Self-Field Effects in Magneto-Thermal Instabilities for Nb-Sn Strands

B. Bordini, E. Barzi, S. Feher, L. Rossi, and A. V. Zlobin

***Abstract*—Recent advancements in the critical current density (J_c) of Nb_3Sn conductors, coupled with a large effective filament size, have drawn attention to the problem of magneto-thermal instabilities. At low magnetic fields, the quench current of such high J_c Nb_3Sn strands is significantly lower than their critical current because of the above-mentioned instabilities. An adiabatic model to calculate the minimum current at which a strand can quench due to magneto-thermal instabilities is developed. The model is based on an 'integral' approach already used elsewhere [1]. The main difference with respect to the previous model is the addition of the self-field effect that allows to describe premature quenches of non-magnetized Nb_3Sn strands and to better calculate the quench current of strongly magnetized strands. The model is in good agreement with experimental results at 4.2 K obtained at Fermilab using virgin Modified Jelly Roll (MJR) strands with a low Residual Resistivity Ratio (RRR) of the stabilizing copper. The prediction of the model at 1.9 K and the results of the tests carried out at CERN, at 4.2 K and 1.9 K, on a 0.8 mm Rod Re-Stack Process (RRP) strand with a low RRR value are discussed. At 1.9 K the test revealed an unexpected strand performance at low fields that might be a sign of a new stability regime.**

***Index Terms*—Instability, magnet, Nb_3Sn, superconductor.**

I. INTRODUCTION

THe high critical current density (J_c) of state of the art Nb_3Sn wires for high energy physics, coupled with large effective filament size (D_{eff}) and high electrical resistivity of the stabilizing copper have drawn attention to the problem of flux jumps [1]. These flux jumps, caused by magneto-thermal instabilities [2]–[5], can quench the superconductor and severely limit the strand performance.

In order to be sufficiently stable the superconducting filaments must be tightly twisted and the product between J_c and D_{eff} must be sufficiently small. The most conservative criterion to establish the dimensions of a stable filament is based on the 'adiabatic' assumption [3] with no stabilizer. If the filament is not adiabatically stable, the first flux jump can occur when the applied magnetic field (B_a) is changed by a value larger than a certain value $(\varDelta B_{fj})$ [3]. For Nb_3Sn at 4.2 K the value of $\varDelta B_{fj}$ is about 0.3 T, independently of the B_a value.

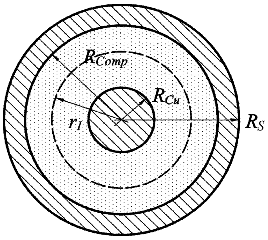

Fig. 1. Simplified cross section of a high J_c Nb_3Sn strand. The strand is composed by three concentric regions : a central copper core with an approximately circular section of radius equal to R_{Cu}, an annular composite of outer radius equal to R_{Comp} and, an external copper shell whose external radius is equal to the strand radius R_S.

Flux jumps can still be present in high J_c strands driven by the 'self-field' instability [4] even with very small and tightly twisted filaments. This type of instability is caused by the uneven distribution of the transport current (I) while increasing the current at a fixed B_a. In these conditions the multifilamentary strand acts like a large monofilament whose radius is equal to the composite radius (R_{Comp}, Fig. 1), with a critical current density equal to λJ_c, where λ is the fraction of the *non-Cu* area (Nb_3Sn, bronze and, barriers) only in the composite. Taking as reference Fig. 1, for a certain I, the current flows, with $J = \lambda J_c$, only in the region delimited by r_I and R_{Comp} [4]. An adiabatic criterion of the self-field stability for a round monofilament was also developed in [5]. It establishes the current value at which a flux jump may happen.

This paper shows that high J_c Nb_3Sn wires may suffer premature quenches due to the self-field instability. This instability is especially dangerous in the intermediate field region where the quench current (I_q) may become lower than the strand design current used in a Nb_3Sn magnet. A model is also presented to compute the quench current of Nb_3Sn strands affected by magneto-thermal instabilities in the cases of non-magnetized and strongly magnetized strands. Since the model is adiabatic, it is mainly applicable to strands whose stabilizing copper has a low Residual Resistivity Ratio (RRR).

