VI. PHYSICAL ACOUSTICS*

Academic and Research Staff

Prof. K. U. Ingard
Prof. L. W. Dean III
Prof. K. W. Gentle

Graduate Students

R. H. Katyl
J. L. Macon
W. M. Manheimer
J. A. Ross
H. M. Schulz III

A. INTENSITY DISTRIBUTION OF LIGHT SCATTERED BY THERMAL SURFACE WAVES ON A LIQUID SURFACE

The intensity distribution of light scattered by a thermal ensemble of surface waves on the plane surface of a metallic conductor was calculated with the use of a vector Kirchhoff integral formula. The calculation is similar to that of Gans[1] and will not be repeated here; the results are presented and expected intensities are shown for an experiment to be undertaken with liquid mercury.

The geometry of the problem is the following. The unperturbed liquid surface is taken to be the x-y plane, and the z-axis extends from the liquid. The direction of the incident plane wave of light is taken to be in the x-z plane, at an angle θ_o to the z-axis, and the observer is at the angles θ, ϕ. This is shown in Fig. VI-1.

With this definition of angles, we find for the electric field at the peak of the diffraction maximum, for scattering off of one surface wave of peak surface displacement ζ_o

$$\begin{bmatrix} E_{S\phi} \\ \\ E_{S\theta} \end{bmatrix} = 8a^2k^2\zeta_o \frac{e^{ikr}}{4\pi r} \begin{bmatrix} \cos\theta_o \cos\phi \cos\theta; & -\cos\theta \sin\phi \\ \\ \cos\theta_o \sin\phi; & (\cos\phi - \sin\theta_o \sin\theta) \end{bmatrix} \begin{bmatrix} E_\perp \\ \\ E_\parallel \end{bmatrix}$$

where the liquid surface has been taken to be a square of side a.

The choice of parameters λ, the light wavelength, and θ and ϕ, the observation direction angles, determines uniquely the surface wave wavelength, Λ, and surface wave direction, ψ, up to an additive constant of 180°. These relations are

$$\tan\psi = -\frac{x \sin\phi}{1 - x\cos\phi}; \qquad x = \frac{\sin\theta}{\sin\theta_o}$$

$$\frac{\Lambda}{\lambda} = \{\sin^2\theta_o + \sin^2\theta - 2\sin\theta\sin\theta_o\cos\phi\}^{-1/2}.$$

---

*This work was supported principally by the U.S. Navy (Office of Naval Research) under Contract Nonr-1841-(42); and in part by the Joint Services Electronics Programs (U.S. Army, U.S. Navy and U.S. Air Force) under Contract DA 36-039-AMC-03200(E).

