Particle Accelerators, Vol. 59, pp. 169–174
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SYNCHROTRON DEPOLARIZING RESONANCES IN THE PRESENCE OF A SOLENOIDAL SIBERIAN SNAKE*

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(Received 19 November 1997; In final form 29 January 1998)

Particle's energy variation arising from synchrotron oscillations modulates the spin tune, which causes synchrotron sidebands around depolarizing resonances. Studying the spin tune modulation in a ring with a single solenoidal Siberian snake, we found that for a given energy it is possible to tune the strength of the snake solenoid to eliminate spin the tune modulation, thus eliminating the synchrotron sideband resonances.

Keywords: Spin dynamics; Siberian snake; Depolarizing resonances

Polarized proton beams encounter many depolarizing resonances during acceleration to high energies. Most of the depolarizing resonances are caused by small horizontal magnetic fields[1] which act on the particles traveling around the ring. In addition, many of these resonances are found to have synchrotron sideband resonances, which are mainly caused by the energy modulated spin tune.[2] The Siberian snake technique[3] was proposed to universally cancel all depolarizing resonances. Each full Siberian snake rotates the proton's spin by 180° around some horizontal axis and has no effect on the beam motion. While full Siberian snakes work best at high energies, one still needs to

* This research was supported by grants from the U.S. Department of Energy and the U.S. National Science Foundation.
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correct a few depolarizing resonances that occur at low energies. In this paper we will study how a solenoidal partial Siberian snake could be used to cancel the synchrotron sideband resonances.

In an accelerator without Siberian snakes, the spin tune ν_s, which is the number of spin revolutions around the spin precession axis in one turn around the ring, is energy dependent

$$\nu_s = G\gamma, \tag{1}$$

where $G = (g-2)/2$ is the gyromagnetic anomaly ($G = 1.792847$ for protons) and γ is the Lorentz energy factor. During acceleration of a polarized proton beam, the energy dependent spin tune crosses many depolarizing resonances, which ultimately destroy the beam polarization.

In synchrotrons, the energy of each particle is modulated due to synchrotron oscillations,

$$\gamma = \gamma_0 + \Delta\gamma \cos(\nu_{\text{syn}}\theta), \tag{2}$$

where $\Delta\gamma$ is an amplitude of the energy modulation, ν_{syn} is the synchrotron tune and $\theta = 2\pi t/\tau$ is time in units of the revolution period τ of the ring. The synchrotron energy modulation causes spin tune modulation, which in turn leads to a spectrum of synchrotron sidebands around the depolarizing resonances.

In a ring with a single Siberian snake, the spin tune ν_s can be expressed in terms of $G\gamma$ and the snake strength s:

$$\cos(\pi\nu_s) = \cos(\pi G\gamma)\cos(s\pi/2). \tag{3}$$

For a full snake ($s = 1$) the spin tune becomes a half-integer at all energies, which eliminates the problem of crossing many depolarizing resonances during the acceleration. In the presence of a full Siberian snake the spin tune is energy independent only if the snake strength is energy independent. This is certainly true for dipole snakes at high energy. At low energies, dipoles cause large orbit excursions, making a dipole snake impractical. Low energy snakes are usually solenoids. The snake strength of a solenoid is given by:

$$s = \frac{e(1+G)}{pc}\int B\,\mathrm{d}l, \tag{4}$$

where p is the proton's momentum. For a constant field in a solenoid the snake's strength is proportional to p^{-1}; particles with slightly different energies will see different snake strengths, and therefore will have different spin tunes. If the energy has synchrotron modulation, so does the snake strength and the spin tune, even for a "full" snake (average $s = 1$).