II. SELF-FIELD INSTABILITY IN Nb_3Sb STRANDS

The criteria for superconductor stability [2]–[5] are generally based on a 'differential' approach which analyzes the development of a perturbation using the assumption that the superconductor properties are constant. However, these properties can change significantly. For example the Nb_3Sn specific heat increases significantly with temperature (T) and that improves the superconductor stability. Thus, there are situations in which a flux jump can start at T_0 but then stop at a temperature below the critical temperature (T_c). Hence, the 'differential' criteria determine the conditions in which a flux jump might start but they do not give any information regarding the possibility of quenching the strand. A model is presented in the next section that estimates the necessary conditions for having a self-field

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flux jump capable of quenching a non-magnetized strand in a constant field (B_a). In Section II-C the effect of strand magnetization will be analyzed.

Flux jumps are stochastic processes. They happen when an external perturbation is applied and the superconductor does not satisfy the 'differential' stability criteria. In this context a perturbation is an energy deposition on the strand much smaller than the minimum quench energy. Therefore, the self-field models presented in this paper do not predict at which current a quench will occur but they do estimate the minimum current values at which a flux jump has sufficient energy to start a quench.

A. Model of Self-field Instability

The model is based on the same hypothesis made by Wilson to derive the 'differential' criterion for the self-field stability [4]. The differences with respect to Wilson's model include the simplification of the strand geometry shown in Fig. 1 and the following assumptions: 1) during a flux-jump the temperature changes only in the composite section carrying the transport current and, in that region, it is uniform; 2) J_c is a function of temperature and of the peak field (including the self-field) described by a scaling law optimized for MJR strands [6]. The most relevant difference with respect to Wilson's model is the use of an 'integral' approach in which the model calculates the energy released by a full penetration of the self-field and not the energy due to an infinitesimal penetration. An 'integral' approach was already used by other authors for different conductor geometries and different current density distributions [1], [7]. In what follows the term 'integral' model will refer to the model described in this paper.

For a certain current value the 'integral' model calculates the energy per unit length that can be released by a complete redistribution of the transport current and of the self-field within the composite. With these assumptions an infinitesimal penetration of the self-field dissipates an infinitesimal amount of energy (per unit length) dQ that can be calculated using Wilson's method [4]. Integrating dQ over the whole penetration of the self-field in the composite gives the total energy released per unit length [8]:

$$Q = -\mu_0 \lambda \frac{I^2}{\pi} \int_{\varepsilon_o}^{\varepsilon_f} \left[\frac{2\varepsilon}{(1-\varepsilon^2)^3} \left\{ -\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right\} \right] d\varepsilon \quad (1)$$

where: $\mu 0$ is the vacuum permeability; $\varepsilon_0 = r_I / R_{Comp}$; r_I is the internal radius of the annular section with the transport current, Fig. 1; $\varepsilon_f = R_{Cu}/R_{Comp}$; ε is the normalized radius.

For the 'integral' model the flux-jump may quench the strand if the composite final temperature due to Q (1) is higher than $T_c(I, B)$ ('first condition of the integral model'). In order to fit the experimental data at 4.2 K in the medium-high field range of 7–11 T, an additional condition for the quench development was introduced in the 'integral' model. The energy released by the flux jump must heat up the composite by more than a certain ΔT ('second condition of the integral model'). Taking into account that the flux jump is a local phenomenon which may or may not propagate longitudinally [9], it is necessary to have a 'complete' flux jump that propagates longitudinally to quench a strand. Since the model described above does not consider the longitudinal strand direction, the additional condition was introduced to take into account the longitudinal propagation of the flux jump. The parameter ΔT was chosen to be equal to 4.8 K based on the experimental data for one Modified Jelly Roll (MJR) strand (Table I—ref. Fig. 3) and was not modified to predict the behavior of the other wires. In a further development of the theory this parameter may become an output rather then a fitting parameter. Thus, in the 'integral' model the flux-jump may quench the strand if the two conditions described above are satisfied. Note that D_{eff} does not play any role in the model.

