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Momentum scale calibration using resonances

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Abstract

The use of resonances to calibrate the momentum scale is discussed. Formula for the relationship between the magnetic field scale and the measured mass are derived and applied to the decays $J/\psi \to \mu^+\mu^-$, $K_S \to \pi^+\pi^-$ and $\Lambda \to p^+\pi^-$.

1 Introduction

In order for the track fit to give an unbiased momentum estimate the effect of the magnetic field seen by the particle must be accounted for. In studies with simulated data it is easy to check that this requirement is met by comparing reconstructed quantities to their true values. With real data other tests have to be found. One check is to compare the reconstructed masses of resonances to their known values. In this note the use of $\mathrm{J}/\psi \to \mu^+\mu^-, \mathrm{K_S} \to \pi^+\pi^-$ and $\Lambda \to p^+\pi^-$ to calibrate the momentum scale is discussed. The structure of this note is as follows. First in Section 1 formulae for the relationship between the magnetic field scale and the measured mass are derived. Then these are applied to the above decays and compared to Monte Carlo studies.

2 Formula

The invariant mass of two particles d_1, d_2 is given by:

$$m_{d_1 d_2}^2 = (E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2}) \cdot (\vec{p_1} + \vec{p_2}) \tag{1}$$

Assuming $m(d_1) = m(d_2)$ and making a first order Taylor expansion this reduces to:

$$m_{d_1 d_2}^2 = m_d^2 R + 2 \cdot (p_1 p_2 - \vec{p_1} \cdot \vec{p_2}) \tag{2}$$

where:

$$R = 2 + \frac{p_1}{p_2} + \frac{p_2}{p_1}. \tag{3}$$

If the scale of the magnetic field is wrong by a factor α then the momentum of each particle needs to be scaled by $1+\alpha$. Assuming the particles originated in the decay $P \to d_1 d_2$ then:

$$m_P^2 = m_d^2 R + 2 \cdot (1+\alpha)^2 \cdot (p_1 p_2 - \vec{p_1} \cdot \vec{p_2}). \tag{4}$$

Subtracting Equation 2 and 4 gives:

$$m_{d_1 d_2}^2 = \frac{m_P^2 - m_d^2 R}{(1+\alpha)^2} + m_d^2 R. \tag{5}$$

For $\alpha \ll 1$ this simplifies to

$$\Delta m = \alpha \cdot \frac{m_d^2 R - m_P^2}{m_P} \tag{6}$$

If in addition $m_d^2 R \ll m_P^2$ then:

$$\Delta m = \alpha \cdot m_P. \tag{7}$$

As will be seen the approximation made in Equation 7 is valid for the case of J/$\psi \rightarrow \mu^+\mu^-$ but not in the case of $\mathrm{K_S} \rightarrow \pi^+\pi^-$.

Another possibility is that the momentum is biased due to a poor tuning of the energy loss correction. In this case $p \rightarrow p' - \beta_{ion}$. Assuming, the opening angle is small, the decay symmetric such that $p_1 \approx p_2$ and the correction β_{ion} is the same for both particle it follows from Equation 2 that:

$$m_{d_1 d_2}^2 - m_P^2 = \beta_{ion} \cdot (p_1 + p_2) \cdot \theta^2 \tag{8}$$

From Equation 8 it follows that a wrong tuning of the energy loss correction results in a bias on the mass that depends on the square root of the parent particle momentum. Furthermore, the bias scales as $\sqrt{\beta_{ion}}$ and is proportional θ. Therefore, for the case of LHCb the effect of energy loss on the mass is small.

3 Results

3.1 J/$\psi \rightarrow \mu^+\mu^-$ studies

The J/ψ mass distribution has been studied with a sample of 110,000 inclusive J/ψ events from the DC 06 production [1]. A J/$\psi \rightarrow \mu^+\mu^-$ selection was made using the LoKi [2] analysis toolkit. First, muon candidates were selected by requiring that $\Delta L_{\mu\pi} < -8$ and pt > 500 MeV. Muon pairs with opposite charge were then fitted to a common vertex and the χ^2 required to be less than 10. Finally, in order to benefit from the improvements in the track fit [3] that have occurred since the DSTs were produced a re-fit of the tracks in selected candidates was made and the vertex refitted. This also allowed the effect of varying the energy loss correction and changing the field scale to be studied. Around 50,000 J/ψ candidates were reconstructed. The momentum spectra for the candidates that are associated to a true J/ψ using the Monte Carlo truth is shown in Fig. 1 whilst Fig 2 shows the distribution of cos θ and R.

