GENERAL PHYSICS

I. MICROWAVE SPECTROSCOPY*

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A. MINIMUM DETECTABLE POWER IN SUPERCONDUCTING BOLOMETERS

There is considerable interest in the development of sensitive, high-speed detectors for the far infrared portion of the electromagnetic spectrum. This is a particularly awkward energy range for the usual types of quantum detector (photomultiplier, photoconductor, etc.) because at these frequencies individual quantum events are completely masked by the statistical fluctuations associated with thermal equilibrium at ordinary temperatures. At cryogenic temperatures, on the other hand, the ultimate sensitivity of both quantum and thermal detectors is increased by several orders of magnitude. Thermal detectors have the added advantage that their response is not characterized by an absorption edge. The successful use of superconducting bolometers for the detection of far infrared lattice vibrations[1,2] has stimulated an interest in examining their theoretical capabilities as sensitive, high-speed, far infrared detectors.

1. Sources of Noise in Radiation Bolometers

The ultimate sensitivity of a radiation bolometer is set by three noise processes: temperature noise, Johnson noise, and current noise. The first two can be easily derived from fundamental considerations. Temperature noise is determined by the statistical nature of thermal equilibrium within the sensing element of a thermal detector. Johnson or Nyquist noise is the manifestation of voltage fluctuations at the terminals of a conductor arising from the random motion of charged carriers. Current noise, on the other hand, is much more complex. Since it is produced by processes that cannot be adequately observed, it is usually determined empirically. Each of these noise processes has been treated in considerable detail by Smith, Jones, and Chasmar.[3]

The mean-square values of statistically independent fluctuations are additive, and the rms noise power arising from the three processes is given by

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$$\sqrt{\overline{W_m^2}} = \epsilon^{-1}\left[4kT^2G + \frac{4kTR}{r^2} + \frac{SR^2i^2}{r^2f}\right]^{1/2} (\Delta f)^{1/2} \text{ watts.} \tag{1}$$

The first term in the bracket on the right is the contribution of thermal noise; the second, Johnson noise; and the third, current noise.

Let us now define the symbols appearing in Eq. 1: ϵ is the emissivity of the receiving area of the bolometer; k (joules/°K) is Boltzmann's constant; T (°K) is the absolute temperature; G (watts/°K) is the thermal conductance of the receiving area of the bolometer to its surroundings; R (ohms) is the electrical resistance of the bolometer; i (amps) is the current flowing through it; f (sec^{-1}) is the noise frequency; and Δf (sec^{-1}) is the bandwidth over which the noise is observed. The quantity S is an intrinsic structure parameter related to the geometry, material, crystal structure, purity and strain state of the conducting element. This is a complex factor that is usually determined empirically.

The quantity r is the responsivity of the bolometer. It is given by

$$r = \frac{F\epsilon Ri\alpha}{G_e(1+\omega^2\tau^2)^{1/2}} \text{ volts/watt,} \tag{2}$$

where F is the "bridge factor" given by

$$F = \frac{R_\ell}{R + R_\ell}, \tag{3}$$

and R_ℓ (ohms) is the load resistance connected to the bolometer. The temperature coefficient of resistivity α is defined as

$$\alpha = \frac{1}{R}\frac{dR}{dT} \text{ °K}^{-1}. \tag{4}$$

The effect of Joule heating within the bolometer produced by the biasing current i is to alter the thermal conductance to an effective value given by

$$G_e = G - \alpha Ri^2\left(\frac{R_\ell - R}{R_\ell + R}\right) \text{watts/°K.} \tag{5}$$

It should be clear from Eqs. 2 and 5 that the stability of the bolometer is closely allied to its operating conditions. If, for example, $R_\ell > R$ and the biasing current i is increased to the point where G_e vanishes, Eq. 2 becomes singular and a thermal runaway condition develops. Within the stable portion of its operating range, the responsivity of a bolometer can be easily adjusted by varying the biasing current i and the load resistance R_ℓ.