TABLE I
STRAND PROPERTIES

Ref.	Strand diam. [mm]	Strand type	J_c @ 4.2 K-12 T [A/mm^2]	B_{c2} @ 4.2 K [T]	D_{eff} [μm]	RRR	λ	R_{Comp} [mm]	R_{Cu} [mm]
Fig. 2	0.7	MJR 54/61	2000	22.73	70	-	0.89	0.289	0.098
Fig. 3	0.7	MJR 54/61	2123	22.73*	70	≤7	0.89	0.289	0.098
Fig. 4	1	MJR 54/61	1671	22.33*	100	≤7	0.89	0.413	0.14
Fig. 5/6	0.8	RRP 54/61	2602	24.54*	80	8	0.87	0.329	0.115

* Determined from critical current measurements

** Measurements by D. Richter, analysis by C. Scheuerlein (CERN)

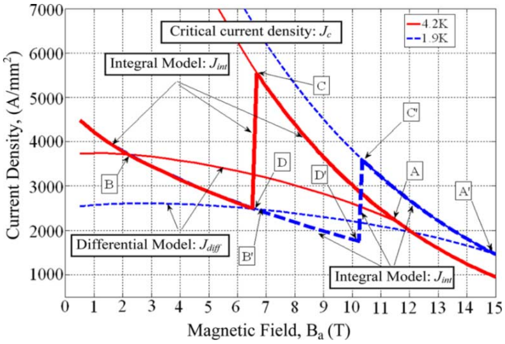

Fig. 2. Modeling self-field instability in a 0.7 mm MJR strand: our model is represented by the 'Integral Model: J_{int}' curves, see text for details.

The results calculated by the 'integral' model for a 0.7 mm MJR strand at 4.2 K and 1.9 K are shown in Fig. 2. The strand properties used in the model are summarized in Table I. In the plot the current densities are averaged over the *non-Cu* area. The 'integral' model is represented by the curves labeled as 'Integral model: J_{int}'; in the plot there are also the intrinsic J_c curves and the curves obtained adapting the self-field 'differential' adiabatic criterion for a monofilament surrounded by a sufficiently thick copper shell [5] at the annular geometry and substituting J_c with λJ_c. These curves are labeled as 'Differential model: J_{diff}' and they represent the minimum current density at which a flux jump can start.

The 'differential' model predicts whether a flux jump can start or not, but one needs the 'integral' model to establish whether the energy of the flux jump is sufficient to initiate a quench or not. The 'Integral' model predicts that: 1) the strand reaches its J_c for high B_a; 2) for B_a lower than a certain value (C or C′ in Fig. 2 depending if one considers $T = 4.2$ K or $T = 1.9$ K respectively) premature quenches occur and the minimum quench current density is determined by the 'first condition of the integral model' (to satisfy the first condition in this field region implies that the second condition is also satisfied). The 'Integral' model also predicts that, for $B_a < B_C$, J_{int} is almost the same at 4.2 K and 1.9 K.

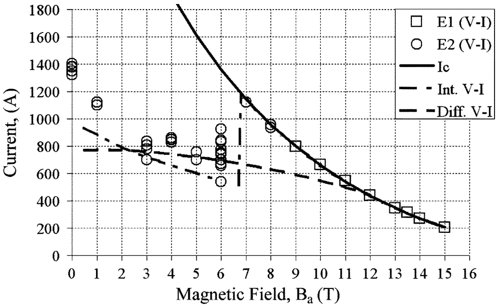

Fig. 3. V-I data (FNAL) and self-field instability model comparison (4.2 K); E1 indicates that the critical current (I_c) was reached; E2 indicates premature quench (the sample magnetization is removed before the measurement).