Fig. 3 shows the resulting invariant mass distribution. Due to QED radiative corrections the distribution has a non-Gaussian tail towards low invariant

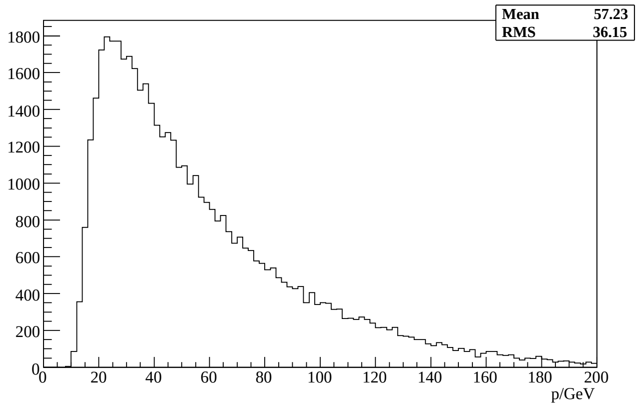


Figure 1: Momentum spectrum of reconstructed J/ψ candidates.

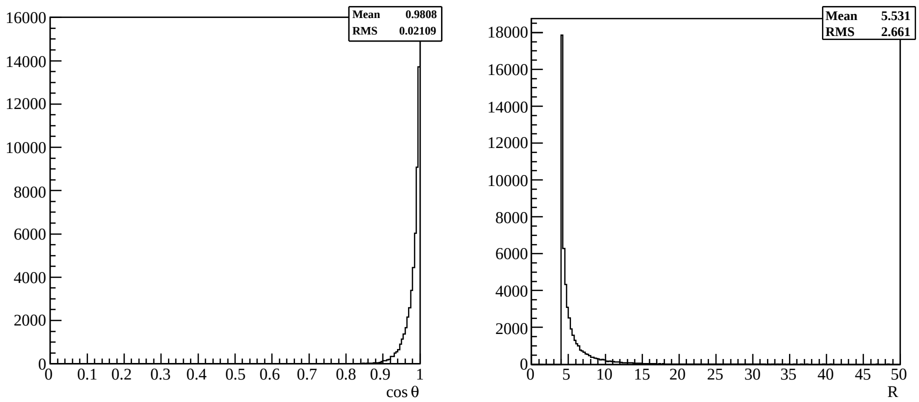


Figure 2: Distribution of cosθ and R for reconstructed $J/\psi \to \mu^+\mu^-$ candidates .

mass. One way to account for this is to fit a Crystal Ball shape [4] which describes the radiative tail using a power law:

This procedure has some disadvantages. First, with small statistics the fit was found to be unstable due to correlations between the parameters. In addition, it is hard to judge how 'Gaussian' the distribution would be in the absence of radiative corrections. Finally, as will be seen a bias towards lower mass is seen with the Crystal Ball fit. These difficulties can be avoided in Monte Carlo studies by fitting the difference between the true and reconstructed invariant mass of the di-muon pair. Fig. 4 shows the distribution of

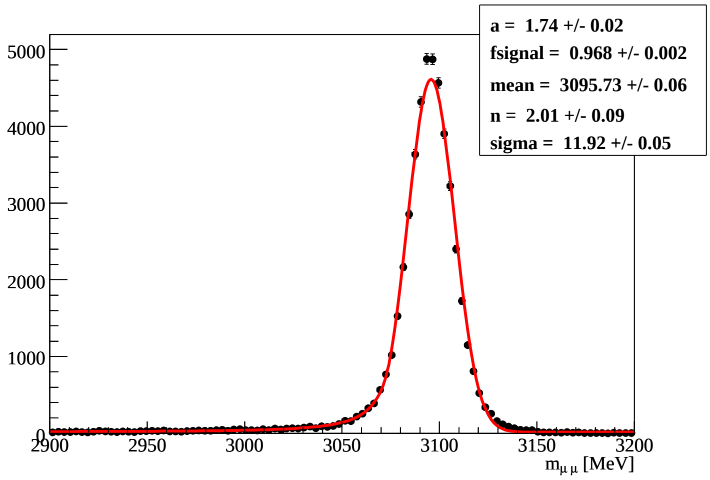


Figure 3: J/ψ mass distribution. The result of a fit to a Crystal Ball plus a flat background component is superimposed.

this variable together with the result of a Gaussian fit. It can be seen that the distribution is well described by a single Gaussian [1].

