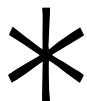


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Quantum Computing

A Short Course from Theory to Experiment

Second, Updated and Enlarged Edition



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