

*Joachim Stolze and Dieter Suter*

# **Quantum Computing**

A Short Course from Theory to Experiment

Second, Updated and Enlarged Edition



WILEY-  
VCH

WILEY-VCH Verlag GmbH & Co. KGaA

# Contents

<b>Preface</b>	<b>XIII</b>
<b>1 Introduction and Survey</b>	<b>1</b>
1.1 Information, Computers, and Quantum Mechanics .....	1
1.1.1 Digital Information .....	1
1.1.2 Moore's Law .....	2
1.1.3 Emergence of Quantum Behavior .....	3
1.1.4 Energy Dissipation in Computers .....	4
1.2 Quantum Computer Basics .....	5
1.2.1 Quantum Information .....	5
1.2.2 Quantum Communication .....	7
1.2.3 Basics of Quantum Information Processing .....	8
1.2.4 Decoherence .....	9
1.2.5 Implementation .....	10
1.3 History of Quantum Information Processing .....	11
1.3.1 Initial Ideas .....	11
1.3.2 Quantum Algorithms .....	12
1.3.3 Implementations .....	13
<b>2 Physics of Computation</b>	<b>15</b>
2.1 Physical Laws and Information Processing .....	15
2.1.1 Hardware Representation .....	15
2.1.2 Quantum vs. Classical Information Processing .....	16
2.2 Limitations on Computer Performance .....	17
2.2.1 Switching Energy .....	17
2.2.2 Entropy Generation and Maxwell's Demon .....	18
2.2.3 Reversible Logic .....	19
2.2.4 Reversible Gates for Universal Computers .....	21
2.2.5 Processing Speed .....	22
2.2.6 Storage Density .....	23
2.3 The Ultimate Laptop .....	23
2.3.1 Processing Speed .....	23
2.3.2 Maximum Storage Density .....	24

3	Elements of Classical Computer Science	27
3.1	Bits of History	.....27
3.2	Boolean Algebra and Logic Gates	.....28
3.2.1	Bits and Gates	..... 28
3.2.2	2-Bit Logic Gates	..... 28
3.2.3	Minimum Set of Irreversible Gates	..... 30
3.2.4	Minimum Set of Reversible Gates	..... 31
3.2.5	The CNOT Gate	..... 31
3.2.6	The Toffoli Gate	..... 32
3.2.7	The Fredkin Gate	..... 33
3.3	Universal Computers	..... 34
3.3.1	The Turing Machine	..... 34
3.3.2	The Church—Turing Hypothesis	..... 35
3.4	Complexity and Algorithms	..... 35
3.4.1	Complexity Classes	..... 35
3.4.2	Hard and Impossible Problems	..... 36
4	Quantum Mechanics	39
4.1	General Structure	..... 39
4.1.1	Spectral Lines and Stationary States	..... 39
4.1.2	Vectors in Hilbert Space	..... 39
4.1.3	Operators in Hilbert Space	..... 40
4.1.4	Dynamics and the Hamiltonian Operator	..... 42
4.1.5	Measurements	..... 43
4.2	Quantum States	..... 44
4.2.1	The Two-Dimensional Hilbert Space: Qubits, Spins, and Photons	44
4.2.2	Hamiltonian and Evolution	..... 45
4.2.3	Coupling to Environment	..... 47
4.2.4	Density Operator	..... 48
4.2.5	Entanglement and Mixing	..... 49
4.2.6	Quantification of Entanglement	..... 50
4.2.7	Bloch Sphere	..... 52
4.2.8	EPR Correlations	..... 54
4.2.9	Bell's Theorem	..... 54
4.2.10	Violation of Bell's Inequality	..... 55
4.2.11	The No-Cloning Theorem	..... 56
4.3	Measurement Revisited	..... 58
4.3.1	Quantum Mechanical Projection Postulate	..... 58
4.3.2	The Copenhagen Interpretation	..... 60
4.3.3	Von Neumann's Model	..... 61
5	Quantum Bits and Quantum Gates	65
5.1	Single-Qubit Gates	..... 65
5.1.1	Introduction	..... 65
5.1.2	Rotations Around Coordinate Axes	..... 65

5.1.3	General Rotations .....	66
5.1.4	Composite Rotations .....	67
5.2	Two-Qubit Gates .....	68
5.2.1	Controlled Gates .....	68
5.2.2	Composite Gates .....	69
5.3	Universal Sets of Gates .....	71
5.3.1	Choice of Set .....	71
5.3.2	Unitary Operations .....	72
5.3.3	Two-Qubit Operations .....	72
5.3.4	Approximating Single-Qubit Gates .....	73
<b>6</b>	<b>Feynman's Contribution</b> .....	<b>77</b>
6.1	Simulating Physics with Computers .....	77
6.1.1	Discrete System Representations .....	77
6.1.2	Probabilistic Simulations .....	78
6.2	Quantum Mechanical Computers .....	79
6.2.1	Simple Gates .....	79
6.2.2	Adder Circuits .....	79
6.2.3	Qubit Raising and Lowering Operators .....	80
6.2.4	Adder Hamiltonian .....	82
<b>7</b>	<b>Errors and Decoherence</b> .....	<b>85</b>
7.1	Motivation .....	85
7.1.1	Sources of Error .....	85
7.1.2	A Counterstrategy .....	86
7.2	Decoherence .....	86
7.2.1	Phenomenology .....	86
7.2.2	Semiclassical Description .....	87
7.2.3	Quantum Mechanical Model .....	89
7.2.4	Entanglement and Mixing .....	90
7.3	Error Correction .....	92
7.3.1	Basics .....	92
7.3.2	Classical Error Correction .....	92
7.3.3	Quantum Error Correction .....	93
7.3.4	Single Spin-Flip Error .....	94
7.3.5	Continuous Phase Errors .....	95
7.3.6	General Single Qubit Errors .....	96
7.3.7	The Quantum Zeno Effect .....	97
7.3.8	Stabilizer Codes .....	100
7.3.9	Fault-Tolerant Computing .....	101
7.4	Avoiding Errors .....	102
7.4.1	Basics .....	102
7.4.2	Decoherence-Free Subspaces .....	103
7.4.3	NMR in Liquids .....	104
7.4.4	Scaling Considerations .....	106