Fig. 5 shows the bias on the J/ψ mass as a function of momentum in four cases:

- A Crystal Ball fit to the J/ψ mass distribution with the standard track fit.
- A Gaussian fit to the difference between the true and reconstructed invariant mass with the standard track fit.
- A Gaussian fit to the difference between the true and reconstructed invariant mass with the energy loss correction turned off in the fit.
- A Gaussian fit to the difference between the true and reconstructed invariant mass with the magnetic field scaled downwards by 0.5 % so that in the reconstruction only 0.995 of the field used in the simulation is seen [2].

[1] A double Gaussian would fit better. However, for the studies in this note there is no need to introduce this additional complication.

[2] The option **MagneticFieldSvc.ScaleFactor** = 0.995

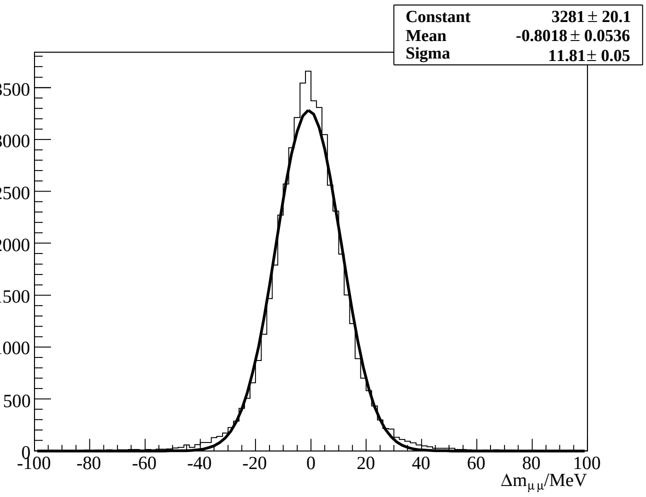


Figure 4: Di-muon invariant mass distribution. The result of a Gaussian fit is superimposed.

A bias of -1.2 MeV is seen with the Crystal Ball fit. Fitting a Gaussian to the difference between the true and reconstructed di-muon mass the bias is reduced to -0.8 MeV. This shows that 0.4 MeV of the bias is due to the effect of radiative corrections. Studies have shown the remaining bias is related to the energy loss correction. However, whether the effect originates in the reconstruction or the simulation is not understood at this time. This bias will not be discussed further here.

The results for runs with the energy loss correction turned off in the fit and with the field scaled behave as expected from Section 2. In the former case the bias depends on the square root of the momentum as expected from Equation 8. For the latter case a 0.5 % bias independent of the J/ψ momentum is seen. This is consistent with the expectation that in this case the approximations made in Equation 5 are valid and that Δm scales with the shift of the field scale. Fig. 6 shows the results of three runs with different scalings of the field together with Equations 5 and 7. In the first formula the average value of R found for true J/ψ candidates of 5.5 is used. It can be seen that in this case the approximations made in Equation 7 are valid and that the simulation behaves as expected.

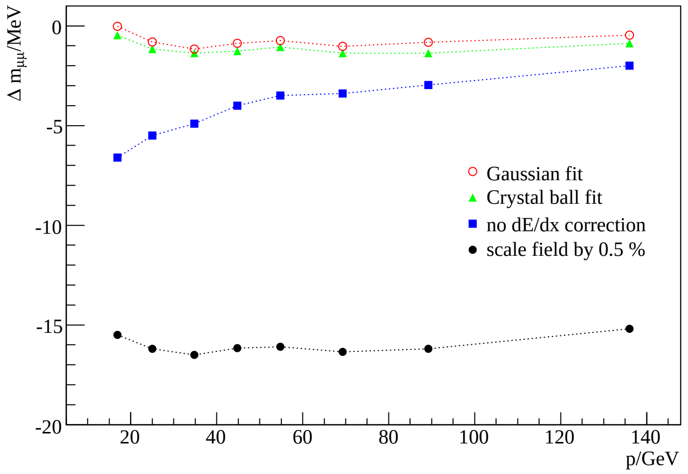


Figure 5: Bias on the J/ψ mass versus p/GeV for the four cases discussed in the text.

