

## The top quark as a calibration tool at the LHC

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**Summary.** — Thanks to the large top quark pair production cross section and the relatively low background at the LHC,  $t\bar{t}$  events can be used for calibration at ATLAS and CMS. Assuming the Standard Model prediction  $BR(t \rightarrow bW)=1$  to be true, the heavy flavour content of  $t\bar{t}$  events is well predicted, which allows to calibrate and measure the efficiency of  $b$ -tagging algorithms directly from the data with a precision of about 5%. The light ( $b$ -) jet energy scale can also be extracted from  $t\bar{t}$  events at the 1% level using  $W$  (and top) hadronic decays.

### 1. – Introduction

The ATLAS and CMS collaborations plan to take advantage of the large  $t\bar{t}$  production cross section and use  $t\bar{t}$  events for calibration purposes. Several techniques have been developed to calibrate  $b$ -tagging algorithms in both the lepton+jets and the fully leptonic channel, and estimate the jet energy scale in the lepton+jets channel. In the following, the basic selection and reconstruction of  $t\bar{t}$  events is similar to the one described in [1].

### 2. – Calibration of $b$ -tagging algorithms with data

Several methods are proposed to estimate the efficiency of  $b$ -tagging algorithms with data. The first method described is a tag counting method that compares the number of selected events with a certain number of  $b$ -tagged jets, leading to a combined measurement of the  $b$ -tagging efficiency and the  $t\bar{t}$  production cross section. Another class of methods is based on the selection of a  $b$ -enriched jet sample to estimate the  $b$ -tagging efficiency, and check the variables used by the  $b$ -taggers against data. Several strategies can be applied to choose the correct jet combination, based on either a topological or a kinematic selection or a selection which makes use of a combined likelihood of several observables. The key in these methods is to control the background in the selected jet sample. Background contributions are either estimated from simulation or using a side-band subtraction technique.



**2.1. Combined  $b$ -tagging efficiency and  $t\bar{t}$  cross section measurement (ATLAS).** – Every  $t\bar{t}$  event contains two  $b$ -jets in the final state so one can naively expect that the number of  $t\bar{t}$  events with two (one)  $b$ -tagged jets is proportional to  $\epsilon_b^2$  ( $2\epsilon_b(1-\epsilon_b)$ ), where  $\epsilon_b$  is the  $b$ -tagging efficiency. Thus comparing the number of events with one and two  $b$ -tagged jets allows to constrain both the total  $t\bar{t}$  event yield and the  $b$ -tagging efficiency. In reality,  $c$ -jets and light jets, either from the hadronic  $W$  decay or from initial and final state radiation (ISR/FSR) are present and contribute to the number of tagged jets in the event. Moreover not all  $b$ -jets coming from the top decays end up being selected, whilst a small number of  $b$ -jets are produced through gluon radiation. To take these effects into account, the event flavour content is estimated from Monte Carlo with a large simulation  $t\bar{t}$  sample. The expected number of events with a certain number of  $b$ -tagged jets is then estimated as a function of the  $t\bar{t}$  cross section, detector acceptance, trigger and reconstruction efficiencies, and  $b$ -,  $c$ -, and light jet tagging efficiencies. Finally, a likelihood is used to fit the  $t\bar{t}$  cross section and the tagging efficiencies:  $L = \Pi(\text{Poisson}(N_n, < N_n >))$  where  $N_n$  ( $< N_n >$ ) is the observed (expected) number of events with  $n$  tags. In practice only events with one, two, or three tags in the lepton+jets channel and one or two tags in the dilepton channel are used. Indeed, events with no tag suffer from significant background, whilst there are few events with more than three tags (two tags in the dilepton channel). In the lepton+jets channel, both  $b$ - and  $c$ -tagging efficiencies are allowed to fluctuate in the fit together with the  $t\bar{t}$  cross-section (hence three variables and three constraints); the light jet tagging efficiency is fixed in the fit and must be measured elsewhere. In the dilepton channel, the  $c$ -jet tagging efficiency is also fixed (hence two variables and two constraints). Figure 1 shows the expected yield of  $t\bar{t}$  and background events with an integrated luminosity of  $100 \text{ pb}^{-1}$  in the  $e\mu$  dilepton channel, the  $ee$  and  $\mu\mu$  dilepton channels, and the lepton+jets channel as a function of the number of tagged jets, for a true  $b$ -tagging efficiency of about 60%. With  $100 \text{ pb}^{-1}$  of data, the counting method allows the  $b$ -tagging efficiency to be measured with a relative precision of  $\pm 2.7(\text{stat.}) \pm 3.4(\text{syst.})\%$  in the lepton+jets channel, and  $\pm 4.2(\text{stat.}) \pm 3.5(\text{syst.})\%$  in the dilepton channel. Sources of systematics are listed in Table I. A better understanding of ISR/FSR could significantly reduce the systematic uncertainty. The uncertainty on the  $t\bar{t}$  production cross-section is  $\pm 2.4(\text{stat.})_{-14.7}^{+12.7}(\text{syst.})\%$  in the lepton+jets channel, and  $\pm 4.8(\text{stat.})_{-7.7}^{+7.2}(\text{syst.})\%$  in the dilepton channel, mostly because of the uncertainties on the jet multiplicity, jet energy scale, and background estimate, uncertainties that can be improved in the long term and have been assessed conservatively here. (In addition to the quoted systematic uncertainties, a 5% uncertainty on luminosity is expected at the beginning of data taking.)

