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MINIMIZING THE PACMAN EFFECT ON THE CLOSED ORBIT

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Pacman bunches, in storage rings, experience two deleterious effects: anomalous tune shifts and orbit displacements. The anomalous tune shifts result in larger effective tune spreads which, as is known, can be compensated by arranging crossing planes 90° relative to each other at successive paired interaction points (IPs) separated by half the ring circumference. This paper evaluates the associated anomalous Pacman orbit displacements for such paired interaction points. The anomalous orbit displacements can in turn be minimized by setting the phase advance between the paired IPs to half the overall phase around the machine. Numerical results are evaluated for the LHC design parameters. The resultant displacements are small and should not significantly affect the LHC performance.

Keywords: Pacman; Beam–beam; LHC; Orbit distortion; Tune shift

1 INTRODUCTION

It is believed that the attainable beam–beam tune shifts in hadron colliders may not be limited by bunches in a standard environment, but by "Pacman" bunches, bunches that in the interaction regions (IRs) are circulating past "gaps" of missing bunches in the counter-circulating beam. Such bunches will suffer anomalous tune shifts and orbit displacements different from the "average" bunches circulating relative to a locally fully filled beam. Therefore if the machine is optimized for average bunches the Pacman bunches will not be in an optimized environment and may suffer enhanced losses. However, loss of a

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Pacman bunch will create new Pacman bunches in the counter-circulating beam, and over the course of time holes will develop in both beams and eventually the beams may be destroyed. Only for the special case (in practice the usual configuration) of beams interacting at IPs symmetrically placed with separations of half the ring circumference, does a circulating bunch encounters the identical pattern of counter circulating bunches at each IP. For this special case the Pacman effects at the paired IPs are related and the IPs can be configured to cancel or minimize the Pacman anomalies. Irrespective of the phase advance between the IPs, the anomalous tune shift is cancellable. For example, in the LHC, this is easily accomplished for equal β^*_{ip} (β-function at the IP) by crossings planes at 90° relative to each other at the two high luminosity IPs 1 and 5. The anomalous tune shift and associated enhanced tune spread is the primary limitation on potentially achievable bunch currents and luminosity, and the anomalous orbit displacements are a secondary effect. Therefore the need to cancel out the anomalous tune shift is the primary consideration, and for this reason we confine our considerations to machines with symmetrically paired IPs. Unfortunately, while it is relatively easy to remove anomalous tune shifts, the anomalous orbit shifts can at best be minimized by a "best" choice of phase separations between the IPs, and this is the problem examined below. Not perhaps surprisingly, the optimum is found for paired IPs symmetrically separated in phase by half the phase advance around the ring. Obviously if the crossings are successively in horizontal or vertical crossing planes orbit cancellation is no longer possible. However even for this case equal phasing minimizes the combined horizontal and vertical displacements at the paired IPs.

Finally we evaluate the physical magnitude of the expected orbit anomalies in the LHC and find them to be small.

2 THEORETICAL DERIVATION

The derivation uses a number of straightforward properties for finding eigen-solutions for the equilibrium orbit in a machine. We first assume a machine with two identical interaction regions, IRs A and B, at the start and midpoint of the circumference around the machine. The IPs are at α (slope of the β-function) equal to zero, and are assumed to have equal $\beta^*_{\rm ip}$. In units of $x/\beta^*_{\rm ip}$ the transfer matrix R around the machine from IP A back to IP A is given by

$$R = \begin{pmatrix} \cos(\mu) & \sin(\mu) \\ -\sin(\mu) & \cos(\mu) \end{pmatrix}, \tag{1}$$

where μ is the phase advance around the machine. We shall use the shorthand notation

$$R = M(\mu), \tag{2}$$

and this has the standard property that

$$M(\mu)M(\phi) = M(\mu + \phi). \tag{3}$$

Orbit displacements are only significant for bunches that suffer head-on collisions. The missing "Pacman gap" in the counter-circulating beam can then be encountered before collision ("IN" configuration) or after collision ("OUT" configuration). In a single passage across the IR to the IP the gap will cause the bunch to deviate from its equilibrium orbit by a deflection as it passes the missing bunch(es). We will content ourselves with the following case (which is a good approximation for the LHC and for other envisaged machines,[1] see Section 2.3 below): The missing parasitic crossings are $\sim 90°$ in phase away from the IP and the net effect is for there to be a transverse displacement at the IP and a close to zero angular displacement. In units of the net displacement (see Section 2.3) we can represent the single pass displacement as a vector, V,

$$V \simeq \begin{pmatrix} 1 \\ \epsilon \end{pmatrix}, \tag{4}$$

$$\epsilon \simeq \beta^*_{\rm ip}/L, \tag{5}$$

where L is the distance of the missing bunch from the IP.