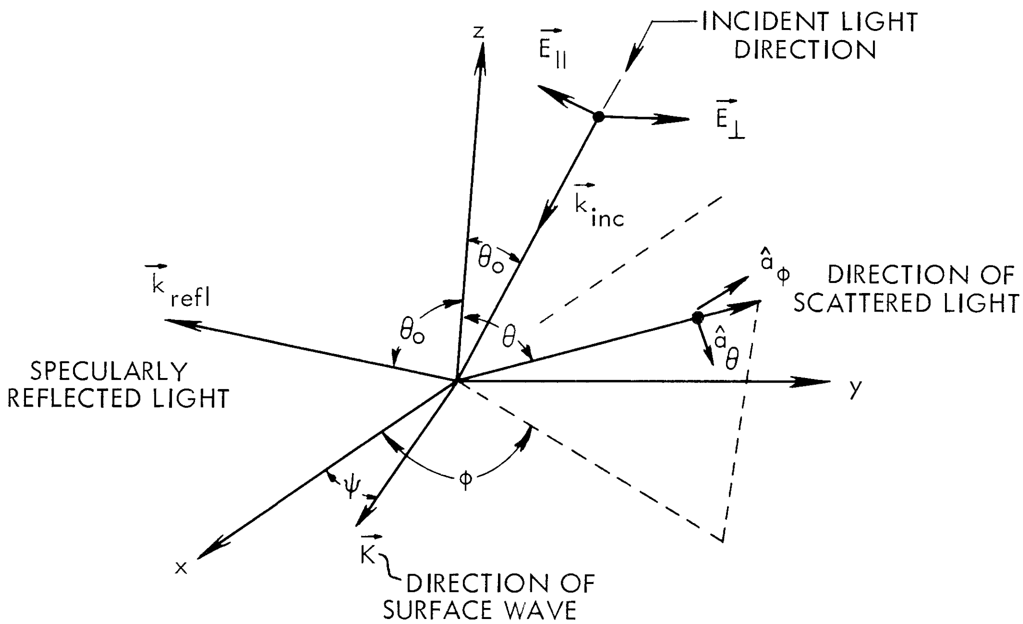


Fig. VI-1. Scattering Geometry

For scattering off of a thermal ensemble of surface waves, the number of waves scattering into solid angle $\Delta\Omega$ is

$$\Delta n = \left(\frac{L}{2\pi}\right)^2 KdKd\psi = \left(\frac{kL}{2\pi}\right)^2 (\cos\theta)(\Delta\Omega).$$

For surface tension waves, the rms surface displacement is

$$\zeta_{o_{rms}} = \sqrt{\frac{4K_BT}{\sigma(KL)^2}},$$

where K_B is Boltzmann's constant, and σ is the surface tension.

Summing the scattered waves incoherently, we find for the total scattered intensity

$$\begin{bmatrix} I_\phi \\ \\ I_\theta \end{bmatrix} = \frac{4K_BT\Delta\Omega}{\sigma\lambda^2} \frac{1}{\sin^2\theta + \sin^2\theta_o - 2\sin\theta\sin\theta_o\cos\phi}$$

$$\cdot \begin{bmatrix} (\cos\theta_o\cos\phi\cos\theta)^2 & (-\cos\theta\sin\phi)^2 \\ \\ (\cos\theta_o\sin\phi)^2 & (\cos\phi - \sin\theta_o\sin\theta)^2 \end{bmatrix} \begin{bmatrix} I_\perp \\ \\ I_\parallel \end{bmatrix}$$

For the case of mercury, $\sigma \approx 500$ ergs/cm^2, and the multiplying constant at room temperature is

$$I_{sc} \sim (8.26\ 10^{-8})\ \Delta\Omega I_{in} f(\theta_o, \theta, \phi),$$

and for a collecting solid angle of 0.1, 10% transmission through the optics, and an incident intensity of 100 μwatts, $I_{sc} \sim 10^{-13} f(\theta, \phi)$ watts. For backscattering at a grazing incident angle $f \approx 1$ for the proper polarization, and the expected signal current is of the order of the photomultiplier dark current.

R. H. Katyl

References

1. R. Gans, Ann. Physik 74, 231 (1924); 79, 204 (1926). See also A. Andronow and M. Leontowicz, Z. Physik 38, 485 (1926).

B. ACOUSTIC WAVE AMPLIFICATION

Theoretical studies have been carried out on the spontaneous amplification of an acoustic wave in a weakly ionized gas. The amplification mechanism is a coherent heating of the neutral gas by the electrons, which move in phase with the neutrals and ions in the acoustic mode. The linearized equations of motion for the three componets of a plasma ionized to the extent of approximately 10^{-6} and with a pressure ≈1 torr lead to the acoustic dispersion relation

$$k^2 \approx (\omega/c_n)^2 \left[1 + i/\omega\tau_\alpha + (i/\omega\tau_\beta)(1 - i/\omega\tau'_\alpha)(1 + i/\omega\tau'_\beta)^{-1}\right]^{-1},$$

where

$$\tau_\alpha = c_n^2/(\gamma_n - 1)\ \alpha_n N_e \approx 1$$

$$\tau_\beta = c_n^2/(\gamma_n - 1)(\gamma_e - 1)\ \beta_n T_e \approx 1$$

$$\tau' = (\gamma_n - 1) m_e N_e c_e^2 \tau / \gamma_e m_n N_n c_n^2 \approx 10^{-4}.$$