**2.2. Selection of a  $b$ -enriched jet sample (ATLAS-CMS).** – In the lepton+jets channel, the  $b$ -jet from the hadronic branch of the top quark decay is tagged to reduce the jet combinatorial background. The  $b$ -jet from the leptonic branch is then used as a probe to measure the  $b$ -tagging efficiency. In the fully leptonic channel, one electron and one muon are required and both  $b$ -jets are used to estimate the  $b$ -tagging efficiency.

**2.2.1. Topological selection (ATLAS).** An attempt is made to reconstruct both hadronic and leptonic top decays in each event by looking at jet combinations that are consistent with a  $W$  and a top hadronic decay, and a top leptonic decay. To reduce background, the two  $W$  jets are both required not to be  $b$ -tagged and to have a mass between 60 and 100 GeV. In case of ambiguity, the combination with the largest scalar sum of the  $p_T$  of the two top quarks is retained for further analysis. The resulting distributions

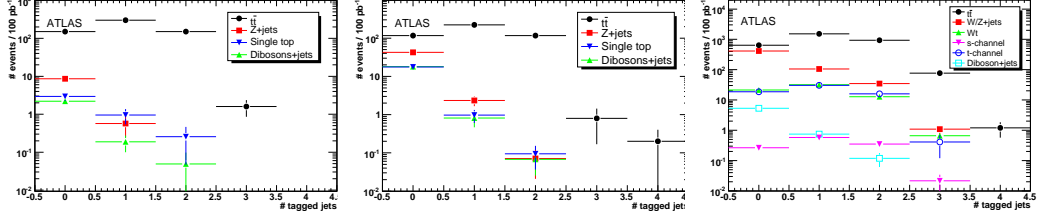


Fig. 1. – Event yield expected with an integrated luminosity of  $100 \text{ pb}^{-1}$  in the  $e\mu$  dilepton channel (left), the  $ee$  and  $\mu\mu$  dilepton channels (center) and lepton+jets channel (right) as a function of the number of tagged jets. The expected background from  $W/Z$ +jets, single top, and diboson production is also shown.

of reconstructed hadronic and leptonic top masses are shown in Fig. 2. Clear top mass peaks are seen in both distributions, though with significant combinatorial background and a small contribution from  $W$ +jet and single top background events. Contributions from non- $t\bar{t}$  events are also shown separately.

In order to correct for the background under the leptonic top mass peak, both its size and flavour composition must be determined. This is done on a statistical basis using the sideband region with high leptonic top mass to normalize the background contribution, and a control sample made of events with high hadronic top mass and no  $b$ -tagged jet on the hadronic side to determine the shape of its mass distribution. The amount of background under the signal is then extracted using a simultaneous fit to both signal and control sample leptonic top mass distributions. The fits are performed for several jet  $E_T$  bins between 20 and 200 GeV. At a true  $b$ -tagging efficiency of 60%,  $200 \text{ pb}^{-1}$  of data are required for this technique to work, leading to a relative statistical error of 6.4%, and a relative systematic uncertainty of 3.4%.