Referring to Eq. 2, the quantity $\omega = 2\pi f$ (sec^{-1}) is the angular frequency of the signal, and the quantity τ (sec) is the thermal time constant of the bolometric element.

$$\tau = C/G \text{ sec} \tag{6}$$

The thermal capacity of the bolometric element is

$$C = \rho C_v Ad \text{ joules/°K}, \tag{7}$$

where ρ (gms/cm^3) is the density of the conducting element, C_v (joules/gm°K) is its specific heat, A (cm^2) is its area, and d (cm) is its thickness.

Referring to Eq. 1, we wish to point out that the current noise spectrum is the only one that is a function of frequency. This f^{-1} law was found experimentally to hold over a wide range of frequencies in carbon resistors[3]:

$$10^{-3} < f < 10^4. \tag{8}$$

But clearly an integration over this noise spectrum would yield a logarithmic singularity at the origin, so it must be conceded that the f^{-1} behavior does not apply at all frequencies. Nevertheless, over the region of practical interest it is likely that current noise could be reduced to a tolerable level by chopping.

Both Johnson and current noises are inversely proportional to the responsivity r. Since the responsivity is directly proportional to the thermal coefficient of resistance α (see Eq. 2), materials with sufficiently high coefficients α can be utilized to effectively eliminate all noise contributions other than thermal noise. Since thermal noise is proportional to the absolute temperature T, the motivation for working at cryogenic temperatures is clear.

It may be argued that fluctuations in signal or background radiation introduce problems that transcend considerations of the ultimate sensitivity of a detector. We readily concede that there may be situations in which the ultimate detector sensitivity may be an irrelevant issue. Nevertheless, we maintain that an accurate knowledge of the Wiener spectra of the radiation of interest should suggest signal-processing techniques (such as chopping or filtering) that substantially reduce such fluctuations to the point where the ultimate detector sensitivity remains a relevant issue. These considerations are considerably more specialized and are outside the scope of the present discussion.

2. Superconducting Bolometer

In normal metals the necessary conditions of high α and low operating temperature T are mutually exclusive. But the superconducting transition exhibits enormous values for α at transition temperatures that can be driven arbitrarily close to absolute zero by means of an external magnetic field. Suppose, for example, we were to approximate the superconducting transition by the following function:

$$\rho(T) = \rho_o\left\{1 + \exp\left[\frac{2(T_c - T)}{\Delta T_c}\right]\right\}^{-1} \text{ ohm cm.} \tag{9}$$

This function is sketched in Fig. I-1. The transition temperature T_c (°K) is defined by

$$\rho(T_c) = \rho_o/2, \tag{10}$$

where ρ_o is the limiting resistivity at absolute zero in the corresponding normal state. The width of the superconducting transition ΔT_c (°K) is so defined that

$$\rho(T_c - \Delta T_c/2) = \frac{\rho_o}{1 + e} = 0.27\rho_o \tag{11}$$

and

$$\rho(T_c + \Delta T_c/2) = \frac{\rho_o}{1 + e^{-1}} = 0.73\rho_o. \tag{12}$$

In this approximation, evaluation of $\alpha(T_c)$ from Eq. 4 yields

$$\alpha(T_c) = (\Delta T_c)^{-1} \ (°K)^{-1}. \tag{13}$$

The transition width ΔT_c is a complex function of metallic purity, perfection of crystal structure, mechanical strain, bias current density, and applied magnetic field. In alloys

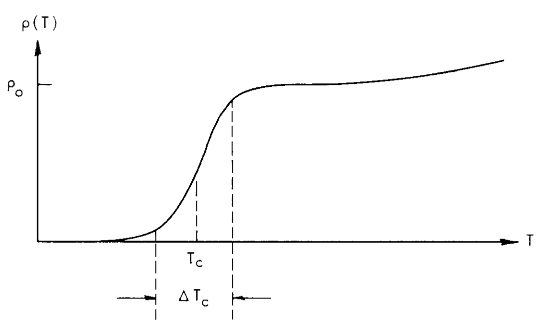

Fig. I-1. Temperature dependence of the resistivity of a superconductor. The quantity ρ_o (ohm cm) is the limiting resistivity at absolute zero in the corresponding normal state.

ΔT_c may amount to several degrees Kelvin. On the other hand, ΔT_c has been measured in extremely pure, unstrained single crystals of tin at very low current densities.[4] Values as low as 5×10^{-4} have been observed, and the authors remarked that even these finite widths may have been caused by small, mechanical strains existing in their samples.

