VIII. GASEOUS ELECTRONICS*

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A. COMPARISON OF MEASURED TIME-DEPENDENT ELECTRON VELOCITY DISTRIBUTIONS WITH A THEORETICAL MODEL

In Quarterly Progress Report No. 80 (page 99) a method was discussed for extracting information on the velocity distribution of electrons in a plasma from measurements of the spectrum of emitted microwave radiation. Briefly, in the neighborhood of ω_B, the electron cyclotron frequency of an applied magnetic field, the departure of the effective radiation temperature, $Tr(\omega)$, from a constant value reflects the departure of the electron velocity distribution, $f(v)$, from a Maxwellian. By inversion of an integral relation, $f(v)$ can in principle be determined from $Tr(\omega)$, providing the velocity-dependent electron-atom collision frequency $\nu(v)$, is known. In practice, a shortened expansion of $f(v)$ in Hermite polynomials is adjusted for best fit of the data. The results previously presented illustrated the time dependence of the distribution function early in the afterglow of a weakly ionized argon discharge. At gas pressure ≈ 0.7 Torr and electron density $\approx 10^{10}\ \text{cm}^{-3}$, it was noted that relaxation to a Maxwellian distribution on a time scale of a few microseconds was accompanied by significant loss of electron energy. Crude analysis seemed to indicate that electron-atom elastic collisions alone could not account for this relatively rapid cooling. It is the purpose of this report to present the results of a more comprehensive attack on that problem.

A theoretical treatment of the behavior of a weakly ionized plasma may be achieved by examination of the Boltzmann equation. We present it here in a form given by Allis.[1]

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$$\frac{\partial}{\partial t} f(v,t) = \frac{1}{3}\left(\frac{eE}{m}\right)^2 \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{v^2}{\nu}\frac{\partial f}{\partial v}\right]$$

$$+ \frac{2m}{M}\frac{1}{v}\frac{\partial}{\partial (v^2)}\left[v^3 \nu f\right]$$

$$+ \sum\left[\nu_x(v_x)\frac{v_x}{v} f(v_x) - \nu_x(v)\, f(v)\right]$$

$$+ n\left(\frac{e^2}{\epsilon_o m}\right)^2 \ln\left(12\pi n \ell_D^3\right)\left\{f^2\right.$$

$$+\left[\frac{1}{v^2}\int_0^v V^2 f dV - \frac{1}{3v^4}\int_0^v V^4 f dV + \frac{2}{3v}\int_v^\infty V f dV\right]\frac{\partial f}{\partial v}$$

$$\left. + \left[\frac{1}{3v^3}\int_0^v V^4 f dV + \frac{1}{3}\int_v^\infty V f dV\right]\frac{\partial^2 f}{\partial v^2}\right\},$$

where e is the charge of the electron, m is its mass, M is the mass of a neutral atom, E is the applied (DC) electric field, ℓ_D is the Debye length, n is the electron density, and $4\pi \int_o^\infty v^2 f dv = 1$. The first term on the right represents the driving force of the applied electric field; the second, elastic recoil of the electron-atom collisions, and the third reflects the transfer of energetic electrons to low velocities through inelastic impacts. Here, $\nu_x(v)$ is the frequency of excitation of an atomic level at an energy eV_x, and v_x is given by $v_x^2 = v^2 + 2eV_x/m$. The last term in brackets describes electron-electron interaction. It is an approximation based on a Fokker-Planck treatment of the fluctuating particle fields. These are the four mechanisms that are thought to be dominant under the conditions of the experiment.

The analysis of the expression above proceeds in two steps. First, the steady-state solution $\left(\frac{\partial f}{\partial t} = 0\right)$ is determined for given values of pressure, density, and electric field $(E \neq 0)$. The calculation is performed on a digital computer using a method similar to that employed by Dreicer[2] in his analysis of discharges in hydrogen. Once the steady state has been found, removal of the applied field $(E = 0)$ provides the instantaneous time derivative of the distribution function $\left(\frac{\partial f}{\partial t} \neq 0\right)$ at the beginning of the afterglow. The constituent parts of $\partial f/\partial t$ are also examined separately and characteristic rates can be determined for electron-electron, recoil, and inelastic processes.