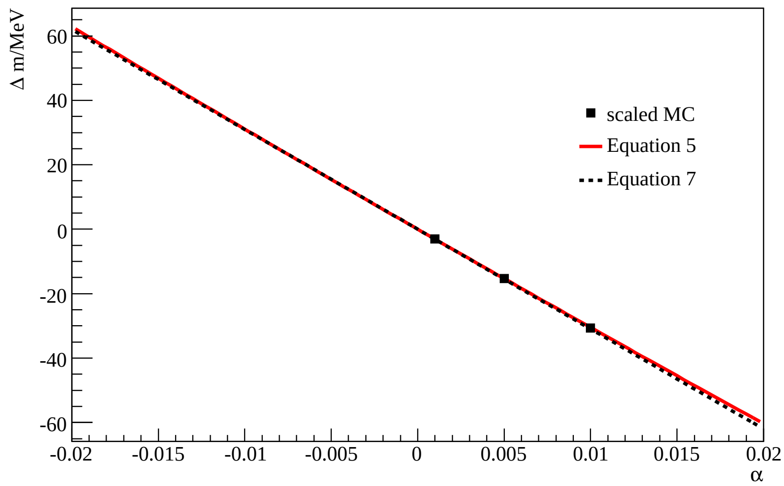


Figure 6: Effect of varying the magnetic field scale on the bias. The points have been corrected for the -0.8 MeV bias seen in the run with nominal field. Equations 5 and 7 are superimposed.

3.2 $\mathrm{K_S} \to \pi^+\pi^-$ studies

The $\mathrm{K_S}$ mass distribution has been studied with a sample of $\sim 100,000$ L0 selected minimum bias events from the DC 06 production [1]. Candidates were selected using the standard loose selection for K_S decays that occur in the VeLo. To simplify the analysis events with only one reconstructed primary vertex were considered. Fig. 7 shows the mass distribution obtained. The S/B is around 0.8. This is increased to 5.9 for a 17 % loss in efficiency by making the additional requirements that the χ^2 of the vertex be less than 20 and that the flight distance between the primary and the K_S decay vertex be greater than 5 cm (Fig. 8).

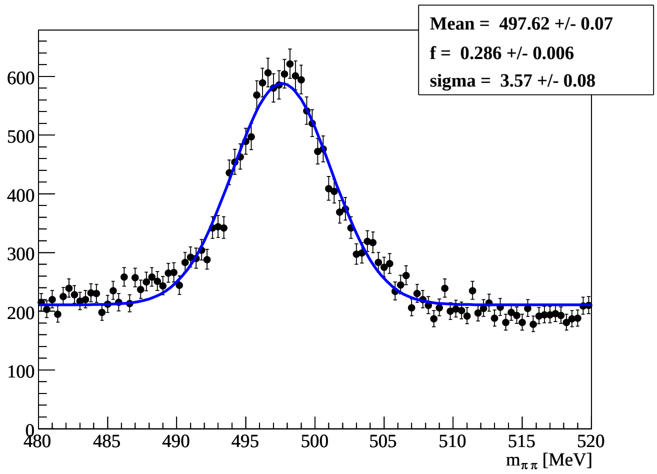


Figure 7: Mass distribution for candidates selected by the loose K_S selection. The result of a fit to a Gaussian plus a flat background component is superimposed.

The momentum spectra for the candidates that are associated to a true K_S using the Monte Carlo truth is shown in Fig. 9 whilst Fig. 10 shows the distribution of cos θ and R. It can be seen that the mean momentum for K_S candidates is 17 GeV (compared to 60 GeV for the J/ψ case). The mean value of R is 4.9 [3].

Fig. 11 shows the bias on the K_S mass for the three scenarios considered in the J/ψ case. For the effects related to energy loss the behaviour is similar to that seen in the J/ψ case with the size of the bias reduced by a factor of $\sim m_K/m_{J/\psi}$. The shift seen in the run with the field scaled is -1.6 MeV to be compared with the value of -2.5 MeV that would be expected from Equation 7. This shows that the approximation made in Equation 7 is not valid in this case.

[3] Counting events within a 3 σ window around the K_S mass the value increases to 5.2.

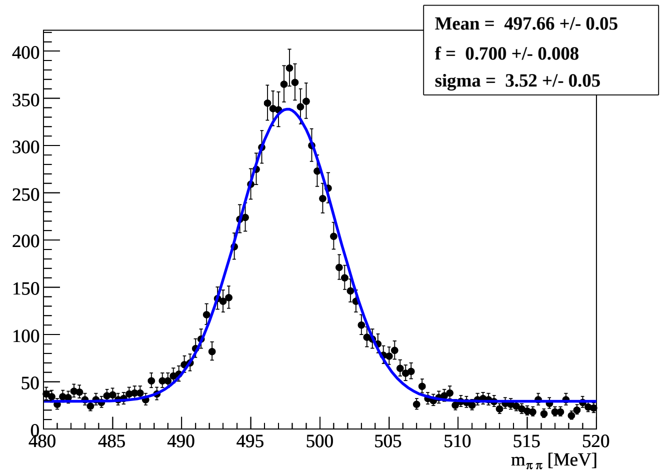


Figure 8: Mass distribution for candidates selected by the K_S selection described in the text. The result of a fit to a Gaussian plus a flat background component is superimposed.

