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# ON THE EFFECTS OF FRINGE FIELDS IN THE LHC RING

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The effects of the dipole and quadrupole fringe fields on such machine parameter as chromaticity, anharmonicity, closed orbit, etc. are investigated by stepwise ray-tracing in the Version 4 of the LHC ring. First the ray-tracing method is described, and the relevant LHC fringe field data and corresponding numerical models are given. Then follows a detailed study of the machine parameters which shows the innocuity of the non-linearities introduced by the fringe fields, at injection conditions.

*Keywords:* Fringe fields; non-linear perturbations; dipole defects; quadrupole defects.

## 1 INTRODUCTION

The effects of the dipole and quadrupole fringe fields on the machine parameters are investigated by stepwise ray-tracing in the Version 4 of the LHC ring.[1,2] First the ray-tracing method is described, then follows an overview of the relevant LHC characteristics and in particular the fringe field data and the corresponding numerical models. Then follows a detailed study of the machine parameters.

## 2 THE RAY-TRACING METHOD

The equation of motion of a particle of charge $q$ and mass $m$ in a magnetic field $\vec{B}$,

$$m\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B} \tag{1}$$

is solved by stepwise Taylor expansions of the position $\vec{R}$ and normalized velocity $\vec{u} = \vec{v}/v$,

$$\vec{R}\,(M_1) = \vec{R}\,(M_0) + \vec{u}\,(M_0)\,ds + ..., \qquad \vec{u}\,(M_1) = \vec{u}\,(M_0) + \vec{u}'\,(M_0)\,ds + ... \tag{2}$$

with $ds = vdt$ = stepsize from point $M_0$ to $M_1$, $\vec{u}' = d\vec{u}/ds$ and $m\vec{v} = qB\rho\vec{u}$. The derivatives of $\vec{u}$ are given by $\vec{u}' = \vec{u} \times \vec{b}$, $\vec{u}'' = (\vec{u} \times \vec{b})' = \vec{u}' \times \vec{b} + \vec{u} \times \vec{b}'$, etc., involving the derivatives of $\vec{b} = \vec{B}/B\rho$ up to the fourth order.[3] A unique generic model for the magnetic field in the dipoles and quadrupoles of the ring is used, namely, the scalar potential[4]

$$V_n\,(x, z, s) = n!^2 \left\{ \sum_{q=0}^{\infty} (-)^q \frac{\alpha_{n,0}^{(2q)}(s)(x^2 + z^2)^q}{4^q q!(n+q)!} \right\} \left\{ \sum_{m=0}^{m=n} \frac{\sin m\frac{\pi}{2} x^{n-m} z^m}{m!(n-m)!} \right\} \tag{3}$$

where $x$, $z$, are the transverse coordinates, $\alpha_{n,0}(s)$ is the longitudinal form factor at $x = z = 0$, and $\alpha_{n,0}^{(2q)}$ its 2q-th order derivative w.r.t. the longitudinal coordinate $s$. If several harmonics are present the superposition theorem is applied. The longitudinal form factor writes[5]

$$\alpha_{n,0}(d) = \frac{1}{1 + \exp[P(d)]} \qquad P(d) = C_0 + C_1\frac{d}{\lambda} + C_2\left(\frac{d}{\lambda}\right)^2 + ..... + C_5\left(\frac{d}{\lambda}\right)^5 \tag{4}$$

where $d$ is the distance to the effective field boundary, and the numerical coefficients $\lambda$, $C_0$–$C_5$ are determined from a matching with the numerical fringe field data.

The prime interest of this integration method lies in its good symplecticity.

## 3 LATTICE AND FRINGE FIELD CHARACTERISTICS

The lattice of concern is the Version 4 at injection conditions.[1] The total machine tunes are $\nu_x/\nu_z = 66.28/66.32$. The normalized emittance is $\gamma\varepsilon/\pi = 3.75\ 10^{-6}$ m.rad in both planes. The four-fold superperiodicity machine is considered in this study.

The fringe fields involved,[6] for the dipole magnets (≪White book≫ design)[7] and the quadrupoles (CEA-Saclay)[8] are shown in Figure 1, with the fitting models (Equation 4) superimposed.