The quantities in these expressions are defined by the following relations:

$$\alpha_n = 2(m_e/m_n)^2\ (KT_e/m_e)^{3/2}\ \sigma(2/\pi)^{1/2}$$

$$\beta_n = 2(m_e/m_n)^2\ (KT_e/m_e)^{3/2}\ \sigma(N_e/T_e)(2/\pi)^{1/2}\ (3/2 + d \ln \sigma / d \ln T_e)$$

T = equilibrium temperature

N = equilibrium particle number density

m = particle mass

γ = specific heat ratio

K = Boltzmann's constant

$c = (\gamma KT/m)^{1/2}$ = speed of sound

k = wave number

$\omega = (2\pi)$ times the frequency, $(10^2 < \omega < 10^5)$

σ = cross section for energy transfer,

with the subscripts n and e referring to the neutrals and electrons.

At high frequencies (for example, $\omega \gg 10^4\ \mathrm{sec}^{-1}$) the process is almost adiabatic and the electron temperature is not significantly affected by the amplification mechanism. The dispersion relation becomes

$$k \approx (\omega/c_n)\left[1-i(\tau_\alpha^{-1}+\tau_\beta^{-1})/2\omega\right].$$

At low frequencies, however, the electron temperature is strongly affected by the process, and the degree of amplification depends on the energy dependence of the electron-neutral cross section. The dispersion relation becomes

$$k \approx (\omega/c_n)\left[1+(\tau_\alpha'+\tau_\beta')/2\tau_\beta-i(\tau_\alpha^{-1}-\tau_\beta^{-1})/2\omega-i\omega^2\tau_\alpha'\tau_\beta'^2/2\tau_\beta\right],$$

and the degree of amplification depends on the energy-dependence of the electron-neutral cross section. For a hard-sphere gas with $\gamma_n = \gamma_e = 5/3$, it develops that $\tau_\alpha = \tau_\beta$. Significant amplification is possible at low frequencies only if σ decreases with energy.

A similar calculation for ion-acoustic waves in a strongly ionized gas yields substantially the same results, but the electron-ion cross section always decreases with energy. The amplification mechanism, however, competes with the attenuation caused by ion-neutral momentum-transfer collisions.

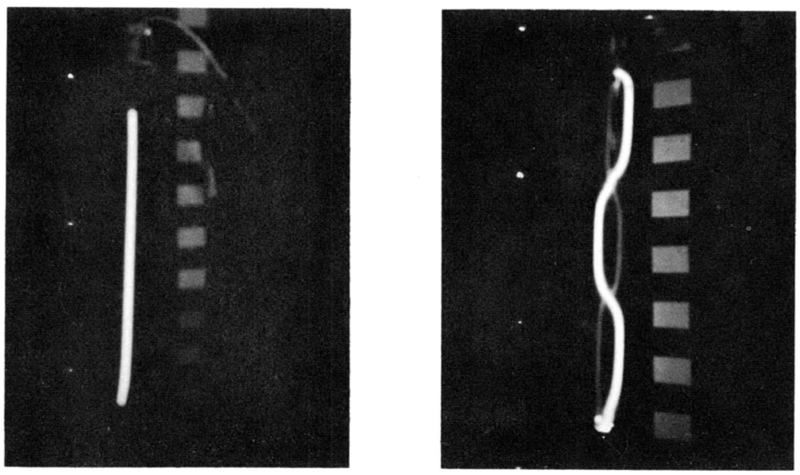
H. M. Schulz III

C. LATERAL ACOUSTIC INSTABILITY

Strickler and Stewart[1] have reported a lateral acoustic instability which results in a pronounced modification of the path of a constricted argon discharge when the current is modulated at a lateral resonant frequency of the neutral gas in the discharge tube.

Futher examination of this effect has recently revealed that the geometric shape of the discharge path depends on the longitudinal pressure wave, as well as on the radical and azimuthal one. Thus all three quantum numbers of the wave play a role in determining the discharge path. In a typical experiment situation the discharge tube has a length much greater than its diameter, and the frequency is determined principally by the radical and azimuthal quantum numbers, that is, by the indices of the Bessel function $J_m(k_n r)$. The longitudinal wave function $\cos \ell\pi z/L$ contributes only a small increment to the frequency but is very important in establishing the path of the discharge. It has been possible to resolve the fine structure of the $m = 1$, $n = 1$ mode, that is, of the lowest mode of the tube, from $\ell = 0$ up to approximately $\ell = 15$.