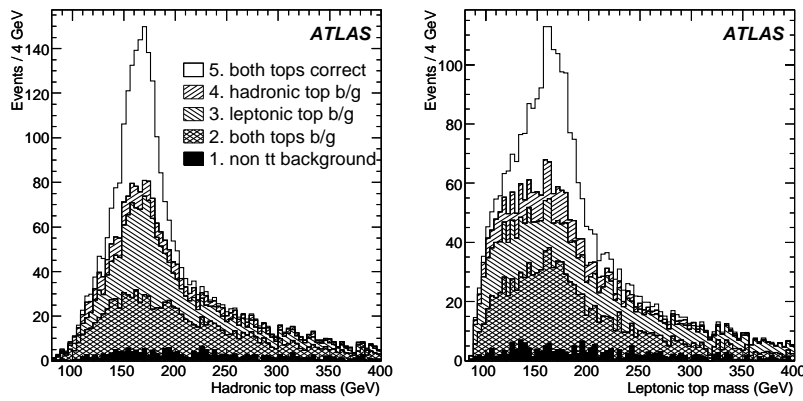


Fig. 2. – Reconstructed hadronic (left) and leptonic (right) top masses for the selected jet combination, showing the contributions from correctly reconstructed  $t\bar{t}$  events, combinatorial and non- $t\bar{t}$  background, normalized to  $100 \text{ pb}^{-1}$ .

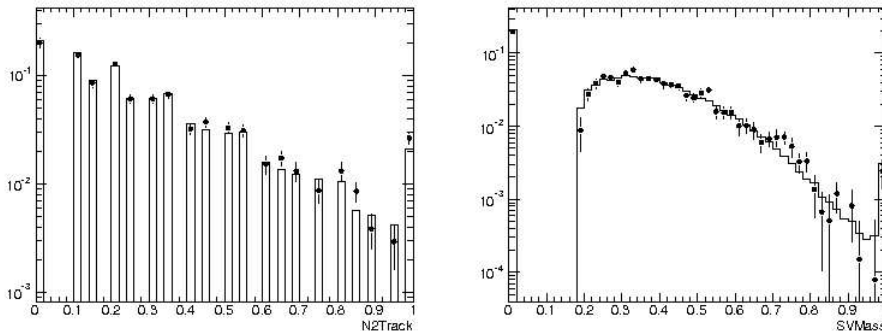


Fig. 3. – Background-subtracted  $b$ -tagging variable distributions derived from the  $b$ -jet sample selected by the topological method, with  $948 \text{ pb}^{-1}$  of simulated  $t\bar{t}$  plus background data. The derived distributions are shown by the points with error bars, and the Monte Carlo truth for an unbiased sample of  $b$ -jets is shown by the solid histograms. Left: number of two-track vertices. Right: mass of the secondary vertex. For convenience, variables have been transformed so as to be in the range  $[0;1]$ .

Other techniques of selection of a pure  $b$ -jet sample such as the one described below (“likelihood”) and one using a kinematic fit to select the correct jet permutation (“kinematic”) have also been studied by the ATLAS collaboration. The performance of each of these methods is summarized in Table I. The topological method leads to smaller systematic uncertainties because it relies less on Monte Carlo and background estimates since both combinatorial and physical backgrounds are estimated from data.

The same technique can be used to extract from data the distributions of elementary variables used by the  $b$ -tagging algorithms, as shown in Fig. 3. This is particularly important to validate and possibly tune the simulation against data.

**2.2.2. Selection using a combined likelihood ratio (CMS).** Several variables are able to differentiate between correct and wrong jet combinations. For each variable  $x_i$  a likelihood is constructed which is defined as:  $\mathcal{L}_i(x_i) = \mathcal{S}_i(x_i)/\mathcal{B}_i(x_i)$ , where  $\mathcal{S}_i(x_i)$  and  $\mathcal{B}_i(x_i)$  are the distributions of variable  $x_i$  for the correct and the wrong jet combinations respectively. The discriminating power of these variables is then exploited by combining them as  $\mathcal{L} = \prod_{i=1}^n \mathcal{L}_i(x_i)$ , where  $n$  is the number of variables. To purify the selected jet sample, a cut on the combined likelihood ratio  $\mathcal{L}$  can be applied. In Fig. 4 the  $b$ -jet purity as a function of the efficiency of the cut on the combined likelihood ratio  $\mathcal{L}$  is shown.