From the 'integral' model and the 'differential' model of the self-field instability one can deduce that there are 3 stability regions: 1) a high field stable region for $B_a > B_{\mathrm{C}}$; 2) an intermediate field region, $B_{\mathrm{B}} < B_{\mathrm{a}} < B_{\mathrm{C}}$, where $J_c > J_{diff} > J_{int}$ and premature quenches may occur as soon as the conditions for starting a flux jump are satisfied; 3) a low field region for $B_a < B_{\mathrm{B}}$ where $J_c > J_{int} > J_{diff}$ and premature quenches can occur when $J > J_{int}$. In the low field region the quench current density, $J_q(B_q)$, decreases with increasing B_a while in the intermediate field region $J_q(B_a)$ does not change significantly with B_a. The minimum value of $J_q(B_a)$ is equal to J_{int} not only in the low field region but also in the intermediate field region. If a severe perturbation occurs, possibly caused by strand motion, the flux jump can be triggered even if $J < J_{diff}$. This means that for the self-field instability, the lowest value of J_q can occur in the intermediate field region.

Comparing the behavior at 4.2 K and 1.9 K, one notices that at the lower temperature: 1) the low field region is extended, $B_{\mathrm{B}'} > B_{\mathrm{B}}$; 2) J_c can only be attained at higher field values, $B_{\mathrm{C}'} > B_{\mathrm{C}}$; 3) the lowest value of J_q is lower, $J_{int-D'} < J_{int-D}$.

B. Comparison Between Model and V-I Measurements

Premature quenches due to the self-field instability were observed at FNAL during critical current measurements of Nb_3Sn strands. The test consisted of measuring the voltage across a length of the strand, in a constant applied magnetic field, as a function of increasing current (V-I measurement). The measurements were performed on non-magnetized Nb_3Sn strands (E2 experiments [9]) mounted on ITER sample holders. In such measurements, having excluded the possibility of mechanical instabilities, the premature quenches must be related to the self-field instability because the strand magnetization is very low. Indeed, for the current values of interest (below 2000 A), the distribution of the magnetic field within the strand is not much different from that of a straight strand. The magnetization energy generated by ramping the current from 0 A to 2000 A in a strand mounted on ITER sample holders is negligible, being almost equivalent to the magnetization energy generated by a 0.22 T change of B_a [8].

Figs. 3 and 4 show comparisons between the measurements carried out on two virgin MJR strands (Table I) and the models described above. The conclusions drawn by the model in the previous section appear to be in good agreement with the experimental data for V-I measurements.

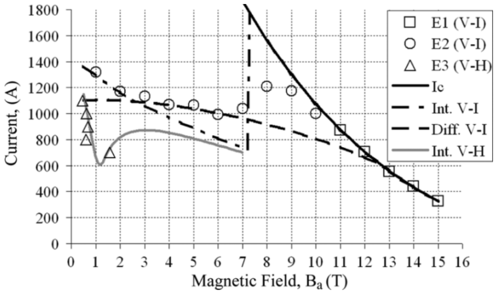

Fig. 4. V-I and V-H data (FNAL) and model comparison (4.2 K); E3 are V-H measurements starting from 0 T with the sample not magnetized.

C. V-H Measurements: A Combination of Self-field Effect and Strand Magnetization

The V-H measurement consists of measuring the voltage across a length of the strand with a constant current while sweeping B_a. During such measurements the strand is strongly magnetized at low B_a and the magnetization energy can not be neglected in calculating the minimum $I_q(B_q)$. For this reason, the stored energy due to magnetization was incorporated in the 'integral' model of the self-field [8]. The magnetization energy was calculated for the case of V-H measurements starting from 0 T with the sample not magnetized (E3 experiment [9]). This more complete model is based on the assumptions that: 1) the transport current flows in the outermost filaments while the inner filaments get magnetized; 2) the energy dissipated during a 'complete' flux jump is the sum of the result of Eq. 1 and of the magnetization energy. To calculate the total magnetization energy, the magnetization energy of a round filament with no transport current was estimated [8] and then, this value was multiplied by the total number of filaments and by the fraction of composite area not occupied by the transport current.

A comparison between the E3 experiment and the model is shown in Fig. 4 (see V-H data). The experiment was carried out at FNAL on a virgin 1 mm MJR strand (Table I) [9]. The model described in this paper predicts a local minimum of 600 A around 1.2 T, which is in good agreement with the experimental results. This local minimum, which occurs during V-H measurements in high-J_c Nb_3Sn strands with large D_{eff}, was also predicted by a previous model [1] assuming that the transport current was equally distributed among sub-elements in the strand cross section and widely studied experimentally [10]–[12]. For the same virgin 1 mm MJR strand the two models predict a local minimum in the same magnetic field region but in one case the lowest value of $I_q(B_a)$ was 1500 A [1] and in the other 600 A.