In the following we define the phase separation between IPs as ϕ. As an example application we evaluate the orbit displacement, x, at IP A (the first IP) where the second crossing, at IP B, is in the same plane as for A. The "vector" displacement, X, is defined by

$$X = \begin{pmatrix} x \\ \theta \end{pmatrix}. \tag{6}$$

Then in matrix notation the eigen-solution for X is found by equating the X at each sequential turn with the X at the previous turn. We then find

$$X = M(\mu)X + M(\mu - \phi)V + V. \tag{7}$$

This can be rearranged as

$$\big(M(0) - M(\mu)\big)X = M(\mu - \phi)V + M(0)V. \tag{8}$$

X is then specified by

$$X = \big(M(0) - M(\mu)\big)^{-1}\big(M(\mu - \phi) + M(0)\big)V. \tag{9}$$

The previous result can be simplified by noting that the term

$$M(0) - M(\mu) = \begin{pmatrix} 1 - \cos(\mu) & -\sin(\mu) \\ \sin(\mu) & 1 - \cos(\mu) \end{pmatrix} \tag{10}$$

or

$$M(0) - M(\mu) = 2\sin\left(\frac{\mu}{2}\right)\begin{pmatrix} \sin(\mu/2) & -\cos(\mu/2) \\ \cos(\mu/2) & \sin(\mu/2) \end{pmatrix}. \tag{11}$$

Taking the inverse

$$\big(M(0) - M(\mu)\big)^{-1} = \frac{1}{2\sin(\mu/2)}\begin{pmatrix} \sin(\mu/2) & \cos(\mu/2) \\ -\cos(\mu/2) & \sin(\mu/2) \end{pmatrix}. \tag{12}$$

This in turn can be written as

$$\big(M(0) - M(\mu)\big)^{-1} = \frac{-1}{2\sin(\mu/2)} M\left(-\frac{\mu}{2} - \frac{\pi}{2}\right). \tag{13}$$

Substituting this into Eq. (9) we get

$$X = \frac{-1}{2\sin(\mu/2)}\left(M\left(\frac{\mu}{2} - \phi - \frac{\pi}{2}\right) + M\left(-\frac{\mu}{2} - \frac{\pi}{2}\right)\right)V = SV, \tag{14}$$

where S is a matrix defined by the above equation.

A small change in the angular component, θ, of X has an insignificant effect on the deflection at the IP, and the only quantity of interest is the spatial component, x, of X. Remembering V is approximated by Eq. (4) and therefore

$$x \simeq S_{11} + \epsilon S_{12}, \tag{15}$$

or substituting we obtain the simple and final result

$$x \simeq \frac{-1}{2\sin(\mu/2)}\left(-\sin\left(\frac{\mu}{2}\right) + \sin\left(\frac{\mu}{2} - \phi\right) - \epsilon\left(\cos\left(\frac{\mu}{2}\right) + \cos\left(\frac{\mu}{2} - \phi\right)\right)\right). \tag{16}$$

In Section 2.1 we generalize the above result to paired IPs with "coherent" cancellation, namely those with a horizontal/vertical crossing plane followed by a horizontal/vertical crossings plane, or those with successive 45° tilted crossing configurations.

In Section 2.2 we examine the configurations for which cancellation cannot occur, namely a horizontal/vertical crossing plane followed by a vertical/horizontal crossing plane. These cases are identified as "incoherent crossings".