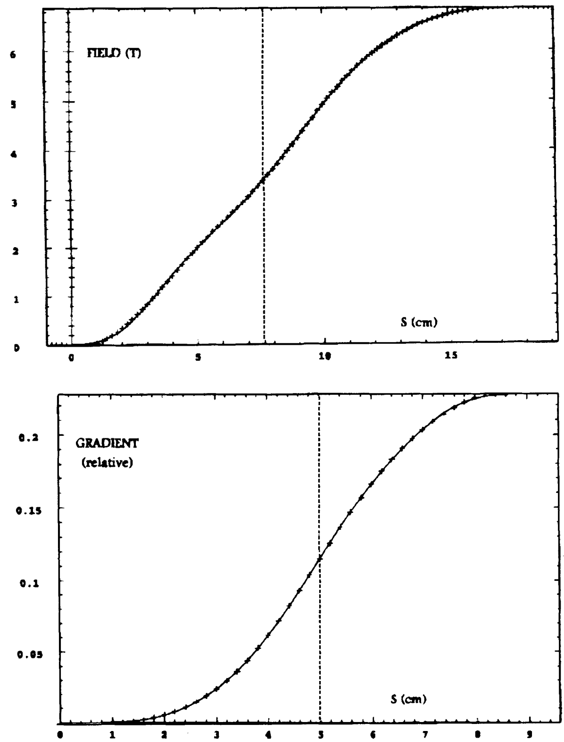


FIGURE 1 Fringe fields in the dipole (top; ≪White Book≫ design) and quadrupole (bottom; CEA-Saclay design). Solid line: the analytical model (Equation 4) used in the ray-tracing.

The values of the corresponding coefficients $\lambda$, $C_0$–$C_5$ are

$$\lambda = 0.112 \text{ m}, \quad C_0 = 0.1553, \quad C_1 = 3.875, \quad C_2 = -2.3622,$$
$$C_3 = 2.9782, \quad C_4 = 12.604, \quad C_5 = 15.026$$

for the dipole (magnetic length = 14.2 m), and also $K_1$.Gap = Fint.Gap = 2.1211 $10^{-2}$ m (in respectively the Transport and MAD[9] notations.) For the quadrupole (magnetic length = 3 m),

$$\lambda = 0.056 \text{ m}, \quad C_0 = -0.01097, \quad C_1 = 5.4648, \quad C_2 = 0.9968,$$
$$C_3 = 1.5688, \quad C_4 = -5.6716, \quad C_5 = 18.506$$

## 4 RAY-TRACING

Two means are used simultaneously to evaluate the machine parameters. On one hand multiturn ray-tracing (of the order of 2000 full 8-octants machine turns for one tune value) followed by Fourier analysis to get the tunes, or elliptical fit to get the optical functions, smear, etc. On the other hand, one-turn first order mapping followed by beam matrix computation (by identification with $\cos\mu I + \sin\mu J$.) In both cases the symplecticity is thoroughly checked, in terms of the smear by a calculation of the dispersion $\sigma(\varepsilon_{x,z}/\pi)$ in the first case, in terms of the second order symplectic conditions[10] in the second case. From both survey means, the chromaticity $d\nu/dp/p$, anharmonicity $d\nu/d\varepsilon/\pi$, the $\beta$ functions and their derivatives w.r.t. $dp/p$, the horizontal closed orbit, the dispersion functions $\eta_x$ and $\eta'_x$, etc., are evaluated. The results of the study are displayed in Figures 2–8 as follows.

The comparison between multiturn tracking and mapping is synthetized in Table I below, which shows the agreement between both means, in terms of the tune values.

TABLE I

| | $\nu_x$ | | $\nu_z$ | |
|---|---|---|---|---|
| $\delta p/p$ $(10^{-3})$ | *Multiturn* | *Mapping* | *Multiturn* | *Mapping* |
| −2 | 0.48834 | 0.48881 | 0.51050 | 0.51052 |
| −1.5 | 0.43700 | | 0.45915 | |
| −1 | 0.38576 | 0.38577 | 0.40790 | 0.40790 |
| −0.5 | 0.33464 | | 0.35678 | |
| $-10^{-4}(10^{-5})$ | 0.29381[1] | (0.28463)[2] | 0.31594[3] | (0.306762)[4] |
| 0 | 0.28362 | 0.28361 | 0.30575 | 0.305746 |
| $+10^{-4}(10^{-5})$ | 0.27342[1] | (0.28259)[2] | 0.29555[2] | (0.304723)[4] |
| 0.5 | 0.23268 | | 0.25480 | |
| 1 | 0.18177 | 0.18177 | 0.20391 | 0.203908 |
| 1.5 | 0.13080 | | 0.15298 | |
| 2 | 0.07937 | 0.07932 | 0.10176 | 0.10177 |

[1] $\Delta\nu_x/\delta p/p = -101.96$; [2] $\Delta\nu_x/\delta p/p = -101.95$; [3] $\Delta\nu_z/\delta p/p = -101.94$; [4] $\Delta\nu_z/\delta p/p = -101.96$