III. STRAND TEST

To study magneto-thermal stability of a high J_c Nb_3Sn strand with a low RRR, a Rod Re-Stack Process (RRP) strand produced by Oxford Superconducting Technology (OST) was appropriately heat treated and tested at CERN. The strand was delivered by OST in the framework of the LHC superconducting undulator upgrade [13]. The strand properties are shown in Table I. Details regarding the sample preparation and test procedure can be found elsewhere [14]. The experimental results at 4.2 K and 1.9 K are summarized in Figs. 5 and 6. The strand reached I_c for $B_a \geq 8$ T at 4.2 K and for $B_a \geq 11$ T at 1.9 K. At lower B_a, premature quenches occurred during V-I measurements with

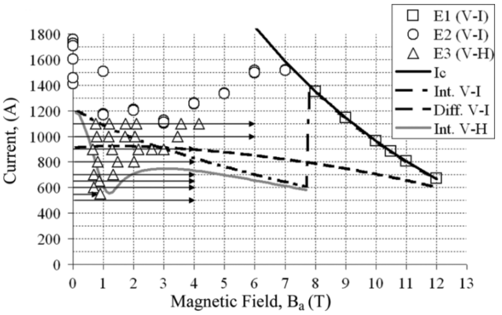

Fig. 5. Test results of the RRP strand at 4.2 K (CERN); arrows indicate the B_a range covered in the E3 experiment; arrows crossing a marker indicate that a quench occurred, the I power supply was tripped and the B_a ramp was stopped, then the same current value was restored and finally the B_a was ramped up again; arrows with no marker indicate a B_a ramp with no quench.

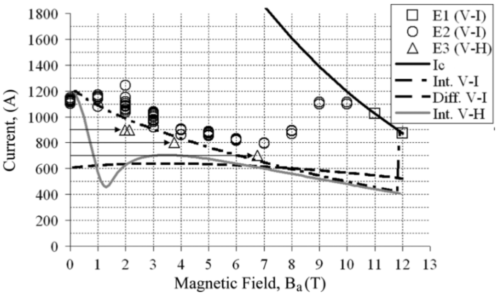

Fig. 6. Test results of the RRP strand at 1.9 K (CERN).

the strand not magnetized (E2 experiment). The lowest value of $I_q(B_a)$ was lower at 1.9 K ($\sim$800 A) than at 4.2 K ($\sim$1100~A).

In V-H measurements at 4.2 K, starting with the strand not magnetized (E3 experiment), an expected significant reduction in I_q was observed with respect to E2 measurements in the very low field region between 0 T and 4 T.

An unexpected behavior was observed in the E3 experiment at 1.9 K (Fig. 6). The above mentioned I_q reduction with respect to E2 measurements was not significant. This behavior at 1.9 K, yet to be understood, is very interesting not only for its practical consequences in magnet performance but also because it is contrary to the increase, from 4.2 K to 1.9 K, of the theoretical maximum magnetization that the strand might have if partial flux jumps were not present.

The comparison of the V-I measurements at 4.2 K and 1.9 K confirms the conclusions drawn from the self-field model in the previous section. Regarding the V-H measurements at 4.2 K the model gives a good estimate of the lowest $I_q(B_a)$ value (554 A) with the local minimum shifted of about 0.2 T. The model does not describe the quench behavior during V-H measurements at 1.9 K most likely because partial flux jumps dissipate the magnetization energy. From this, our preliminarily conclusion is that the self-field instability may be the predominant instability mechanism at 1.9 K.

IV. CONCLUSIONS

An adiabatic model was developed to calculate the minimum quench current due to the self-field instability in high J_c Nb_3Sn strands. This type of magneto-thermal instability, which strongly depends on the J_c and the strand diameter and which is not directly related with the D_{eff} of the filaments, can significantly reduce the current carrying capability of high J_c Nb_3Sn strands.