2.1 Paired Coherent Crossings

We have evaluated above the offset at IP A for a gap in parasitic crossings on the incoming side of the IP. There are four cases, incoming and outgoing gaps and IPs A and B. We summarize the basic equations and solutions for the four cases. Successive crossings may both start from above, same sign crossings, or may start from above/below before the IP and below/above the IP, opposite sign crossings. A similar convention applies to crossings in the horizontal plane. The results are evaluated for same sign crossing angles at the two IPs. The four additional cases for opposite sign crossings can be found by replacing ϕ by $\phi + \pi$. These additional cases are degenerate to an interchange of IPs A and B and therefore do not modify the following results:

IN for IP A

$$X = \left(M(0) - M(\mu)\right)^{-1}\left(M(\mu - \phi) + M(0)\right)V; \tag{17}$$

IN for IP B

$$X = \left(M(0) - M(\mu)\right)^{-1}\left(M(\phi) + M(0)\right)V; \tag{18}$$

OUT for IP A

$$X = \left(M(0) - M(\mu)\right)^{-1}\left(M(\mu) + M(\mu - \phi)\right)V; \tag{19}$$

OUT for IP B

$$X = \left(M(0) - M(\mu)\right)^{-1}\left(M(\mu) + M(\phi)\right)V. \tag{20}$$

The results evaluated for these cases are

$$x_{\mathrm{inA}} \simeq \frac{-1}{2\sin(\mu/2)}\left(-\sin\left(\frac{\mu}{2}\right) + \sin\left(\frac{\mu}{2} - \phi\right) - \epsilon\left(\cos\left(\frac{\mu}{2}\right) + \cos\left(\frac{\mu}{2} - \phi\right)\right)\right), \tag{21}$$

$$x_{\mathrm{inB}} \simeq \frac{-1}{2\sin(\mu/2)}\left(-\sin\left(\frac{\mu}{2}\right) - \sin\left(\frac{\mu}{2} - \phi\right) - \epsilon\left(\cos\left(\frac{\mu}{2}\right) + \cos\left(\frac{\mu}{2} - \phi\right)\right)\right), \tag{22}$$

$$x_{\mathrm{outA}} \simeq \frac{-1}{2\sin(\mu/2)}\left(+\sin\left(\frac{\mu}{2}\right) + \sin\left(\frac{\mu}{2} - \phi\right) + \epsilon\left(\cos\left(\frac{\mu}{2}\right) + \cos\left(\frac{\mu}{2} - \phi\right)\right)\right), \tag{23}$$

$$x_{\mathrm{outB}} \simeq \frac{-1}{2\sin(\mu/2)}\left(+\sin\left(\frac{\mu}{2}\right) - \sin\left(\frac{\mu}{2} - \phi\right) + \epsilon\left(\cos\left(\frac{\mu}{2}\right) + \cos\left(\frac{\mu}{2} - \phi\right)\right)\right). \tag{24}$$

Using the approximation that ϵ is small, order ϵ terms in cosine can be neglected. We then obtain the following simple results:

$$x_{\mathrm{inA}} + x_{\mathrm{inB}} \simeq +1 \tag{25}$$

and

$$x_{\text{outA}} + x_{\text{outB}} \simeq -1. \tag{26}$$

Equivalently, for either the IN or OUT case

$$|x_{\text{A}}| \text{ and/or } |x_{\text{B}}| \geq \tfrac{1}{2}. \tag{27}$$

For the symmetric case where $\phi = \mu/2$

$$|x_{\text{A}}| = |x_{\text{B}}| = \tfrac{1}{2}. \tag{28}$$

The symmetric case represents the optimum configuration.

2.2 Paired Incoherent Crossings

If the crossing planes are 90° relative to each other, horizontal or vertical, then the resultant orbit displacements in the two planes add in quadrature and not linearly. Assuming a horizontal crossing for IP A and vertical crossing for IP B the horizontal orbit displacement at IP A will be given at A from

$$X_{\text{A}} = M(\mu)X_{\text{A}} + M(0)V \tag{29}$$

equivalent to

$$X_{\text{A}} = \left(M(0) - M(\mu)\right)^{-1} V, \tag{30}$$

or evaluating as in the previous subsection

$$x_{\text{A}} = \frac{1}{2}\left(1 + \epsilon \cdot \cot\left(\frac{\mu}{2}\right)\right), \tag{31}$$

or neglecting terms in ϵ

$$x_{\text{A}} \simeq \tfrac{1}{2}. \tag{32}$$

The corresponding displacement at B is

$$X_B = M\left(\frac{\mu}{2}\right)X_A. \tag{33}$$

The horizontal component x_B that adds in quadrature to the vertical orthogonal deflection at B is again set close to zero for equal phases A to B and B to A. For this optimal phase choice

$$X_B = \frac{-1}{2\sin(\mu/2)} M\left(\frac{\mu}{2}\right) M\left(-\frac{\mu}{2} - \frac{\pi}{2}\right) V, \tag{34}$$

or equivalently

$$x_B = \frac{\epsilon}{2\sin(\mu/2)} \simeq 0. \tag{35}$$

Thus for symmetric phasing, in units of V, the deflection at the IPs is one half, and is identical to the orbit deflection obtained for the coherent cases in Section 2.1.