## 5 CONCLUSION

A thorough, well behaved (symplectic) ray-tracing through the 4-periodic LHC ring in the presence of the dipole fringe fields (White Book data) and quadrupole fringe fields (CEA-Saclay data), at injection conditions, shows that these have no effect on the chromaticity, anharmonicity and other relevant

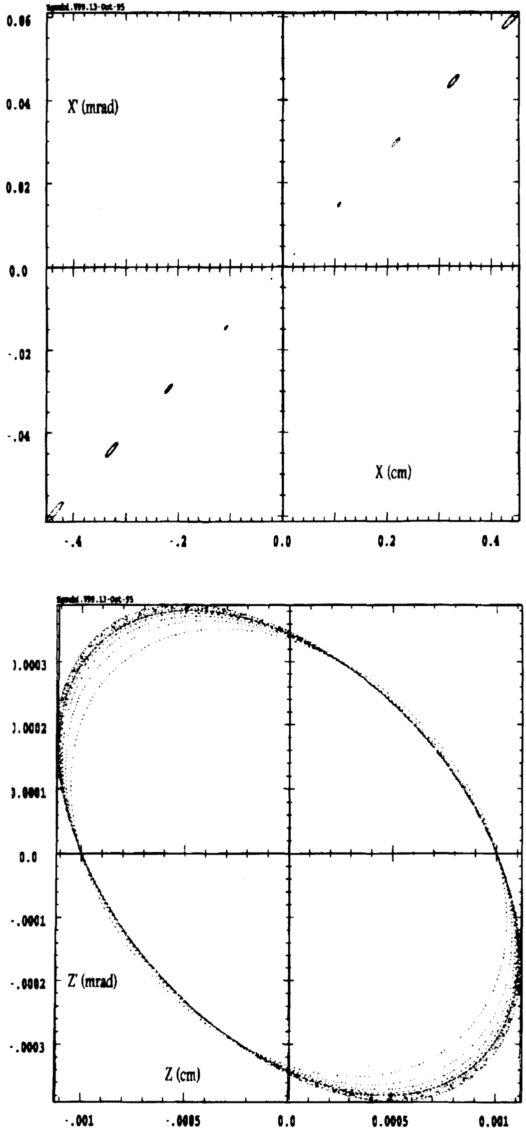


FIGURE 2 **Chromaticity calculations**, horizontal (top) and vertical (bottom) phase-spaces at S17LO (odd-type arc end), for $-210^{-3} \leq \delta p/p \leq 210^{-3}$, including $\delta p/p = \pm 10^{-4}$; $\varepsilon_x$ and $\varepsilon_z \ll \varepsilon_{\text{inj}} \approx 8.33\ 10^{-9}\ \pi$.m.rad. The ray-tracing provides high precision: the $z$ motion is resolved at much better than $10^{-7}$ m/$10^{-7}$ rad.

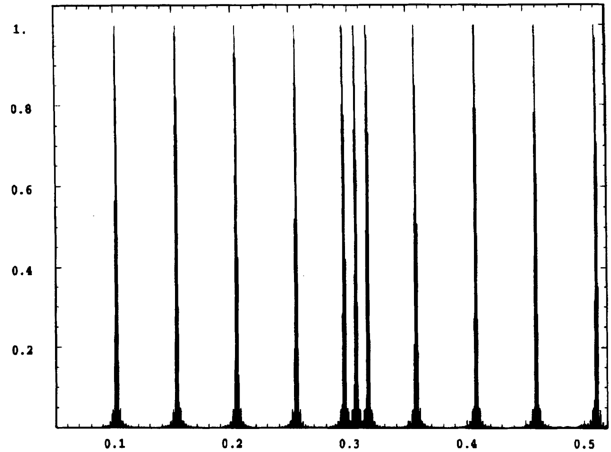


FIGURE 3 Vertical fractional tunes, depending on $\delta p/p$, after Fourier analysis of the vertical motions of Figure 2 (bottom). Similar results are obtained for the horizontal motion. From the tune values at $\delta p/p = \pm 10^{-4}$, the chromaticities $\nu_x'/\nu_z' = -102/-102$ come out.

machine parameters. The only sensitive effect is, as expected, a slight drift of the tunes ($\nu_x/\nu_z$ : 0.28/0.30 $\rightarrow$ 0.2836/0.3057) when the fringe fields are switched on, which can be fixed by slight re-tuning of the dipole field. More details on this study can be found in Reference 11.

