Spin-tensor decomposition:

A useful tool for shell model effective interaction

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Abstract. The spin-tensor decomposition is employed to construct a new interaction, named CKHeN, for $0p$-shell. This new interaction is used to calculate the effective single-particle energies of $\pi 0p_{3/2}$ and $\pi 0p_{1/2}$ orbitals in Li isotopes, and the level structures of ^{7,8,9}Li isotopes. The calculated level structures are found in good agreement with experimental data.

1 Introduction

Spin-tensor decomposition (STD) is a useful tool to decompose the model-space dependent shell model effective two-nucleon interaction into its central, spin-orbit and tensor force structure [1]. For last one and half decades, it has been used with the aim to understand the role of different components of two-nucleon interaction in the shell evolution in neutron-rich nuclei [2–4]. It has also been used to show why microscopic shell model interactions fail to describe the shell evolution in neutron-rich nuclei [3]. In this study, we use STD for the CK(8-16) interaction derived for $0p$-shell nuclei, and examine the properties of its total spin (J) averaged proton-neutron central and tensor force matrix elements;

$$\bar{V}^{\pi\nu}_{jj'} = \frac{\sum_J (2J+1) V^{\pi\nu}_{jj'}}{(2j+1)(2j'+1)}, \quad (1)$$

where, sum runs only over the Pauli principal allowed J values.

For bare-tensor force, Otsuka *et. al.,* [6], showed that proton-neutron interaction $\bar{V}^{\pi\nu}_{jj'}$ corresponding to proton spin-up orbital ($j_> = l + 1/2$) and neutron spin-down orbital ($j'_< = l' - 1/2$) (or vice-versa) is attractive, whereas, if both proton and neutron orbitals are spin-up (or spin-down), the interaction is repulsive. It is also demonstrated that bare-tensor force matrix elements barely change and hold its nature after dealing with short-range repulsion part of two-nucleon problem and in-medium effects [7]. Furthermore, the numerical analysis shows that tensor interaction of well-established shell model effective interaction, *e.g.,* USDB, has same nature as for bare-tensor force.

The proton-neutron central component $\bar{V}^{\pi\nu}$ of shell model effective interaction is found to possess strong-orbital node *(nl)* and weak-spin *(j)* dependency [8]. It means that proton-neutron central force matrix elements $\bar{V}^{\pi\nu}_{j_>j'_>}$, $\bar{V}^{\pi\nu}_{j_<j'_<}$, and $\bar{V}^{\pi\nu}_{j_>j'_<}$ or ($\bar{V}^{\pi\nu}_{j_<j'_>}$) corresponding to proton l-orbital and neutron l'-orbital are nearly same. The strong-orbital node property of central interaction has been analytically demonstrated by Smirnova *et. al.,* [9] using the spin-exchange zero-range δ potential.

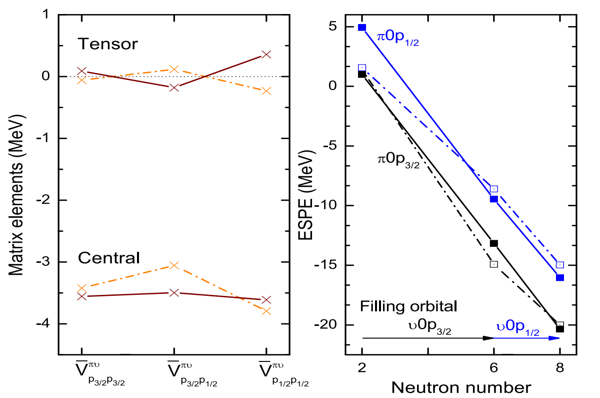

Figure 1. Left: J-averaged proton-neutron central and tensor force matrix elements in $0p-$shell. Right: ESPEs of $\pi - 0p$ orbitals in Li isotopes. The dot-dash and solid line are used for CK(8-16) and CKHeN interactions, respectively

In Fig. 1, we show proton-neutron central and tensor matrix elements of the CK(8-16) interaction. Here, central force matrix elements do not have similar strength which manifest that central component of CK(8-16) lacks weak-spin dependency. Further, tensor force matrix elements are present with opposite nature than its regular nature. In present work, we construct a new interaction in which these discrepancies are not present.