Preliminary results are consistent with a model in which the discharge follows a path of maximum pressure variation. Both spiral and planar discharge paths have been observed. It would appear that spiral paths must be associated with modes having $m \geq 2$, while planar displacements are most logically associated with models having $m \leq 1$.



MODULATED

MODULATION FREQUENCY
9036 cps

Fig. VI-2. Lateral displacement of an intensity-modulated plasma filament at modulation frequencies close to the acoustic lateral resonances in the tube. Each white rectangle and each black rectangle on the scale has a height of 2 cm, and the tube is 2.5 cm in diameter and 40 cm in length.

Figure VI-2. shows an unmodulated discharge and a discharge for which $m = 1$, $n = 0$, and $\ell \approx 4$ or 5. The less intense trace is a reflection of the discharge in the tube wall. The frequencies correspond to a gas temperature $\approx 350°$ K.

H. M. Schulz III, K. W. Gentle

References

1. S. D. Strickler and A. B. Stewart, Phys. Rev. Letters 11, 527 (1963).

D. CONSERVATION EQUATIONS FOR A PLASMA

The object of the present report is to derive for the general case the equations for conservation of particles, momentum, and energy for a plasma, and to examine the physical nature of the various terms in these equations. The treatment will include the relativistic and nonrelativistic cases, consider gravitational and electromagnetic (but not nuclear) forces, and allow for the existence of field sources external to the plasma.

The starting point for the discussion is the Boltzmann equation

$$\frac{\partial f_\alpha}{\partial t} + \frac{\partial}{\partial \vec{x}} \cdot (\vec{v} f_\alpha) + \frac{\partial}{\partial \vec{p}} \cdot \left\{ q_\alpha \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) f_\alpha + m_\alpha \vec{g} f_\alpha \right\} = \left(\frac{df_\alpha}{dt} \right)_{\text{collisions}}. \tag{1}$$

Here the subscript α refers to the α^{th} species contained in the plasma ($\alpha = 1, 2, \ldots, n$), and the gravitational force per unit mass is assumed to be $\vec{g}(\vec{x})$. Note that (1) is appropriate for both the relativistic and nonrelativistic cases. To obtain the equation for the conservation of particles, we merely integrate (1) over all momentum space. If there is no excitation, ionization, or recombination during collisions the result is

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial \vec{x}} \cdot \vec{J}_\alpha = 0 \tag{2}$$

$$n_\alpha = \int d^3p\, f_\alpha \qquad \vec{J}_\alpha \int d^3p\, \vec{v} f_\alpha$$

Thus n_α is just the particle density of the α^{th} species, and $\vec{J}_\alpha$ is the particle current. Equation 2 is clearly a simple continuity equation; the mass and charge continuity equations for the α^{th} species may easily be obtained by multiplying (2) by m_α and q_α, respectively.

The momentum conservation equation is obtained by multiplying (1) by $\vec{p}$, integrating the result over all momentum space, and then summing over all α. The result, since momentum is conserved in any collision, is

$$\frac{\partial}{\partial t} \sum_\alpha \int d^3p\, \vec{p} f_\alpha + \frac{\partial}{\partial \vec{x}} \cdot \sum_\alpha \int d^3p\, \vec{p}\, \vec{v} f_\alpha =$$

$$\sum_\alpha \int d^3p\, f_\alpha \left\{ q_\alpha \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) + m_\alpha \vec{g} \right\}. \tag{3}$$

