XVI. COMPUTATION RESEARCH*

Research Staff

Martha M. Pennell, Heather Davis, Gail M. Fratar, Joan Harwitt, Elaine Isaacs, Eleanor River

A. LINEARIZING THE ROOTS OF A POLYNOMIAL

Probably the most common application of the method of least squares is the determination of constants in an empirical formula whose form is to be inferred from the results of experimental data. The method, however, can equally well be used to approximate known functions by less complicated formulae. The following problem submitted to us by the Microwave Spectroscopy Group is an example of the latter. Find three sets of constants $\phi_1, \ldots, \phi_5$ such that one of the following expressions:

1) $\phi_1\omega_o + \phi_2\omega_4$
2) $\phi_1\omega_o + \phi_2\omega_4 + \phi_3\omega_6$
3) $\phi_1\omega_o + \phi_2\omega_4 + \phi_3\omega_6 + \phi_4\omega_8$
4) $\phi_1\omega_o + \phi_2\omega_4 + \phi_3\omega_6 + \phi_4\omega_8 \ddagger \phi_5\omega_{10}$

would best approximate the three positive real roots of the following sixth-degree polynomial in V in the range $0 \leqslant \theta \leqslant 45°$, $0 \leqslant \gamma \leqslant 45°$:

$$V^6 - 28.38V^4 - (47.38\omega_4 - 226.14)V^2 - (550.33 - 259.09\omega_4 + 183.55\omega_6) = 0 \tag{1}$$

$$\omega_o = 1$$

$$\omega_4 = x^4 + y^4 + z^4 - 3/5$$

$$\omega_6 = x^6 + y^6 + z^6 - 15/11\ \omega_4 - 3/7$$

$$\omega_8 = x^8 + y^8 + z^8 - 28/15\ \omega_6 - 210/143\ \omega_4 - 1/3$$

$$\omega_{10} = x^8 + y^{10} + z^{10} - 45/19\ \omega_8 - 42/17\ \omega_6 - 210/143\ \omega_4 - 3/11$$

$$x = \sin\theta \cos\gamma$$

$$y = \sin\theta \sin\gamma$$

$$z = \cos\theta$$

Equation 1 was solved for all possible combinations of $\theta, \phi = 0, 9, 18, \ldots, 45°$.

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Table XVI-A.

Equation Number	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
8 points					
1)	4.11098	.496842			
2)	4.11108	.496822	-.0717939		
3)	4.11130	.506908	-.0509087	-.405050	
4)	4.11137	.506378	-.0517689	-.403467	.217128
12 points					
1)	4.11129	.4960810			
2)	4.11111	.498560	-.0365156		
3)	4.11126	.507561	-.0459987	-.420735	
4)	4.11127	.507441	-.0464354	-.421370	.0686554

Table XVI-B.

Equation Number	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
8 points					
1)	4.11037	.498241			
2)	4.11105	.496856	-.0283327		
3)	4.11123	.506867	-.0331734	-.432111	
4)	4.11108	.508400	-.0518164	-.442301	.348897
12 points					
1)	4.11108	.500282			
2)	4.11120	.500578	-.0331930		
3)	4.11126	.507069	-.0390402	-.432419	
4)	4.11128	.507350	-.0437185	-.438920	.185722

n arbitrary roots on the same roots locus were then used as the data to a least squares analysis whose fitting functions were the ω's given above. Both n and the roots used as data were varied. The results are summarized below. Table XVI-A approximates the root with the largest magnitude using 8 and 12 points. Table XVI-B repeats the same calculations of a different set of 8 and 12 points.

Martha M. Pennell, Heather Davis