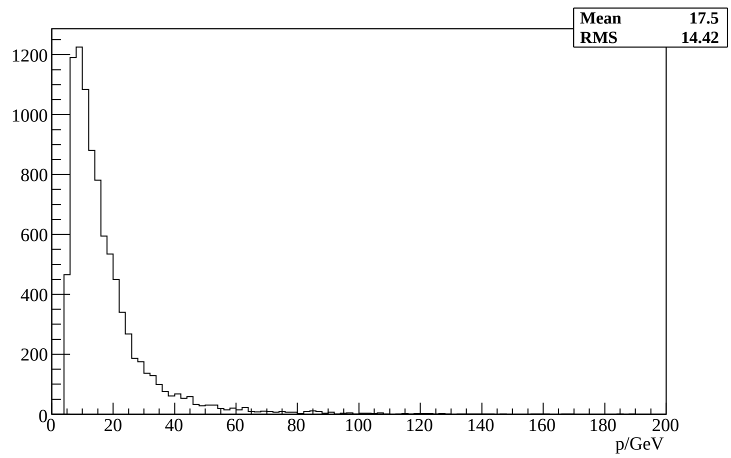


Figure 9: Momentum spectra for selected K_S candidates.

Fig. 12 shows the results of three runs with different scalings of the field. In addition, Equations 5 - 7 are superimposed. In Equations 5 and 6 the average value of R found for true K_S candidates of 4.9 is used. For this case Equation 6 gives Δ m [MeV] = - 304 $\times$ α. It can be seen that Equation 6 agrees well with the Monte Carlo runs.

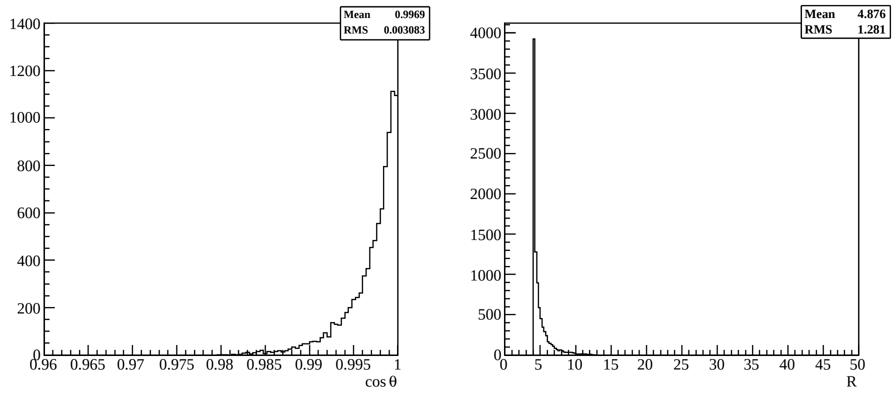


Figure 10: Distribution of cosθ and R for reconstructed $K_S \rightarrow \pi^+\pi^-$ candidates .

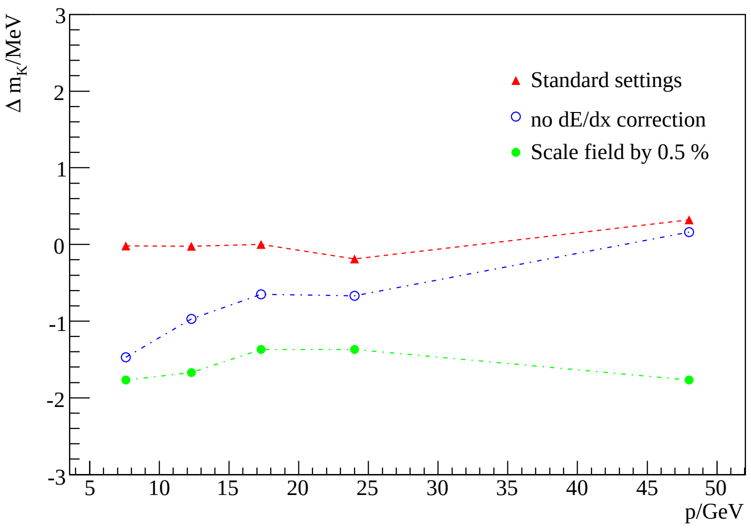


Figure 11: Bias on the K_S mass versus p/GeV for the three scenarios discussed in the text.