When a  $b$ -tagging algorithm is applied on the selected jet sample a fraction  $x_{tag}$  of the jets will be tagged, with  $x_{tag} = \epsilon_b x_b + \epsilon_0(1 - x_b)$  where the mis-tag efficiency  $\epsilon_0$  and the fraction of  $b$ -jets  $x_b$  in the jet sample are determined from simulation at a certain cut on the combined likelihood ratio. The total uncertainty is calculated as a function of the cut on the combined likelihood ratio, including systematics. The optimal cut on  $\mathcal{L}$  is defined as the value that minimizes the total uncertainty. At this cut value,  $\epsilon_0$  and  $x_b$  are determined and the  $b$ -tagging efficiency is calculated for all decay channels. The measurements of the  $b$ -tagging efficiencies in the different decay channels can be combined to reduce the uncertainty. In order to do this, all systematic uncertainties are treated as being fully correlated. To obtain a sample independent  $b$ -tagging efficiency

Systematic	Counting		Topological	Likelihood	Kinematic
	lepton+jet	dilepton			
Light jets and $\tau$	0.1	0.7	0.5	5.2	0.6
Charm jets	0.0	0.8	0.7	4.6	2.2
Jet energy scale	0.9	0.5	0.5	2.5	1.1
$b$ -jet definition	1.4	1.4	-	-	-
MC generators	0.1	2	0.2	5.9	5.5
ISR/FSR	2.7	2	1	2.2	0.5
W/Z/Diboson+jets bgd	1.2	0.3	2.8	9.6	0.3
Single top bgd	0.1	0.1	1.2	-	1.2
Top quark mass	0.3	0.5	-	4.1	-
Total systematic	3.4	3.5	3.4	14.2	6.2
Statistical (100 pb <sup>-1</sup> )	2.7	4.2	-	5.0	7.7
Statistical (200 pb <sup>-1</sup> )	1.9	3.0	6.4	4.4	5.5

TABLE I. – Summary of systematic and statistical uncertainties on the measurement of the  $b$ -tagging efficiency at a true efficiency of 60%, for the counting method in lepton+jets and dilepton channels, and the topological, likelihood and kinematic jet selection methods. The uncertainties are expressed as relative errors (in %).

the measurement is performed as a function of the transverse energy  $E_T$  and the pseudorapidity  $\eta$  of the jet. Two bins in  $\eta$  were chosen defining the barrel ( $|\eta| < 1.5$ ) and the endcap ( $|\eta| > 1.5$ ) region, while 5 bins in  $E_T$  were chosen between 25 and 250 GeV. The potential of the method is shown in Fig. 5, where the relative uncertainty is shown as a function of the transverse energy  $E_T$  of the jet for both barrel and endcaps.

With 1 fb<sup>-1</sup> of integrated luminosity the expected relative uncertainty is about 6% in the barrel and 10% in the endcaps. The main systematic uncertainties come from ISR/FSR and the uncertainty on the cross sections for signal and background processes in case of the fully leptonic decay channel. For the lepton+jets channel the main contributions to the total systematic uncertainty come from the ISR/FSR as well as the  $b$ -tagging efficiency when tagging the  $b$ -jet from the hadronic branch in the top quark pair decay.

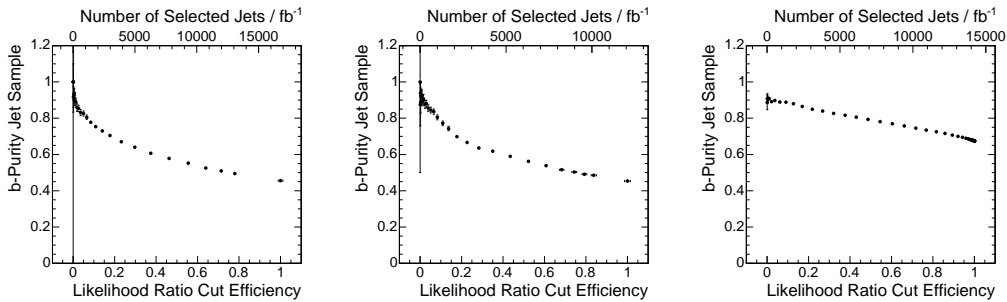


Fig. 4. – The  $b$ -jet purity as a function of the efficiency of the cut on the combined likelihood ratio  $\mathcal{L}$ . On the left for the muon+jets decay channel, in the middle for the electron+jets decay channel and on the right for the fully leptonic decay channel.