Using the approach discussed by Montague,[4] it is easy to show that the energy modulation of the spin tune naturally leads to synchrotron sideband resonances. Assuming a localized horizontal spin kick of strength ϵ_0 at a frequency ν_r, the resonance strength can be computed as the spin perturbation averaged over many turns ($N \rightarrow \infty$):

$$\epsilon = \frac{\epsilon_0}{N} \sum_{n=1}^{N} \exp\left\{ \mathrm{i} \int_0^{2\pi n} \mathrm{d}\theta [G\gamma_0 - \nu_r + G\Delta\gamma \cos(\nu_{\text{syn}}\theta)] \right\}. \quad (5)$$

Without energy modulation, the sum in Eq. (5) would be equal to N whenever $G\gamma_0 - \nu_r$ is equal to an integer. The energy modulation term in the exponent can be expanded in series using,

$$\mathrm{e}^{\mathrm{i}x \sin(y)} = \sum_{m=-\infty}^{\infty} J_m(x) \mathrm{e}^{-imy}, \quad (6)$$

where J_m is the mth Bessel function of the first kind. The resonance strength equation then becomes,

$$\epsilon = \sum_{m=-\infty}^{\infty} \epsilon_0 J_m \left(\frac{G\Delta\gamma}{\nu_{\text{syn}}} \right) \frac{1}{N} \sum_{n=1}^{N} \mathrm{e}^{\mathrm{i}2\pi n (G\gamma_0 - \nu_r - m\nu_{\text{syn}})}. \quad (7)$$

Thus, along with the main resonance at $G\gamma_0 = k + \nu_r$, the spin tune modulation creates synchrotron sideband resonances at $G\gamma_0 = k + \nu_r + m\nu_{\text{syn}}$, where k and m are integers. The strength of each synchrotron sideband resonance is

$$\epsilon_m = \epsilon_0 J_m (G\Delta\gamma / \nu_{\text{syn}}). \quad (8)$$

This analysis can be easily repeated for the case of a single partial or full Siberian snake in a ring. One finds that the strength of the mth synchrotron sideband is then equal to $\varepsilon_0 J_m(\Delta\nu_s/\nu_{\text{syn}})$, where $\Delta\nu_s$ is the spin tune modulation amplitude. To find the strength of the synchrotron sideband resonances one only needs to compute the amplitude of the spin tune modulation in the presence of a Siberian snake.

Using Eq. (3), the spin tune modulation amplitude $\Delta\nu_s$ can be written:

$$\Delta\nu_s = \frac{d\nu_s}{d\gamma}\Delta\gamma = \left(\frac{\partial\nu_s}{\partial\gamma} + \frac{\partial\nu_s}{\partial s}\frac{ds}{d\gamma}\right)\Delta\gamma. \tag{9}$$

The second term takes into account the energy dependence of the snake strength, which can be obtained from Eq. (4). Substituting the derivatives of Eq. (3), the amplitude of the spin tune modulation becomes:

$$\Delta\nu_s = \frac{\Delta\gamma}{\sin(\pi\nu_s)}\left[G\sin(\pi G\gamma)\cos(\pi s/2) - \frac{s}{2}\frac{\gamma}{(\gamma^2-1)}\cos(\pi G\gamma)\sin(\pi s/2)\right]. \tag{10}$$

Note that with no snake ($s=0$), the spin tune modulation amplitude reduces to $\Delta\nu_s = G\Delta\gamma$, and for a full solenoidal snake ($s=1$),

$$\Delta\nu_s = \frac{-1}{2\beta^2}\frac{\Delta\gamma}{\gamma}\cos(\pi G\gamma), \tag{11}$$

which is nonzero except when $G\gamma$ is a half-integer. The depolarizing resonances that occur even in the presence of a Siberian snake (so called "snake resonances"[5]) could then have synchrotron sidebands. The first-order sidebands ($m=\pm 1$) would have strength $\varepsilon_1 \approx \varepsilon_0\Delta\nu_s/(2\nu_{syn})$. The sidebands of a "snake resonance" are strongest when $G\gamma$ is close to an integer; their maximum strength is $2G\gamma\beta^2$ times smaller than the strength of the synchrotron sidebands without a snake.