4 $\Lambda \rightarrow p^+\pi^-$ studies

The decay $\Lambda \rightarrow p^+\pi^-$ is different to the others considered in this note in that the decay products have different mass. In addition, the Q-value of the decay is only 39 MeV. Therefore, it is expected that this decay has a good

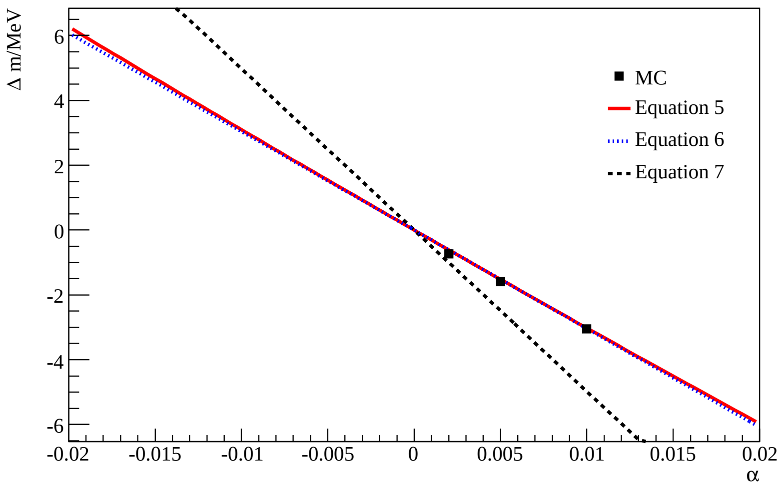


Figure 12: Effect of varying the magnetic field scale on the bias for the K_S case. Equations 5 - 7 are superimposed.

resolution but is relatively insensitive to variations of the magnetic field and energy loss. The formula developed in Section 2 are easily extended to the Λ case. Equation 5 becomes:

$$m_{p\pi}^2 = \frac{m_\Lambda^2 - f}{(1+\alpha)^2} + f. \tag{9}$$

and Equation 6:

$$\Delta m = \alpha \cdot \frac{f - m_\Lambda^2}{m_\Lambda} \tag{10}$$

where:

$$f = m_p^2 + m_\pi^2 + \frac{p_\pi}{p_p} \times m_p^2 + \frac{p_p}{p_\pi} \times m_\pi^2 \tag{11}$$

The Λ mass distribution has been studied with a sample of 1.1 millions L0 selected minimum bias events from the DC 06 production [1]. Candidates were selected using the standard loose selection for Λ decays occurring in the VeLo. To increase the statistics $\overline{\Lambda}$ were also used in the analysis. As in the K_S case only events with one reconstructed primary vertex were considered. Fig. 13 shows the mass distribution obtained. The S/B is around 3.7. This is increased to 5.6 for a 12 % loss in efficiency by making the additional requirements that the χ^2 of the vertex be less than 20 and that the flight distance between the primary vertex and the Λ decay vertex be greater than 5 cm (Fig. 14). As expected given the small Q value of the decay the resolution on the mass is good. A value of 1.2 MeV is found.