The model is in good agreement with the V-I measurements of virgin MJR and RRP strands with low RRR. The model and the experimental strand data show that the self-field instability is especially dangerous for Nb_3Sn magnets in the 'intermediate field region' and at a lower bath temperature.

Combining the self-field model with the magnetization energy in the filaments, the model can also calculate the minimum I_q during V-H measurements, in good agreement with the experimental results at 4.2 K carried out on virgin MJR and RRP strands with low RRR.

In general the model is well adapted to round composite wires where the superconducting filaments are embedded in a matrix with a low thermal and electrical conductivity. For Nb_3Sn strands with a sufficiently large RRR the model underestimates the minimum I_q, especially in the low field region where the effect of the magneto-resistance is reduced. This underestimation is most likely due to having neglected the diffusion of the current and of the heat in the outer Cu shell and in the inner Cu core.

During the testing of a RRP strand at 1.9 K at CERN, an unexpected result was observed for the first time during V-H measurements. The local minimum of the I_q at low field, confirmed at 4.2 K, disappeared at 1.9 K. The I_q was still lower than the I_q measured during V-I tests but the difference was not significant. This behavior might be a sign of a new stability regime and suggests that premature quenches at 1.9 K due to magneto-thermal instabilities in Nb_3Sn magnets should occur in the magnet high field region and they would be mainly caused by the self-field instability.

REFERENCES

[1] V. V. Kashikhin and A. V. Zlobin, "Magnetic instabilities in Nb_3Sn strands and cables," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 1621–1624, Jun. 2005.

[2] R. Hancox, "Stability against flux jumping in sintered Nb_3Sn," *Phys. Lett.*, vol. 16, p. 208, 1965.

[3] P. S. Swartz and C. P. Bean, "A model for magnetic instabilities in hard superconductors: The adiabatic critical state," *J. Appl. Phys.*, vol. 19, p. 4991, 1968.

[4] M. N. Wilson, *Superconducting Magnets*. Oxford: Clarendon Press, 139 141, , 1983.

[5] R. G. Mints and A. L. Rakhmanov, "Critical current of a superconductor," *Sov. Tech. Phys. Lett.*, vol. 2, no. 6, Jun. 1976.

[6] S. A. Keys and D. P. Hampshire, "A scaling law for the critical current density of weakly- and strongly-coupled superconductors, used to parameterize data from a technological Nb_3Sn strand," *Supercond. Sci. Technol.*, vol. 16, pp. 1097–1108, 2003.

[7] R. Hancox, "Enthalpy stabilized superconducting magnets," *IEEE Trans. Magn.*, vol. 4, no. 3, pp. 486–488, Sep. 1968.

[8] B. Bordini, "Thermo-Magnetic Instabilities in Nb_3Sn Superconducting Accelerator Magnets," Fermilab-thesis, , -2006-45.

[9] B. Bordini *et al.*, "Voltage spikes in Nb_3Sn and NbTi strands," *IEEE Trans. Appl. Superconduct.*, vol. 16, no. 2, pp. 366–369, Jun. 2006.

[10] B. Barzi *et al.*, "Instability in transport current measurement," *IEEE Trans. Appl. Superconduct.*, vol. 15, no. 2, pp. 3364–3367, Jun. 2005.

[11] A. K. Ghosh, L. D. Cooley, and A. R. Moodenbaugh, "Investigation of instability in high J_c Nb_3Sn strands," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 3360–3363, Jun. 2005.

[12] D. R. Dietderich *et al.*, "Correlation between strand stability and magnet performance," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 1524–1528, Jun. 2005.

[13] S. Bettoni, Electromagnetic Design of a Nb_3Sn Superconducting Undulator for the LHC Beam Diagnostics, *CERN AT/MAS Internal Note*, 2006-01.

[14] B. Bordini, Magneto-Thermal Instability of Superconducting Strands: Preliminary Results On a New Behavior Observed in Nb_3Sn Strands at 1.9 K, *CERN AT/MCS Internal Note*, 2007-05.