Equation (10) can be used to predict the snake strength required for zero spin tune modulation at a given energy; if a depolarizing resonance occurs at this energy, its synchrotron sidebands would then vanish. Setting the right side of Eq. (10) to zero gives an implicit equation for canceling the spin tune modulation, which can be solved numerically. The predicted snake strength is plotted in Figure 1 against fractional $G\gamma$ for the integer part of $G\gamma$ from 2 to 6, or an energy range of 0.1–2.7 GeV. Note that for half-integer $G\gamma$, a full snake is required to

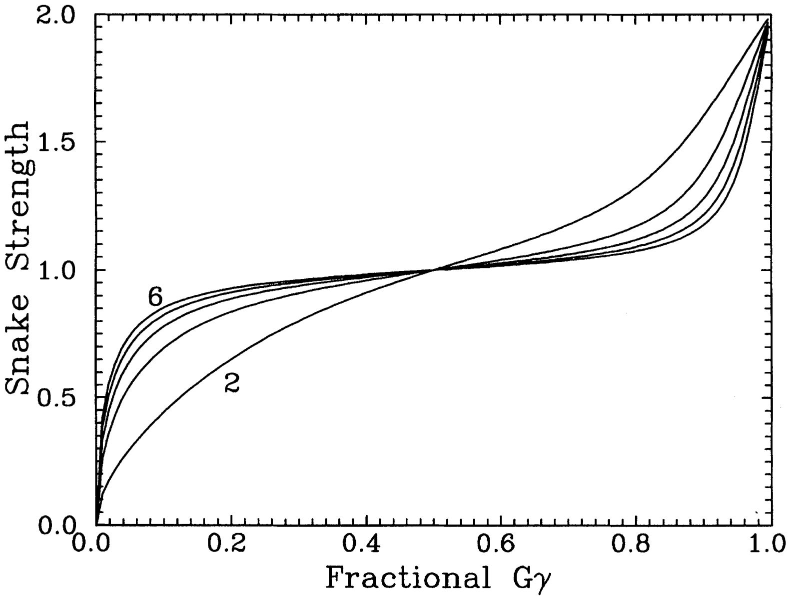

FIGURE 1 The snake strength calculated for cancelling the spin tune modulation is plotted as a function of fractional $G\gamma$. The curves are for the integer part of $G\gamma$ of 2 through 6. The sharpness of the curves increases with the integer part of $G\gamma$.

cancel the spin tune modulation. Also note that for the upper half of each fractional range, one must use a snake strength greater than 1; this could limit practical application of this method. Why cannot a snake's spin precession of $\phi > \pi$ be replaced with $2\pi - \phi$, which gives the same spin tune? Although the spin tune would be the same for these two snake's spin precessions, the energy modulation of $2\pi - \phi$ spin precession is in opposite phase to the modulation of ϕ; the spin tune modulation due to the direct energy effect would no longer cancel the modulating snake spin precession. Finally, note that the profile of s as a function of fractional $G\gamma$ is increasingly sharp for larger integer values (higher energies); tuning the snake for values near integer $G\gamma$ would be more difficult at higher energies.

Canceling the spin tune modulation could have application in the acceleration of polarized beams. One could arrange for a particular snake strength at some point in an energy ramp where it is suspected that strong synchrotron sidebands are causing polarization losses. This might also be useful for intrinsic resonance jumping in low energy accelerators, where spin tune modulation limits the efficiency of the tune jump method.[6] It should also be noted that nowhere in the derivation of Eq. (10) is it necessary to assume that $\Delta\gamma$ be a modulation. The same approach could be used to stabilize the spin tune against random energy fluctuations. Besides the energy modulation of the spin tune, synchrotron resonances could be caused by vertical synchrotron oscillations due to a non-zero vertical dispersion function.[7] The synchrotron depolarizing resonances caused by this effect cannot be compensated by solenoidal snake; however, their strength is expected to be significantly weaker than the one caused by the energy modulated spin tune.[2]

In summary, it has been shown that in the presence of a solenoidal Siberian snake depolarizing resonances can have synchrotron sidebands. The spin tune modulation which causes the synchrotron sidebands can be eliminated by adjusting the solenoidal snake strength. This technique may be helpful in accelerating polarized protons in the low and medium energy machines such as the Brookhaven AGS, the Fermilab Booster, and DESY3 at DESY.

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