In the approximation of Eq. 9, the corresponding coefficient of resistivity for the tin samples of de Haas and Voogd would amount to

$$\alpha(T_c) = 2 \times 10^3 \ °K^{-1}. \tag{14}$$

But the values of α that have actually been observed in superconducting bolometers[3,5] are in the range 2-100 $°K^{-1}$.

The effective suppression of Johnson noise in a superconducting bolometer by the enormous value for α inherent in the superconducting transition has been experimentally demonstrated by Andrews.[6] With an observed $\alpha = 50\,°K^{-1}$, the ratio of Johnson to thermal noise was found to be 0.12.

Probably the most sensitive superconducting bolometer that has been demonstrated is that of Martin and Bloor.[5] The observed minimum detectable power was somewhat less than 10^{-12} watt in the far infrared within a bandpass of 10^{-1} sec^{-1}. The observed signal was one hundred times larger than that observed by means of a Golay cell. Martin and Bloor remark that their noise level was actually set by their amplifier and, in fact, their calculated value for the bolometer was 3.5×10^{-14} watt with a bandpass of 10^{-1} sec^{-1}. The extremely long thermal time constant (1.25 sec) of this bolometer, however, precludes its utility in many practical applications.

3. Thermal Boundary Resistance

Recent experiments on the propagation of heat between dissimilar solids[7] at low temperatures have revealed that a temperature discontinuity develops at the interface which is proportional to the normal component of the thermal flux. The magnitude of the temperature discontinuity is proportional to the so-called "thermal boundary resistance" R^* ($°K\ cm^2\ watts^{-1}$) which is defined by

$$\dot{Q} = \frac{A\Delta T}{R^*} \text{ watts,} \tag{15}$$

where $\dot{Q}$ is the rate of flow of thermal energy, A (cm^2) is the area of the interface, and ΔT (°K) is the magnitude of the discontinuity developed at the interface.

Experiments on the thermal boundary resistance have resulted in an empirical relationship describing the temperature dependence of R^*.

$$R^* = \eta T^{-n}\ °K\ cm^2\ watts^{-1}. \tag{16}$$

The exponent n is dependent upon the nature of the interface, but it is usually very close to 3. The numerical factor η ($cm^2\ °K^{n+1}\ watts^{-1}$) is also related to the nature of the interface and to the particular materials in contact. Typical values range between 0.3 and 30. The lower values are for intermetallic interfaces; the higher values are usually associated with metal-insulator interfaces.

Since the defining equation for thermal conductance G is

$$\dot{Q} = G\Delta T \text{ watts,} \tag{17}$$

upon comparing Eqs. 15 and 16, we have

$$G = A\eta^{-1}T^{n} \text{ watts } {}^{\circ}K^{-1}. \tag{18}$$

4. Recent Experimental Developments

The thermal boundary resistance phenomenon has suggested a novel approach to the design of high-speed thermal detectors. A thin film (~1000 Å) of a superconducting metal such as indium is evaporated directly upon a single-crystal substrate such as sapphire, which exhibits unusually high thermal conductivity at cryogenic temperatures. The thermal conductance between the film and substrate is sufficiently high to provide an extremely fast thermal response time. On the other hand, the sensitivity of the device in narrow-band applications is competitive with the best radiation detectors available,[8] and is not limited to any particular portion of the electromagnetic spectrum.

Returning to the expression for the minimum detectable power (Eq. 1), let us assume that the thermal coefficient of resistance α is sufficiently high so that the thermal noise contribution dominates. Substitution of Eq. 18 in Eq. 1 yields

$$\sqrt{\overline{W_m^2}} = \frac{2}{\epsilon}\left(\frac{AkT^{n+2}}{\eta}\right)^{1/2} (\Delta f)^{1/2}. \tag{19}$$

The thermal boundary resistance of an indium-sapphire interface has been measured by Neeper and Dillinger.[7] Their experimental value is given by

$$R^{*} = 26T^{-2.8} \; {}^{\circ}K \text{ cm}^2 \text{ watt}^{-1}. \tag{20}$$

The transition temperature[9] of indium is 3.4°K. Assuming a perfectly absorbing surface (suitably blackened with $\epsilon = 1$), we evaluate the minimum detectable power of an indium-film bolometer with an effective area of $(0.25 \text{ mm})^2$, deposited upon a sapphire substrate, and connected to an amplifier with an effective bandwidth of 1 cps. Equation 19 yields

$$\sqrt{\overline{W_m^2}} = 7 \times 10^{-13} \text{ watts} \qquad \text{(indium)}. \tag{21}$$

This value is approximately equal to that measured by Martin and Bloor[5] for their tin bolometer.