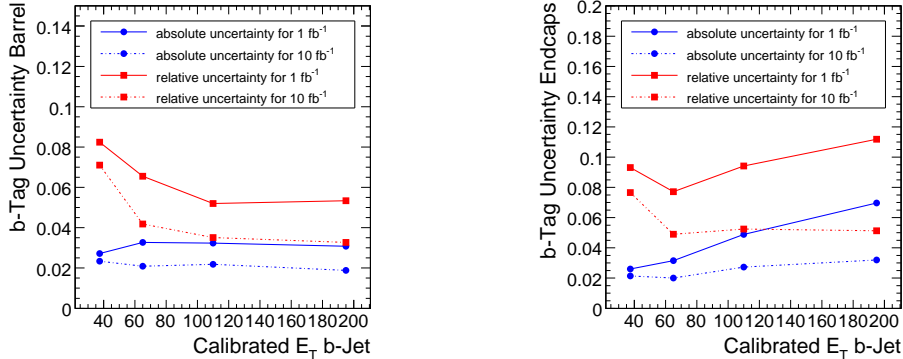


Fig. 5. – Expected uncertainty on the measurement of the  $b$ -tagging efficiency  $\epsilon_b$  as a function of the transverse energy of the jets. Left for the barrel region, right for the endcaps.

### 3. – Calibration of the Jet Energy Scale with data

The  $W$  boson and the top quark mass have been measured in previous experiments with a precision of respectively 0.03% and 0.8%. This high precision provides the possibility to estimate the jet energy scale corrections from data using the  $W$  boson (and the top quark) mass as a constraint in lepton+jets events. The  $W$  boson mass constraint can be applied on the reconstructed  $W$  boson mass spectrum to obtain the jet energy scale corrections. Another method makes use of a kinematic fit which forces the  $W$  boson and top quark mass constraints event-by-event to obtain the jet energy scale for both light and  $b$ -jets.

**3.1. Hadronic  $W$  decays (ATLAS).** – This method is similar to the method described in [2] for the electromagnetic energy scale determination using  $Z^0 \rightarrow e^+e^-$  events. Templates of the hadronic  $W$  decay reconstructed mass are made as a function of light jet energy scale ( $\alpha$ ) and jet energy resolution ( $\beta$ ) by smearing generator-level quark 4-momenta. The templates are then fitted to the full-simulation jet-jet reconstructed mass and a  $\chi^2$  minimization gives the best estimates of  $\alpha$  and  $\beta$ . In order to select the correct jet-jet combination, exactly two jets are required to be  $b$ -tagged, and the reconstructed hadronic top mass is required to be between 150 and 200 GeV. Figure 6 (left) shows the full simulation reconstructed mass compared to the templates for two sets of parameters (default and best fit). The fit gives a jet energy scale of  $\alpha = 0.96$ , in agreement with the true jet energy scale ( $0.961 \pm 0.003$  for the particular simulation version used here). Several checks of the stability of the method have been performed: the influence of the combinatorial background, the choice of using or not events with more than two jets, the smearing used to make templates, and the dependence on top mass; each effect amounts to a variation of 0.5% or less on the jet energy scale estimate. The effect of ISR/FSR was also studied by comparing samples with different radiation tunings: although the raw jet-jet mass is affected (as shown on Fig. 6 (right)), the measured  $W$  mass after calibration is found to be stable within statistical uncertainties of 0.2%. Thus a 1% systematic uncertainty seems achievable. With an integrated luminosity of  $50\text{pb}^{-1}$ , a statistical uncertainty of 2% is expected.

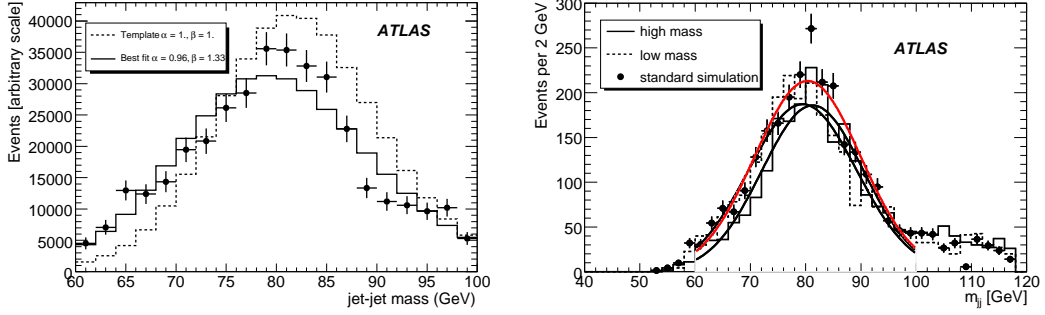


Fig. 6. – Left: Reconstructed  $W$  mass in fully simulated events (dotted line), of template with  $\alpha = 1$  and  $\beta = 1$  (error bars), and of template for best fit (solid line). Right: Reconstructed  $W$  mass in events with different gluon radiation settings.