As an illustration of this method we consider the analysis of the discharge conditions presented in the previous report: argon pressure = 0.723 Torr, electron density = 1.1×10^{10} cm^{-3}. The mean electron energy, U, was found to have a value of 4.56 ev during the pulse and its decay upon removal of the applied field at time t = 0 is illustrated in Fig. VIII-1. The initial rate of energy loss as given by the slope of the straight

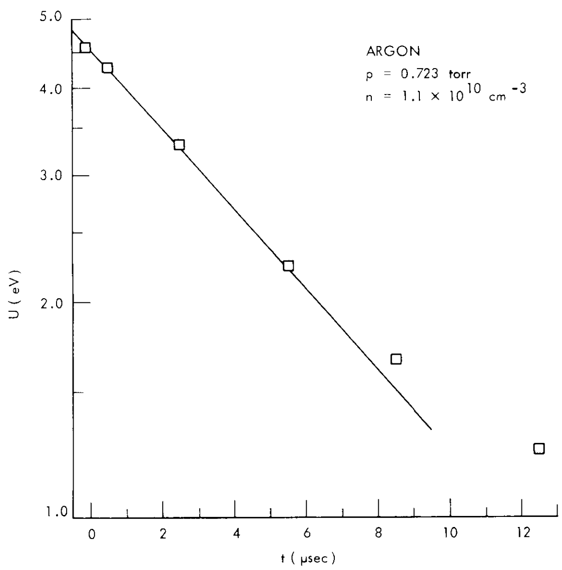

Fig. VIII-1. Afterglow energy decay.

line is 0.59 ev/μsec. The measured argon pressure and electron density were fed directly into the program for computation of the steady-state distribution function. A value of the applied electric field was so chosen that the calculated mean energy would be close to the measured value of 4.56 ev.

A solution obtained with U = 4.54 ev is indicated by the dashed curve in Fig. VIII-2. The solid curve is the velocity distribution inferred from radiation measurements taken during the pulse.

It is also of interest to compare the Tr(ω) peaks generated by these two functions. In Fig. VIII-3, Tr is plotted in terms of a normalized frequency difference

$$\Delta = \left(\frac{m}{2e}\right)^{1/2} \frac{\omega - \omega_B}{100\ p}.$$

The solid and open circles are data points at ω above and below ω_B, respectively. This logarithmic plot tends to emphasize the disparity near Δ = 0, where

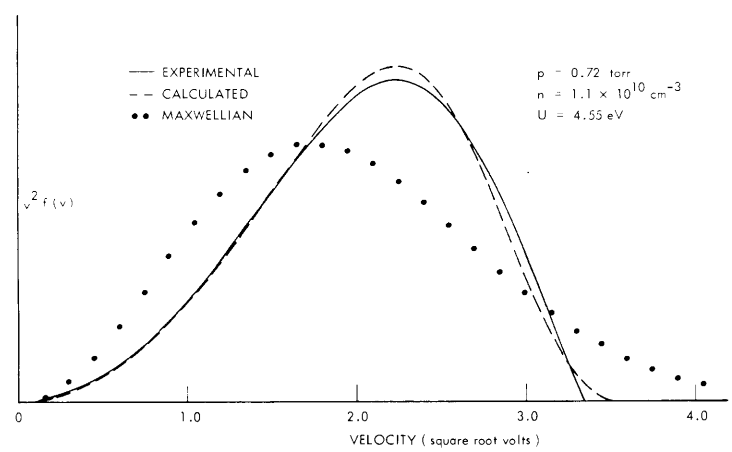

Fig. VIII-2. Comparison of experimentally determined velocity distribution and solution of the Boltzmann equation.