Equation 19 indicates that considerable advantage is secured by low-temperature operation. Consider a zinc-film bolometer[9] with a transition temperature of 0.92°K. Since no experimental thermal boundary resistance data are available for zinc-sapphire interfaces, we shall approximate the value with Eq. 20. The minimum detectable power for a zinc-film bolometer, evaluated under the same conditions as those for indium (Eq. 21), is

$$\sqrt{\overline{W_m^2}} = 1.2 \times 10^{-14} \text{ watts} \qquad \text{(zinc)}. \tag{22}$$

Thus, by lowering the temperature 2.5°K, nearly two orders of magnitude are gained in theoretical sensitivity.

The theoretical thermal time constants of these bolometers can be evaluated from Eqs. 6, 7, and 18. The low-temperature specific heat of a metal can be represented by

$$C_v = \beta T^3 + \frac{\gamma T}{W} \text{ joules gm}^{-1}\ {}^\circ\text{K}^{-1} \tag{23}$$

$$\beta = \frac{1941}{W\theta^3} \text{ joules gm}^{-1}\ {}^\circ\text{K}^{-4}, \tag{24}$$

where γ (joules mole^{-1} °K^{-2}) is the coefficient of the electronic specific heat, β represents the lattice contribution, W (gms mole^{-1}) is the molecular weight, and θ (°K) is the Debye temperature. Experimentally determined values of the constants β and γ have been tabulated in Rosenberg.[10] The resulting expression for the thermal time constant of a superconducting film bolometer deposited upon an insulating crystal is given by

$$\tau = \rho\left(\beta T_c^3 + \frac{\gamma T_c}{W}\right) \eta d T_c^{-n} \text{ sec.} \tag{25}$$

Equation 25, evaluated for indium and zinc films, 1000 Å thick, deposited upon a sapphire substrate, yields

$$\underset{\text{(indium)}}{\tau = 35 \text{ n sec}} \qquad \underset{\text{(zinc)}}{\tau = 22 \text{ n sec.}} \tag{26}$$

These time constants indicate a degree of performance that greatly exceeds that of the more conventional thermal detectors.

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B. FERMI SURFACES OF GALLIUM FROM SIZE EFFECT

The wave vectors corresponding to the cross section of the Fermi surfaces have been found for gallium single crystals whose normals are along the b and c axes (see Figs. I-2 and I-3). The experimental techniques used for these measurements were similar to those reported previously.[1]