2.3 Numerical Results for the LHC

The units of V are easily evaluated for the LHC. The orbit displacement at the IP in a single pass around the machine is[1]

$$\Delta x_s = \frac{8\pi N_p \Delta\nu_{ho}}{\theta_{cross}}, \tag{36}$$

where Δx_s is in units of σ_x (the rms beam transverse size), $\Delta\nu_{ho}$ is the head-on tune shift per IP, θ_{cross} is the full crossing angle in units of $\sigma_{x'}$ (the rms beam angular size), and N_p is the effective number of parasitic crossings, evaluated below. For a circulating beam the equilibrium orbit displacement at the IP for equal phasing between IPs is $1/2\Delta x_s$.

The number of parasitic crossings in the drift space around the IP is 12, within the high beta quads is 16, and in the space prior to separation is 8. Therefore, the actual number of parasitic crossings is 36. To obtain the effective number of crossings we must modify these numbers to take into account the varying ratio of β_x/β_y. Following Irwin[1]

$$N_p = \sum_{i=1}^{36} \frac{\beta_{x/y}^{1/2}}{\beta_{y/x}^{1/2}} = 38. \tag{37}$$

Thus the effective number of crossings is increased by roughly 5%. However for simplicity in the following argument we neglect this 5% effect and assume all 36 crossings are equally effective, an approximation good, on average, to 5% (though for individual bunches it can be in error by $\sim\pm 25\%$).

At the LHC circulating particle bunches pass one another about every 3.75 m as they approach and depart the IP. For a nominal $\beta^*_{\rm ip}$ of 50 cm the closest parasitic crossing corresponds to 81° in phase from the IP and the next closest to 85° in phase from the IP. There are in all 18 parasitic crossings prior to beam separation on either side of the IP with phases increasingly close to 90° and therefore, to a good approximation for the LHC, the missing parasitic crossings are $\sim 90°$ in phase away from the IP.

The effects of anomalous orbit displacement are small compared to the beam separation for the parasitic crossings and only minimally modify the forces on a given bunch. However at the IP, small anomalous offsets comparable to the IP beam size will qualitatively modify the effective potential of the IP beam–beam forces. If for any given bunch at the IP the corresponding counter circulating bunch is missing, the bunch will not of course suffer the direct beam–beam interaction and such a bunch will in general be stable.

Consider a beam 1 and a counter-circulating beam 2 and label the bunches in beam 1 with indices $i-n, \ldots, i, \ldots, i+n$ and that in beam 2 with indices $j-n, \ldots, j, \ldots, j+n$ such that bunch i in beam 1 encounters bunch j in beam 2 at the IP and bunch $j+1$ at the next downstream parasitic crossing, etc., as shown in Figure 1. A maximal Pacman effect will occur for bunch i in beam 1 when bunch j in beam 2 is present but a gap in the bunch pattern occurs either before or after bunch j. This is equivalent to all bunches in beam 2 being missing from $j+1$ through to $j+18$ or beyond or alternatively to bunches in beam 2 are absent from $j-1$ through to $j-18$ or beyond. If the beam intensity is only sufficiently large to render an unstable condition then indeed bunch i in beam 1 will be lost. But the existence of a single missing bunch in beam 1 will be far from a worst case and will not cause further losses in beam 2. However if bunch i is unstable when half or more of the corresponding parasitic crossings on one side of the IP are missing in beam 2 then missing bunches from $j+1$ onwards in beam 2 will cause not only bunch i to be unstable in beam 1 but also bunches $i-1$ through to