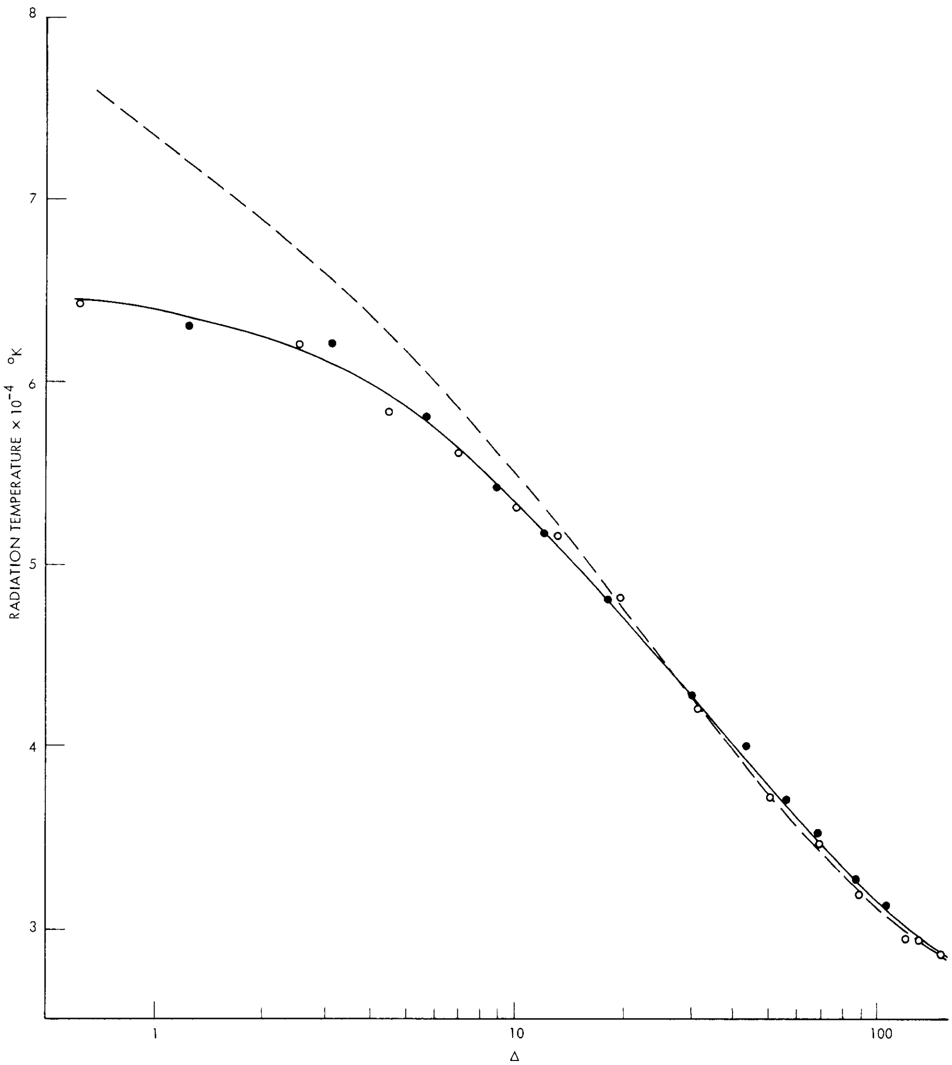

Fig. VIII-3. Comparison of experimental and calculated emission spectra.

Tr(ω) is a very sensitive function of f(v). Under the conditions of the experiment, Δ is usually certain only within ± 1.

Figure VIII-4 shows the computed instantaneous time derivative of $v^2f(v,t)$ at time t = 0. The total derivative shown in the top curve offers a rather complex structure, but it is easily understood in terms of the constituent processes presented beneath. The recoil term exhibits a smooth transfer of electrons to lower energies, in contrast with the dumping caused by the first excitation level at 11.5 ev. Integration over velocities reveals that recoil and excitation represent energy loss rates of 0.409 and 0.128 ev/μsec for a combined total of 0.537 ev/μsec. In comparing this figure with the measured value, 0.59 ev/μsec, it should be pointed out that the loss rate is a sensititve function of the