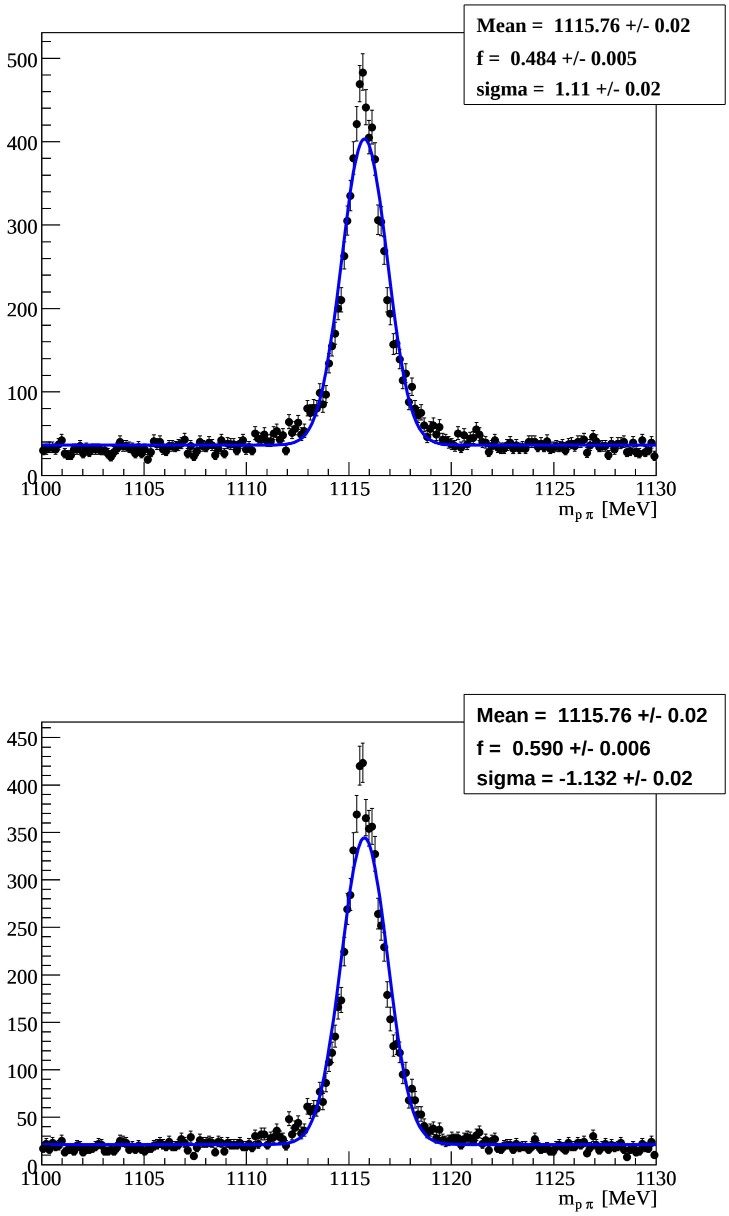
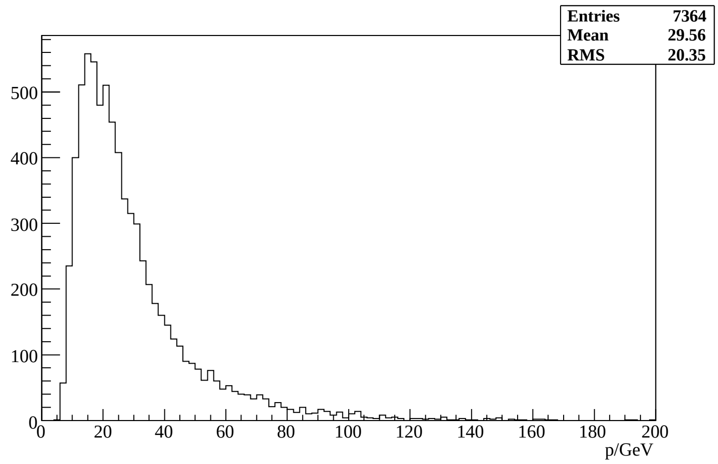


Figure 14: Mass distribution for candidates selected by the Λ selection described in the text. The result of a fit to a Gaussian plus a flat background component is superimposed.

The momentum spectra for the candidates that are associated to a true Λ using the Monte Carlo truth is shown in Fig. 15 whilst Fig. 16 shows the distribution of $\cos\theta$ and f. The mean momentum for Λ candidates is 29 GeV whilst the mean value of f is 1.193 [4].



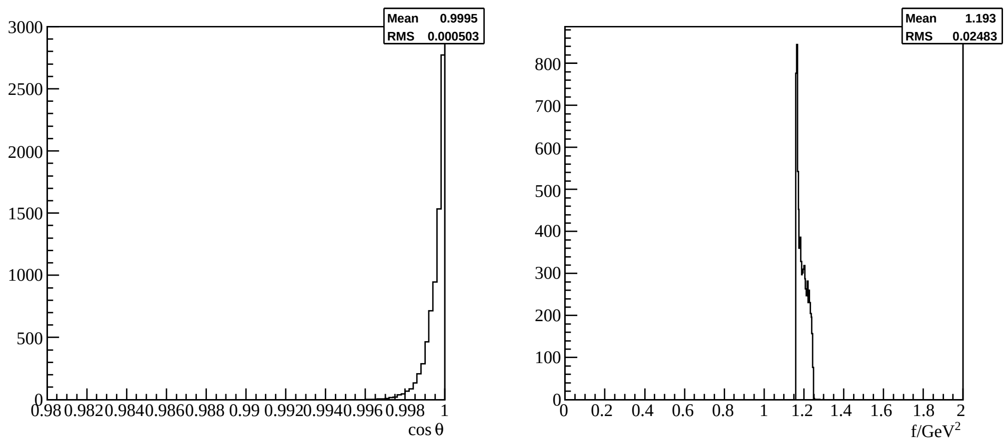


Figure 16: Distribution of $\cos\theta$ and f/GeV^2 for reconstructed $\Lambda \rightarrow p^+\pi^-$ candidates .

