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PLASMA-FILLED WAVEGUIDE FOCUSING LENS

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Large transverse wakefields can be generated via the interaction between a relativistic electron or positron bunch and a plasma, based on this mechanism Chen [P. Chen, *Part. Accel.* **20**, 171 (1987)], proposed the concept of unbound plasma lens. In this paper, we discuss the plasma filled waveguide focusing lens theory and find that the waveguide boundary can strengthen the focusing effect.

Keywords: Wakefield; Focusing lens; Linear collider

1 INTRODUCTION

Large transverse wakefields can be generated via the interaction between a relativistic electron or positron bunch and a plasma, and the bunch will therefore be self-pinched. Based on this effect, Chen[1] suggested a plasma lens theory for future linear colliders; there are also many experimental studies on plasma focus devices.[2–4] In this paper, we study this problem further. Instead of unbound plasma, we suppose that the plasma is bounded by cylindrical waveguide with ideally conducting wall. We find that the boundary can strengthen the focusing effect on the charged particle beam. Finally, a numerical example is given.

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2 THE PLASMA WAKEFIELD

As in Figure 1, R, a are the radii of cylindrical waveguide and bunch, $2b$ is the bunch length. Following Chen's method, the wakefields can be expressed as

$$w_{\|}(r,\xi) = \partial_\xi(A_{1z} - \phi_1), \tag{1}$$

$$w_{\perp}(r,\xi) = \partial_r(A_{1z} - \phi_1), \tag{2}$$

where $\xi = z - ct$ (c is the light speed), A_{1z} is the longitudinal part of magnetic vector potential $\vec{A}_1$, ϕ_1 is the electric potential. Supposing n_0 and n_1 are the zero order (unperturbed) and first order (perturbed) plasma density, we can use the linearized equation of motion and the linearized equation of continuity to have

$$\partial_\xi^2 n_1 + k_p^2 n_1 = \mp k_p^2 \sigma, \tag{3}$$

$$(\nabla_\perp^2 - k_p^2)(A_{1z} - \phi_1) = -4\pi e n_1, \tag{4}$$

where $k_p = (4\pi e^2 n_0/mc^2)^{1/2}$, $\mp e\sigma$ are current densities for electron and positron bunches, and σ is expressed as

$$\sigma = \rho_b f(r) g(\xi), \quad r \leq a, \ \xi \leq 0. \tag{5}$$

Using Green's function method,[5] n_1 is the same as in Ref. [1],

$$n_1 = \begin{cases} \mp\rho_b f(r) G(\xi), & \xi \leq 0, \\ 0, & \xi > 0, \end{cases} \tag{6}$$

$$G(\xi) = k_p \int_\xi^0 \mathrm{d}\xi' \, g(\xi') \sin k_p(\xi' - \xi). \tag{7}$$

But $A_{1z} - \phi_1$ is different from (1), in order to solve $A_{1z} - \phi_1$, we should first solve the following Green function $\tilde{G}(r,r')$:

$$\begin{gathered} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - k_p^2\right)\tilde{G}(r,r') = -\frac{1}{r}\delta(r - r'), \\ \tilde{G}(r,r')\,|_{r=R} = 0. \end{gathered} \tag{8}$$

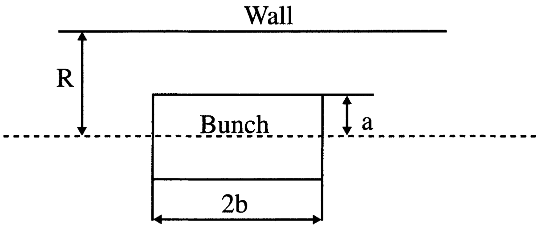

FIGURE 1 The schematic diagram of plasma-filled waveguide system.

We can get

$$\tilde{G}(r,r') = \begin{cases} \left(K_0(k_p r') - \dfrac{K_0(k_p R)}{I_0(k_p R)} I_0(k_p r')\right) I_0(k_p r), & 0 < r < r', \\ I_0(k_p r')\left((K_0 k_p r) - \dfrac{K_0(k_p R)}{I_0(k_p R)} I_0(k_p r)\right), & r' < r < R, \end{cases} \tag{9}$$

so that

$$\begin{aligned} A_{1z} - \phi_1 &= 4\pi e \int_0^\infty n_1 \tilde{G}(r,r')\,\mathrm{d}r' \\ &= \mp \frac{4\pi e \rho_\mathrm{b}}{k_p^2} G(\xi) k_p^2 \int_0^\infty f(r')\tilde{G}(r,r')\,\mathrm{d}r'. \end{aligned} \tag{10}$$

Define $F(r) = k_p^2 \int_0^\infty f(r')\tilde{G}(r,r')\,\mathrm{d}r'$, the wakefield can be expressed as

$$w_\parallel = \mp \frac{4\pi e \rho_\mathrm{b}}{k_p^2} \partial_\xi G(\xi) F(r), \tag{11}$$

$$w_\perp = \mp \frac{4\pi e \rho_\mathrm{b}}{k_p^2} G(\xi) \partial_r F(r). \tag{12}$$