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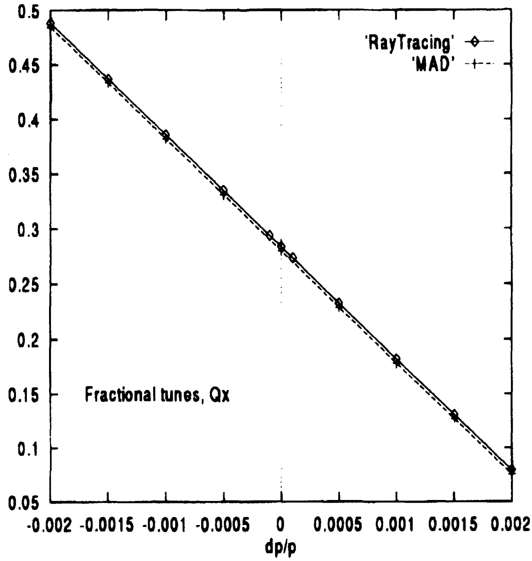


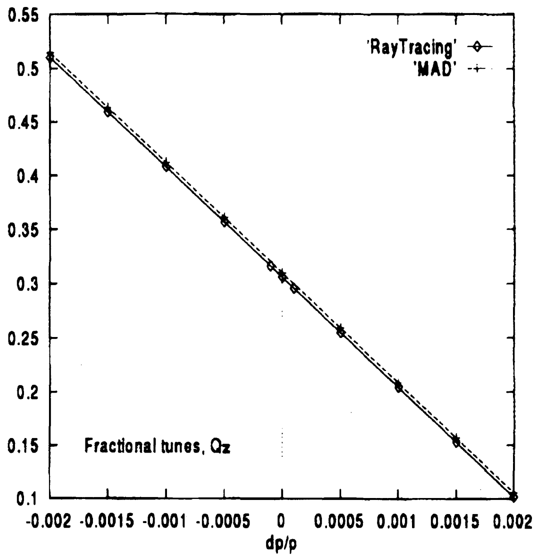


FIGURE 4 Horizontal (top) and vertical (bottom) tunes v.s. $dp/p$, from the survey of Figure 3. The tunes at $dp/p = 0$ are $\nu_x/\nu_z = 0.2836/0.3057$, to be compared to the fringe field free case, $\nu_x/\nu_z = 0.28/0.32$. Results obtained with MAD[9] and the FINT value given in Section 3 are provided, for comparison.

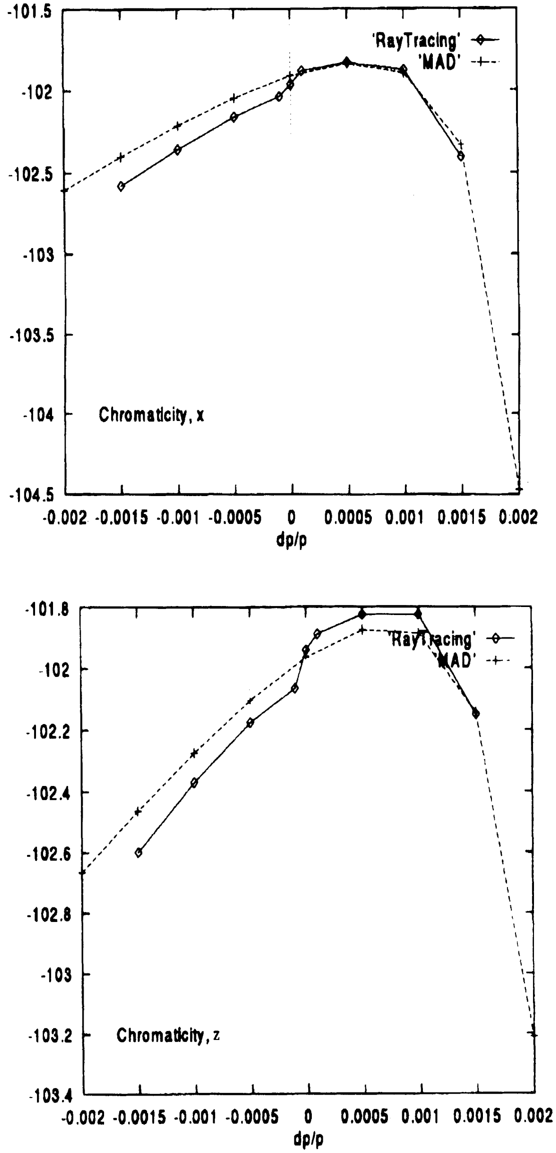


FIGURE 5 Horizontal (top) and vertical (bottom) tune derivatives $d\nu/dp/p$, v.s. $dp/p$. The chromaticities are $\nu_x'/\nu_z' = -102/-102$, identical to the fringe field free case (Figure 3).

