waveguide is the sum of vortex-free and solenoidal parts

arising from TM and TE modes, respectively:

$$\vec{E} = \vec{E}_{\text{TM}} + \vec{E}_{\text{TE}} = ike^{ikz-i\omega t}\left(\nabla_{\perp}\Pi_{\text{TM}} + \text{rot}(\Pi_{\text{TE}}\vec{e}_z)\right). \qquad (9)$$

The scalar functions $\Pi_{\text{TM}}$ and $\Pi_{\text{TE}}$ satisfy two-dimensional Laplace equations which follow from (9):

$$ike^{ikz-i\omega t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Pi_{\text{TM}} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}, \qquad (10)$$

$$ike^{ikz-i\omega t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Pi_{\text{TE}} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}. \qquad (11)$$

The functions $\Pi_{\text{TM}}$ and $\Pi_{\text{TE}}$ obtained from (10) and (11) should then be expanded[15] into Fourier–Bessel series ($\Pi_{\text{TM}}$) or Dini series ($\Pi_{\text{TE}}$) to yield the amplitudes of TM or TE modes, respectively.

The boundary condition for (10) is

$$\Pi_{\text{TM}}(R, \varphi) = 0; \qquad (12)$$

otherwise the Fourier–Bessel series will not converge. The problem (10), (12) thus have the unique solution. The solution to (11) is chosen so that to give correct electric fields (7) after substitution into (9) and to provide $\Pi_{\text{TE}}(0, \varphi) = 0$ (for convergence of the series). It is also unique.

For the electric field (7),

$$\Pi_{\text{TM}} = \cos\varphi\,\frac{E_0 a^2}{2}\left(\frac{1}{r}(1 - e^{-r^2/a^2}) - r(1 - e^{-1/a^2})\right), \qquad (13)$$

$$\Pi_{\text{TE}} = \sin\varphi\,\frac{E_0 a^2}{2}\left(\frac{1}{r}(1 - e^{-r^2/a^2}) + r(1 - e^{-1/a^2})\right). \qquad (14)$$

Only the modes with the azimuthal number $n = 1$ are excited. The fractions of the laser energy falling into dominant waveguide modes are shown in Figure 1. The structure of the electric field for these modes is shown in Figure 2. It is seen from Figure 1 that for $a \approx 0.75\,R$ up to 85% of the incident energy falls into $TE_{11}$ mode.
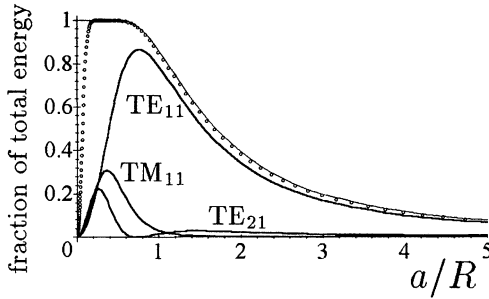
FIGURE 1   Laser energy falling into separate waveguide modes as functions of the incident beam radius. Thin curve shows the total energy falling into the hole. It nearly coincides with the total energy contained in first five TM and TE modes (shown by dots).
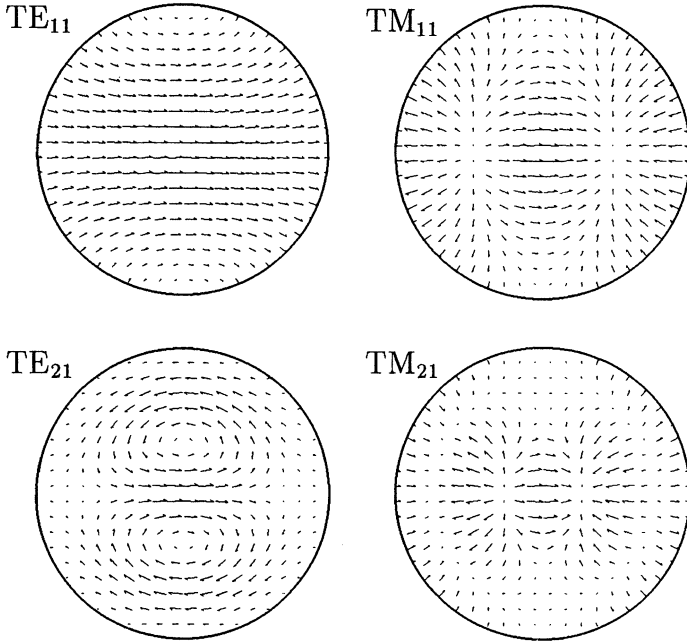


FIGURE 2   The structure of the electric field for dominant waveguide modes.

If the incident laser pulse is short, the electromagnetic field of excited modes drives the plasma wave inside the waveguide in almost the same manner as in infinite plasma.[1] Namely, in the linear regime, the perturbation of plasma electron density $\delta n$ is related to the normalized

vector potential $\vec{a}$ of the driver as

$$\frac{\partial^2 \delta n}{\partial t^2} + \omega_p^2 \delta n = \frac{n_0 c^2}{2} \Delta a^2 \qquad (15)$$

(where $\Delta$ is the Laplacian); behind the driver the force $\vec{F}$ acting on accelerated ultra-relativistic electrons is the gradient of some scalar function $\Phi$,

$$\vec{F} = -e(\vec{E} + [\vec{e}_z \times \vec{H}]) = \nabla\Phi, \qquad (16)$$

which is determined by $\delta n$:

$$\Delta_\perp \Phi - \frac{\omega_p^2}{c^2} \Phi = 4\pi e^2 \delta n. \qquad (17)$$

The only difference between the waveguide and the infinite plasma is in the boundary condition for (17): here $\Phi$ should turn to zero at $r = R$.

For $R \gtrsim c/\omega_p$, the second term in the left-hand side of (17) is dominant, and the accelerating electric field is little affected by the presence of the metal walls:

$$-eE_z = \frac{\partial \Phi}{\partial z} \approx \frac{i\omega_p}{c} \Phi \sim -i\frac{4\pi e^2 c\,\delta n}{\omega_p}. \qquad (18)$$

Otherwise ($R \ll c/\omega_p$), the electric field in the plasma wave is mainly transversal, and $E_z$ is roughly $(R\omega_p/c)^2$ times less than in infinite plasma with the same $\delta n$.

Electromagnetic wavelets in waveguides propagate slower than in the free space. For $R \gtrsim c/\omega_p$, the waveguide contributes to decrease of the group velocity roughly as much as the plasma. Thus, the waveguide does not drastically diminish the length of dephasing between the driver and accelerated particles.

In summary, the metal waveguide filled with a rare plasma can completely suppress the driver diffraction in the laser wakefield accelerator and provide the control of driver trajectory with a precision inaccessible by other proposed channeling schemes. For the channel radius $R$, driver radius $a$, and the inner plasma skin-depth $c/\omega_p$ being of the same order of magnitude (20–200 μm), the wakefield acceleration inside the waveguide occurs as efficient as in the infinite plasma.[1]

The proposed concept contains several points that need further studies. Among them are the wakefield excitation by relativistically strong laser pulses (which, due to nonlinearity, may occur differently than in the infinite plasma), the driver depletion length in the presence of the dense walls (the wall reflectivity was experimentally shown to be low,[8,9] but no detailed theory is developed yet), the rate of inner wall erosion for solid waveguides, and practicability of liquid waveguides. Particle-in-cell simulations and theoretical studies are underway to clarify these points.

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