2 Spin-tensor decomposition and New effective interaction - CKHeN

Spin-tensor decomposition: Nucleons are intrinsic spin 1/2 fermions; therefore, the interaction between two-nucleon can be written as the linear sum of scalar product of configuration space operator Q and spin space operator S of

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rank k [1];

$$V = \sum_{k=0}^{2} V(k) = \sum_{k=0}^{2} Q^k.S^k \tag{2}$$

where rank k = 0, 1 and 2 represent central, spin-orbit and tensor force, respectively. Using the LS-coupled two-nucleon wave functions, the matrix element for each V^k can be calculated from matrix element V [10].

CKHeN interaction: To construct a new effective interaction for $0p$-shell, we have considered the single-particle matrix elements and isospin T = 1 two-body matrix elements of the interaction developed in Ref. [11], and T = 0 two-body matrix elements of the CK (8-16) interaction. The two-body matrix elements of this integrated interaction, mainly diagonal matrix elements, have been modified to gain common features for central and tensor matrix elements. Note that the spin-tensor decomposition has been performed at each step of modification to check the properties of central and tensor force matrix elements. The final interaction is named CKHeN, and its proton-neutron central and tensor force matrix elements are shown in Fig. 1. In this new interaction, central force matrix elements are present with good weak-spin dependency, and tensor force matrix elements are present with its characteristic properties.

3 Shell model calculations

The CKHeN interaction consists of $p_{3/2}$ and $p_{1/2}$ orbitals for protons and neutrons above ^{4}He core, and is tested for the effective single-particle energies (ESPEs) of $\pi 0p_{3/2}$ and $\pi 0p_{1/2}$ orbitals in Li isotopes and the level structures of ^{7,8,9}Li isotopes. The ESPE (ε') of π-j orbital in Li isotopes is given by [8];

$$\varepsilon'^{\pi}_{j} = \varepsilon^{\pi}_{j} + \sum_{j'} n^{\nu}_{j'} \bar{V}^{\nu\pi}_{j'j} \tag{3}$$

where, ε_j is unperturbed single-particle energy of orbital j, and $n_{j'}$ is number of neutrons in orbital j'. The level structures of ^{7,8,9}Li isotopes are calculated using shell model code-NUSHELLX@MSU [12]. The calculated ESPEs and level structures are shown in Fig. 1 and 2, respectively. The results obtained using the CK(8-16) interaction are also shown in these figures.

With the CKHeN interaction, the energy gap between spin-orbital partners, $\pi 0p_{3/2}$ and $\pi 0p_{1/2}$, in Li isotopes is found to remain nearly constant (see Fig. 1). This is similar to the observation seen in F and Sc isotopes where the energy gap between $\pi 0d_{5/2}$ and $\pi 0d_{3/2}$ orbitals, and $\pi 0f_{7/2}$ and $\pi 0f_{5/2}$ orbitals, respectively, remains nearly constant [8, 13]. However, with the CK(8-16) interaction, the energy gap between spin-orbital partners in Li isotopes increases when neutrons occupy $\nu 0p_{3/2}$ orbital, which is not consistent with the systematic.

In Fig. 2, the CKHeN interaction is shown to reasonably predict the experimental low-lying states of ^{7,8,9}Li relative to the CK(8-16) interaction.

4 Summary

In this work, spin-tensor decomposition is employed to discuss the discrepancies of CK(8-16) interaction and to construct a new effective interaction, named CKHeN, for $0p$−shell. The new interaction is tested for Li isotopes and found reasonably predicting their spectroscopic properties.

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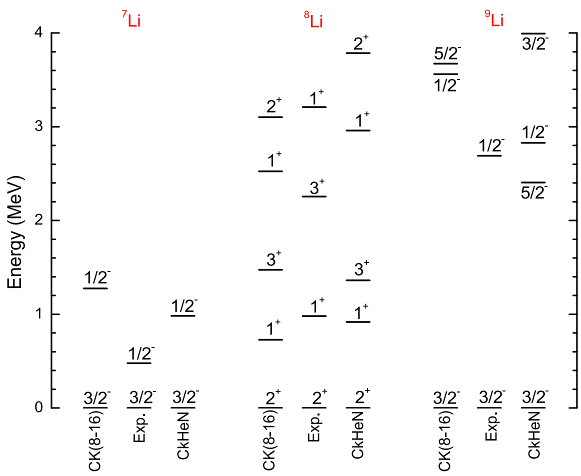

Figure 2. Level structure of ^{7,8,9}Li. Experimental data is taken from Ref. [14].

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