**3.2. Event-by-event using a kinematic with top quark and  $W$  boson mass constraints (CMS).** – The three jets from the hadronic top quark decay are used in an event-by-event kinematic fit forcing the reconstructed  $W$  boson and top quark masses to be equal to their world averages. The momenta of the jets are parametrized in the  $(E_T, \theta, \phi)$  dimensions and the resolutions of these parameters are obtained from simulation as a function of the pseudo-rapidity and transverse energy of the object. The correct jet combination is identified by constructing a combined likelihood ratio from four observables, using the same method as described in 2.2.2. The best jet combination is chosen as the one with the highest value of the combined likelihood ratio. A cut at 0 is made on the logarithm of the combined likelihood ratio variable to purify the event sample. Jet energy corrections are applied in a 3D range of  $\pm 50\%$  around the nominal jet energies. On the remaining events and for the chosen jet association the kinematic fit is applied for each combination of jet energy corrections returning a fit probability  $P_{fit}(\Delta E_b, \Delta E_q, \Delta E_{\bar{q}})$  which reflects the probability that the applied constraints are fulfilled for the event given certain corrections. The maximum of this fit probability  $P_{fit}^{max}$  is searched for in the 3D space, requiring that the 2 light jet energy corrections are equal, thus reducing the 3D range to a 2D range.

A cut of  $P_{fit}^{max} > 0.98$  is made requiring that the correct jet energy scale corrections to fulfill the constraints are found in the scanned energy range. Mis-reconstructed events are removed by requiring that the probability of the fit when no corrections are applied exceeds 0.01. The maximum fit probability for each event  $i$  corresponds to an estimate of the jet energy scale corrections  $\Delta E_{b,i}, \Delta E_{l,i}$  for event  $i$ . If one of these estimates deviates more than 20% from the first estimate  $\Delta E_{b,incl}, \Delta E_{l,incl}$ , the event is removed.  $\Delta E_{l,incl}$  is calculated from the relative difference between the fitted expectation value of the  $W$  boson mass distribution and the world average for the  $W$  boson mass. The difference between the Monte Carlo expectation values of light and  $b$ -jet energy scale corrections (7%) is used to obtain the first estimate for  $b$ -jets  $\Delta E_{b,incl}$ . The fit probabilities in the 2D range of the remaining events are translated into  $\chi^2$ -values and combined. The minimum of the combined  $\Delta\chi^2$ -distribution is searched for to obtain the best estimates  $\Delta E_l, \Delta E_b$  for both the light and  $b$ -jet energy scale corrections for  $100 \text{ pb}^{-1}$  of data. In Fig. 7 the  $5\sigma$  confidence interval for the best estimates is shown.

The pull distributions were calculated using 300 pseudo-experiments each with an

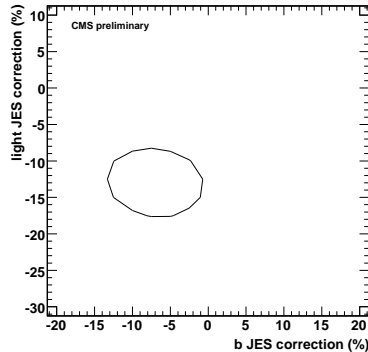


Fig. 7. – The distribution of the  $\Delta\chi^2$  presented as a  $5\sigma$  confidence interval.

integrated luminosity of  $100 \text{ pb}^{-1}$ . The uncertainty on the estimates for the light and  $b$ -jet energy scale corrections is corrected for the width of the pull distribution resulting in an uncertainty of 0.9% for both estimates for an integrated luminosity of  $100 \text{ pb}^{-1}$ . It was checked that the method is linear with respect to the input of the jet energy scales. Also the robustness versus several possible systematic effects has been checked, e.g. the uncertainty on the cross sections of background processes, effect of jet combinatorial background and smeared jet resolutions.

#### 4. – Conclusion

Several methods have been developed to calibrate the  $b$ -tagging and the jet energy scale at ATLAS and CMS using  $t\bar{t}$  events. The  $b$ -tagging efficiency can be measured at the level of 5% with about  $100 \text{ pb}^{-1}$  and specific variables used by the  $b$ -tagging algorithms can be checked against data in order to validate the simulation. A 1% light jet energy scale seems achievable with an integrated luminosity of a few hundreds  $\text{pb}^{-1}$ . An additional constraint on the top mass also allows to calibrate the  $b$ -jet energy scale at the 1% level.

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