With the aid of Maxwell's equations

$$\frac{\partial}{\partial \vec{x}} \cdot \vec{E} = 4\pi\rho \qquad \frac{\partial}{\partial \vec{x}} \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \frac{4\pi\vec{J}}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \tag{4}$$

and the analogous gravitational equations

$$\frac{\partial}{\partial \vec{x}} \cdot \vec{g} = -4\pi G\rho_m \qquad \nabla \times \vec{g} = 0, \tag{5}$$

where ρ_m is the mass density, and G is Newton's gravitational constant, (3) becomes

$$\frac{\partial}{\partial t}\vec{P} + \frac{\partial}{\partial \vec{x}} \cdot \overleftrightarrow{T} = 0 \tag{6}$$

$$\vec{P} = \sum_a \int d^3p\, \vec{p}\, f_a + \frac{\vec{S}}{c^2}$$

$$\overleftrightarrow{T} = \sum_a \int d^3p\, \vec{p}\,\vec{v}\, f_a - \left\{\frac{\vec{E}\vec{E}}{4\pi} - \frac{E^2}{8\pi} + \frac{\vec{B}\vec{B}}{4\pi} - \frac{B^2}{8\pi}\right\} - \left\{-\frac{\vec{g}\vec{g}}{4\pi G} + \frac{g^2}{8\pi G}\right\}$$

$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B} = \text{Poynting vector.}$$

Thus (6) is also a continuity equation, similar in form to (2), except that in (6) we have summed over all the various species. The vector $\vec{P}$ is obviously the total momentum density of the entire system, while $\overleftrightarrow{T}$ is an effective pressure tensor equal to the "particle" pressure tensor minus the electromagnetic (Maxwell) stress tensor minus the analogous gravitational stress tensor.

Finally, we obtain the energy conservation equation by multiplying (1) by $[m_a c^2(1-v^2c^2)^{-1/2} - m_a c^2]$, integrating the result over all momentum space, and then summing over all a. If we assume that all collisons are elastic, the result is

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial \vec{x}} \cdot \vec{Q} = \vec{g} \cdot \sum_a \int d^3p\, m_a f_a \vec{v} \tag{7}$$

$$U = \sum_a \int d^3p\, f_a m_a c^2[(1-v^2/c^2)^{-1/2} - 1] + \frac{E^2}{8\pi} + \frac{B^2}{8\pi}$$

$$\vec{Q} = \sum_a \int d^3p\, f_a m_a c^2[(1-v^2/c^2)^{-1/2} - 1]\,\vec{v} + \vec{S}.$$

Equation 7 is almost an energy continuity equation; U is clearly the total energy density, while $\vec{Q}$ is apparently the total energy flow. The term on the right-hand side seems to correspond to some kind of gravitational flow, although its form certainly does not fit in well with the other terms of (7). This difficulty would quickly disappear if we postulated the existence of a gravitational magnetic field $\vec{h}$ which was coupled to $\vec{g}$ by a set of "gravitational Maxwellian equations":

$$\frac{\partial}{\partial \vec{x}} \cdot \vec{g} = -4\pi\rho_m \qquad \frac{\partial}{\partial \vec{x}} \cdot \vec{h} = 0$$

$$\nabla \times \vec{g} = \mp \frac{1}{c_g} \frac{\partial \vec{h}}{\partial t} \qquad \nabla \times \vec{h} = -\frac{4\pi}{c_g} \vec{J}m \pm \frac{1}{c_g} \frac{\partial \vec{g}}{\partial t}. \tag{8}$$

Equations 8 are written in units in which G = 1; and c_g, the speed of "gravitational waves" in vacuo is presumably c. The "gravitational Lorenz force" would be

$$F_g = m\left(\vec{g} + \frac{\vec{v}}{c_g} \times \vec{h}\right), \tag{9}$$

and the "gravitational Poynting vector" is

$$\vec{S}_g = \frac{c_g}{4\pi} \vec{g} \times \vec{h}. \tag{10}$$

For this case, Eq. 7 would now have no term at all on the right-hand side, while $\vec{Q}$ would contain the additional term $\vec{S}_g$, and U would contain the term $-g^2/8\pi$.

It seems quite possible that the set of equations (8) is actually valid. It would certainly be very difficult to measure the force attributable to $\vec{h}$ directly in any experiment, since it would be greatly masked by the force resulting from $\vec{g}$. In electromagnetic theory, this problem can be circumvented by the use of currents having no net charge density; the (apparent) nonexistence of negative mass precludes this technique in gravitational experiments, however. The release of energy through radiation of gravitational waves is presumably negligible, except in stars. Since gravitational effects tend to completely dominate electromagnetic effects in such bodies, however, it seems likely that this is the chief source of energy loss from asteral systems.

J. A. Ross