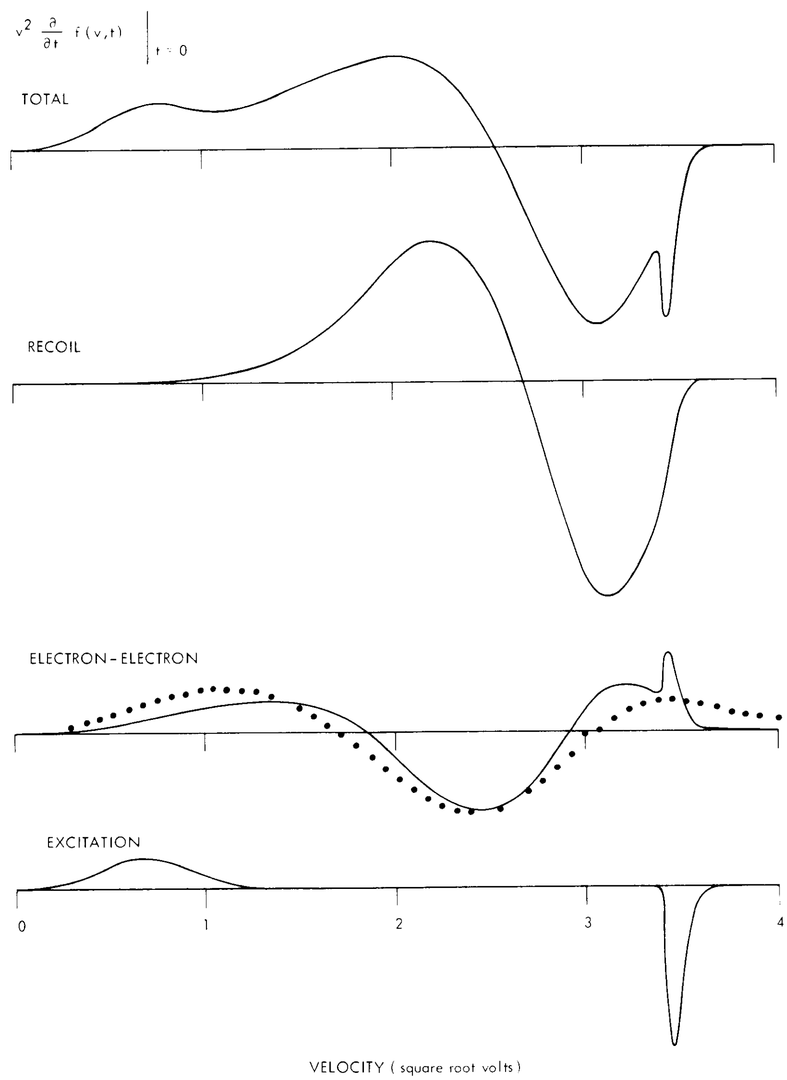

Fig. VIII-4. $v^2 \frac{\partial f}{\partial t}$ at t = 0, showing breakdown into component mechanisms.

mean energy. For instance, a change here in E of 6 per cent would affect U by only 1.75 per cent, but dU/dt would be increased by 10 per cent.

Although the electron-electron interaction term conserves energy, as well as particle number, it clearly participates in the cooling by continuously feeding electrons into the excitation region and thus prevents rapid depopulation. In an attempt to obtain a characteristic electron-electron relaxation time, we have shown for comparison with $v^2 \left.\frac{\partial f}{\partial t}\right|_{ee}$ the dotted curve given by $v^2(f_M-f)/\tau$, where f_M is a Maxwellian distribution with the same mean energy. τ, which represents a time scale for Maxwellization, is chosen for the best fit. In this case τ = 4.6 μsec, which may be compared with

Spitzer's[3] slowing-down time for a test electron of energy U; $t_s = 13.8$ μsec. A rough estimate of the experimental rate of Maxwellization suggests a figure between 5 and 6μsec.

In conclusion, the various comparisons cited above between observed and expected behavior of the electron velocity distribution seem to substantiate both the experimental method and the theoretical model.

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References

1. W. P. Allis "Motions of Ions and Electrons," Technical Report 299, Research Laboratory of Electronics, M.I.T., June 13, 1956.
2. H. Dreicer, Phys. Rev. 117, 343 (1960).
3. L. Spitzer, Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York, 1956).