The signal obtained in these experiments has a complete line shape. The magnetic

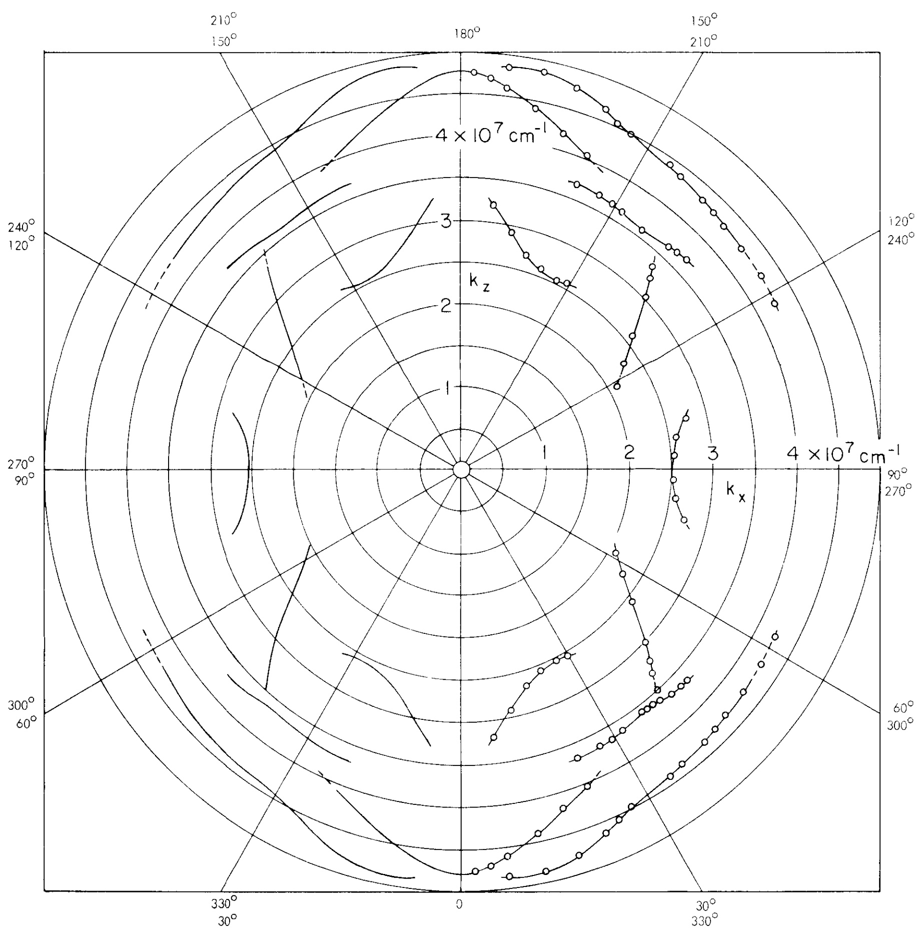

Fig. I-2. Wave vectors for b crystal.

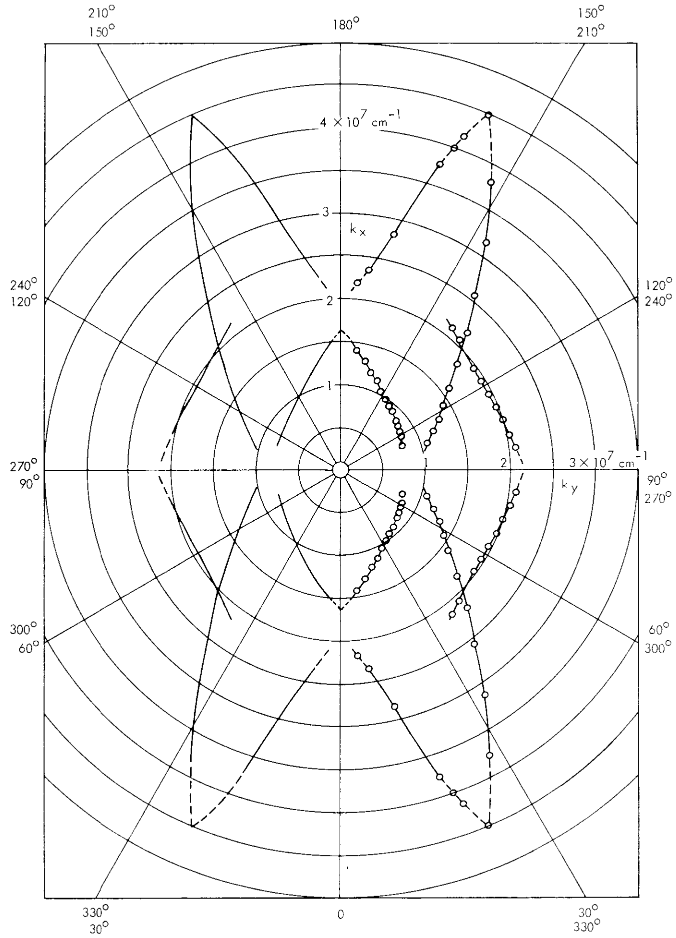

Fig. I-3. Wave vectors for c crystal.

field dependence of the signal as the radiofrequency changes was used to ascertain the portion of the curve for determining the external Fermi surface wave vector in terms of the magnetic field and the thickness of the sample. The portion of the line shape which had the smallest shift of magnetic field with frequency is identified in these curves as the orbit corresponding to the thickness of the sample. Although the line shape is not well understood, these data may be used with some confidence.

A. Fukumoto

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