The transverse focusing force on a particle at (r,ξ) within the bunch is

$$F_\perp(r,\xi) = \mp e w_\perp. \tag{13}$$

Consider the standard parabolic beam density distribution as in Ref. [1]

$$\sigma = \rho_\mathrm{b}\left(1 - \frac{r^2}{a^2}\right)\left(1 - \frac{(\xi+b)^2}{b^2}\right), \tag{14}$$

from Eqs. (7) and (10), we get

$$A_{1z} - \phi_1 = \mp \frac{8\pi e \rho_b}{k_p^2} \left(I_0(k_p r) \left(K_2(k_p a) - \frac{K_0(k_p R)}{I_0(k_p R)} I_2(k_p a) \right) \right. \\ \left. + \frac{1}{2}\left(1 - \frac{r^2}{a^2}\right) - \frac{2}{k_p^2 a^2} \right) \\ \times \left(1 - \frac{(\xi + b)^2}{b^2} + \frac{2}{k_p b} \sin k_p \xi + \frac{2}{k_p^2 b^2}(1 - \cos k_p \xi)\right), \quad (15)$$

then the transverse focusing force is

$$F_\perp = \frac{8\pi e^2 \rho_b}{k_p} G(\xi) \left(I_1(k_p r) \left(K_2(k_p a) - \frac{K_0(k_p R)}{I_0(k_p R)} I_2(k_p a) \right) - \frac{r}{k_p a^2} \right). \quad (16)$$

In the limit $k_p r \leq k_p a \ll 1$,

$$F_\perp \simeq -2\pi e^2 \rho_b G(\xi) \left(1 + \frac{k_p^2 a^2}{4} \frac{K_0(k_p R)}{I_0(k_p R)} \right) r. \quad (17)$$

The term $(k_p^2 a^2/4)(K_0(k_p R)/I_0(k_p R))$ in the above formula is due to the boundary condition in comparision with unbound plasma case,[1] when $R \to \infty$, it vanishes, and we get Chen's result. When $0 < k_p R < 10$, the sign of this term is always positive, so it is a focusing term. Although we have adopted Chen's simplified theory,[1] it is obvious that the focusing force is increased due to the appearance of the waveguide condition, this additional focusing effect exists due to the mapping charges of ions of plasma in conducting wall.

3 A PLASMA-FILLED WAVEGUIDE FOCUSING LENS

In a plasma-filled waveguide lens configuration, we adopt the same beam density distribution σ as:[1]

$$\sigma = \rho_b \left(1 - \frac{r^2}{a^2}\right) \left(k_p^{-1} \delta(\xi) + \theta\left(\xi + b + \frac{\pi}{2k_p}\right) - \theta\left(\xi + \frac{\pi}{2k_p}\right) \right). \quad (18)$$

From Eq. (7), we get

$$G(\xi) = \begin{cases} 1, & -(\pi/2k_p + b) \le \xi \le -\pi/2k_p, \\ \sin k_p\xi, & -\pi/2k_p < \xi < 0. \end{cases} \tag{19}$$

Under the case $k_p r \le k_p a \ll 1$, the focusing force along the main bunch produced by the precursor thin disk

$$F_\perp \simeq -\frac{4Ne^2k_p^2 r}{a^2(1+k_p b)}\left(1+\frac{k_p^2a^2}{4}\frac{K_0(k_pR)}{I_0(k_pR)}\right), \quad -\left(\frac{\pi}{2k_p}+b\right) \le \xi \le -\frac{\pi}{2k_p}, \tag{20}$$

where N is the number of particles in the total beam bunch. As a numerical example we take the same set of parameters of a 5 TeV + 5 TeV e^+e^- linear collider discussed by Chen, $a = 3\,\mu\text{m}$, $b = 100\,\mu\text{m}$, $k_p = 0.2\,(\mu\text{m})^{-1}$, $N = 4.1 \times 10^8$ and we take $R \sim a$ to get the focusing strength of the lens, K,

$$K = \frac{F_\perp/r}{\gamma mc^2} = \frac{4Nk_p r_e}{\gamma a^2(1+k_p b)}\left(1+\frac{k_p^2a^2}{4}\frac{K_0(k_pR)}{I_0(k_pR)}\right) = 5.33 \times 10^{-2}\text{cm}^{-2}, \tag{21}$$

and the field gradient G,

$$G = \frac{1}{r}w_\perp = \frac{4Nek_p}{a^2(1+k_p b)}\left(1+\frac{k_p^2a^2}{4}\frac{K_0(k_pR)}{I_0(k_pR)}\right) = 921\,\text{MG/cm}; \tag{22}$$

K and G increase by about 11% in comparision with Ref. [1].

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