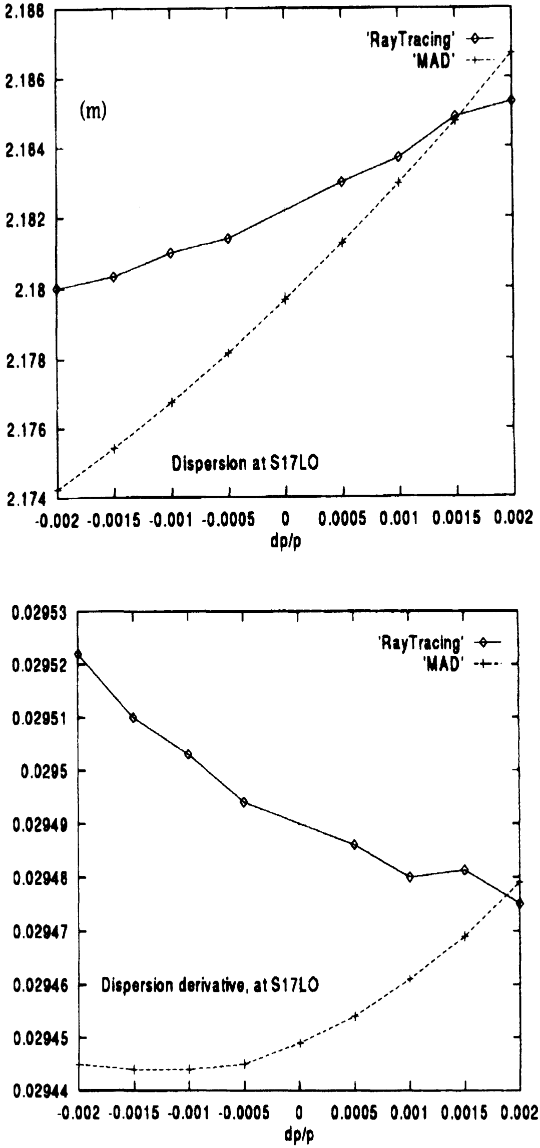


FIGURE 6 Dispersion function $\eta_x$ (top) and its derivative $d\eta_x/ds$ (bottom) v.s. $dp/p$, at S17LO.

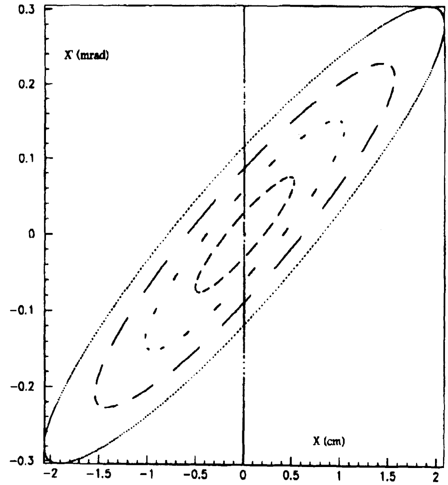


FIGURE 7 **Anharmonicities**: horizontal phase-space, from the ray-tracing of five particles on $x$, $z$ invariants ranging in $\varepsilon_{x,z}$ : $10^{-4} \rightarrow 200\varepsilon_{\text{inj}}$. The smear is negligible $[\sigma(\varepsilon_{x,z}/\pi) < 10^{-4}\varepsilon_{x,z}/\pi]$. The Fourier analysis gives $\nu_x/\nu_z = 0.2836/0.3057$, for any $\varepsilon_{x,z}$ that is, zero anharmonicities $d\nu_{x,z}/d\varepsilon_{x,z}$: whatever the amplitude the non-linearities due to the fringe fields have but negligible effect on the tunes.

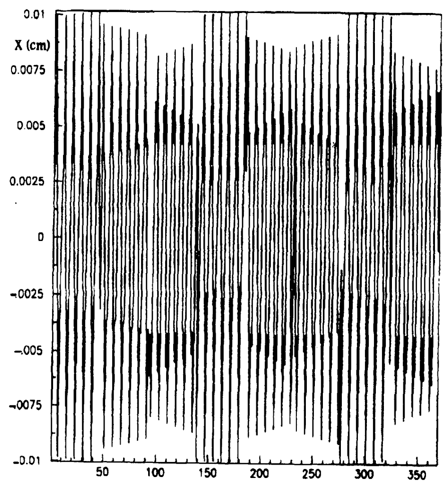


FIGURE 8 **Close orbit**: non-zero horizontal closed orbit along the machine as induced by the fringe fields. The horizontal axis represents the pick-up number (from the MAD files). The closed orbit excursion does not exceed $10^{-4}$ m.