Fig. 12 shows the results of six runs with different scalings of the field. In

[4]Counting all events within a 3 σ window around the Λ mass is included the value increases slightly - to 1.196.

addition, Equation 9 and Equation 10 are superimposed. In both cases f = 1.193 is used. With this value Equation 10 becomes Δ m [MeV] $= -46.7 \times \alpha$ It can be seen that the approximation made in Equation 6 is reasonable for $\alpha < 0.02$.

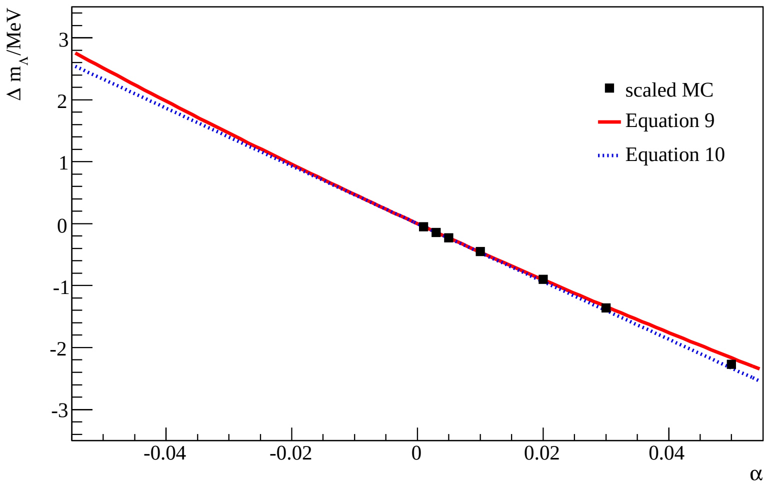


Figure 17: Effect of varying the magnetic field scale on the bias. Equations 9 - 10 are superimposed.

5 Summary

In this note the use of resonances to calibrate the momentum scale has been discussed. It has been shown that for the case of $J/\psi \to \mu^+\mu^-$ that the relative shift in the observed mass from the true value allows the field scale to be extracted. In contrast for the decays $K_S \to \pi^+\pi^-$ and $\Lambda \to p^+\pi^-$ the effect of relativistic kinematics must be taken into account. In both cases the relationship between the shift in the mass the field scale and the remains linear allowing its value to be easily extracted. Since the size of the shift is largest for the J/ψ case and its mass is closest to the B mass it can be considered as the golden mode for momentum scale calibration. The possibility of using muonic Υ's and $\psi(2S)$ decays for momentum scale calibration is also under study. These will behave in the same way as the J/ψ in that the shift in mass will be directly proportional to the change in the field. Therefore, they will allow to make a powerful cross check of the calibration made with J/ψ decays.

This note has concentrated on the extraction of the overall field scale from the data. Given the large data sample that will be available at LHCb it may also be possible to infer a discrepancy in the field shape from the data.

References

[1] Gauss v25r7, Boole v12r10, Brunel v30r14, XmlDDDB v30r14.

[2] I. Belyaev. Loki: Smart and Friendly C++ Physics Analysis Toolkit. LHCb-note 2004-023.

[3] M. Needham. Performance of the LHCb Track Reconstruction Software. LHCb-note 2007-144.

[4] J. Gaiser. *Charmonium Spectroscopy from Radiative Decays of the J/ψ and ψ'*. PhD thesis, Stanford University, 1982.

A Standard K_S selection

The standard K_S selection [5] for the DC' 06 production makes the following requirements on the daughter particles:

- The momentum should be greater than 2 GeV.
- The χ^2/ndof given by the track fit should be less than 20.
- The minimum impact parameter to a primary vertex should be greater than 3.

In addition the χ^2 of the vertex fit should be less than 30.

[5] See the CommonParticles package v3r10.

B Standard Λ selection

The standard Λ selection for the DC 06 production makes the following requirements on the daughter particles:

- The pt of the pion should be greater than 100 MeV.
- The pt of the proton should be greater than 300 MeV
- The minimum impact parameter to a primary vertex should be greater than 3.

In addition the following cuts are applied to Λ:

- The χ^2 of the vertex fit should be less than 50.
- The pt of the Λ should be greater than 500 MeV
- The minimum flight distance significance should be greater than 10.