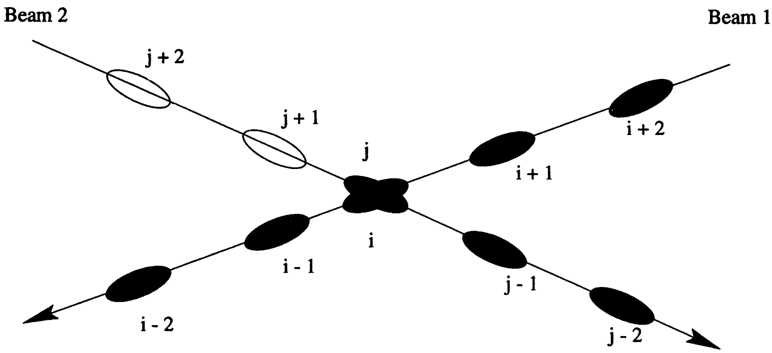

FIGURE 1 Beams 1 and 2 are two counter-circulating beams. There is a gap in Beam 2: bunches from $j+1$ through to $j+18$ or beyond are missing. Depending on machine parameters, this gap may cause two different types of Pacman effects: (1) The localized effect, when only bunch i in Beam 1 will get lost; (2) The run away effect, when bunches i, $i-1$ through to $i-9$ or beyond will be lost. The latter could destroy the machine luminosity.

$i-9$. Now beam 1 will have a gap sufficient to destabilize bunch $j-10$ in beam 2 and the process can continue. This is a run away Pacman effect that will eventually destroy the machine luminosity. Provided that the machine parameters, for operation at maximum luminosity, do not result in a run away Pacman effect, anomalous orbit displacements are not a problem. We therefore calculate below for the nominal parameters the orbit displacement at the IP corresponding to a pattern in the counter-circulating beam with half the bunches missing, or $N_p \sim 9$ on one or other side of the IP and show that it is small.

Using the LHC parameters in Table I and the number of parasitic crossings equal to 9, the spatial component of $\frac{1}{2}V$ (the symmetric case) is $0.06\sigma_x$ or $1\,\mu$m. Such an orbit displacement is very small and will contribute minimally to instability. For all practical purposes it is negligible and the orbit displacement will not contribute to any appreciable extent to a Pacman effect. Of course the additional tune spread from Pacman bunches, comparable to the head-on beam–beam tune shift, do play a major role and we assume that the machine design will permit the use of crossing planes rotated by 90°.

Herr[2] has previously investigated the impact of the Pacman effect on LHC running. Our results agree with his with the exception of our distinction of a localized loss of bunches (where we agree with Herr) and a run away Pacman effect, where two times higher bunch currents

TABLE I LHC parameters

Bunch spacing	7.5 m
Crossing separation	3.75 m
Emittance	5×10^{-10} m-rad
β-function at the IP β^*_{ip}	0.5 m
Head-on tune shift per IP $\Delta\nu_{\text{ho}}$	0.0034
Full crossing angle θ_{cross}	200 μrad ($6\sigma_{x'}$)
RMS beam transverse size σ_x	16 μm
RMS beam angular size $\sigma_{x'}$	32 μrad

are required to initiate losses. Herr points out that if more IPs than IPs 1 and 5 are run simultaneously at high luminosity the Pacman orbit effects are substantial and might require a bunch by bunch feedback control system. However it is presently envisaged that high LHC luminosity running will only occur simultaneously for IPs 1 and 5. Therefore with symmetric phasing both our and Herr's results show that feedback control of Pacman orbit displacements will be unnecessary.

3 CONCLUSIONS

Minimization or cancellation of anomalous Pacman induced tune spread and orbit displacement requires the use of paired IPs separated symmetrically by half a circumference and in phase by half the phase advance around the machine. The spatial separation by half a circumference is required so that the patterns of missing bunches in the counter-circulating beam as seen by a given bunch is identical at the two IPs. For crossing planes 90° relative to each other the anomalous tune spread is cancelled. The requirement of symmetric phasing minimizes the anomalous Pacman orbit displacements.

For LHC parameters the orbit displacement contribution to Pacman instability is small and, even for a 200 μrad "worst case" crossing angle is, for all effective purposes, negligible.

References

[1] John Irwin, SSC-233 (1989). (This reference can be retrieved on the web at the address: http://ccdb1.kek.jp/cgi-bin/img-index?8911363.)
[2] Werner Herr, LHC Project Report 39, CERN (1996).