<b>8</b>	<b>Tasks for Quantum Computers</b>	<b>109</b>
8.1	Quantum Versus Classical Algorithms .....	109
8.1.1	Why Quantum? .....	109
8.1.2	Classes of Quantum Algorithms .....	110
8.2	The Deutsch Algorithm: Looking at Both Sides of a Coin at the Same Time .....	111
8.2.1	Functions and Their Properties .....	111
8.2.2	Example: One-Qubit Functions .....	111
8.2.3	Evaluation .....	112
8.2.4	Many Qubits .....	113
8.2.5	Extensions and Generalizations .....	115
8.3	The Shor Algorithm: It's Prime Time .....	115
8.3.1	Some Number Theory .....	116
8.3.2	Factoring Strategy .....	117
8.3.3	The Core of Shor's Algorithm .....	118
8.3.4	The Quantum Fourier Transform .....	121
8.3.5	Gates for the QFT .....	124
8.4	The Grover Algorithm: Looking for a Needle in a Haystack .....	125
8.4.1	Oracle Functions .....	126
8.4.2	The Search Algorithm .....	127
8.4.3	Geometrical Analysis .....	128
8.4.4	Quantum Counting .....	130
8.4.5	Phase Estimation .....	130
8.5	Quantum Simulations .....	132
8.5.1	Potential and Limitations .....	132
8.5.2	Simulated Evolution .....	134
8.5.3	Implementations .....	135
<b>9</b>	<b>How to Build a Quantum Computer</b>	<b>137</b>
9.1	Components .....	137
9.1.1	The Network Model .....	137
9.1.2	Some Existing and Proposed Implementations .....	138
9.2	Requirements for Quantum Information Processing Hardware .....	139
9.2.1	Qubits .....	139
9.2.2	Initialization .....	140
9.2.3	Decoherence Time .....	140
9.2.4	Quantum Gates .....	141
9.2.5	Readout .....	142
9.3	Converting Quantum to Classical Information .....	143
9.3.1	Principle and Strategies .....	143
9.3.2	Example: Deutsch—Jozsa Algorithm .....	144
9.3.3	Effect of Correlations .....	145
9.3.4	Repeated Measurements .....	145
9.4	Alternatives to the Network Model .....	146
9.4.1	Linear Optics and Measurements .....	146

9.4.3 Quantum Cellular Automata .....	148
9.4.4 One-Way Quantum Computer .....	148
<b>10 Liquid State NMR Quantum Computer</b> .....	<b>151</b>
10.1 Basics of NMR .....	151
10.1.1 System and Interactions .....	151
10.1.2 Radio Frequency Field .....	153
10.1.3 Rotating Frame .....	154
10.1.4 Equation of Motion .....	155
10.1.5 Evolution .....	156
10.1.6 NMR Signals .....	157
10.1.7 Refocusing .....	158
10.2 NMR as a Molecular Quantum Computer .....	160
10.2.1 Spins as Qubits .....	160
10.2.2 Coupled Spin Systems .....	162
10.2.3 Pseudo/Effective Pure States .....	163
10.2.4 Single-Qubit Gates .....	164
10.2.5 Two-Qubit Gates .....	166
10.2.6 Readout .....	167
10.2.7 Readout in Multispin Systems .....	169
10.2.8 Quantum State Tomography .....	170
10.2.9 DiVincenzo's Criteria .....	172
10.3 NMR Implementation of Shor's Algorithm .....	173
10.3.1 Qubit Implementation .....	173
10.3.2 Initialization .....	174
10.3.3 Computation .....	174
10.3.4 Readout .....	176
10.3.5 Decoherence .....	177
<b>11 Trapped Ions and Atoms</b> .....	<b>179</b>
11.1 Trapping Ions .....	179
11.1.1 Ions, Traps, and Light .....	179
11.1.2 Linear Traps .....	180
11.2 Interaction with Light .....	181
11.2.1 Optical Transitions .....	181
11.2.2 Motional Effects .....	182
11.2.3 Basics of Laser Cooling .....	183
11.3 Quantum Information Processing with Trapped Ions .....	185
11.3.1 Qubits .....	185
11.3.2 Single-Qubit Gates .....	187
11.3.3 Two-Qubit Gates .....	188
11.3.4 Readout .....	189
11.4 Experimental Implementations .....	190
11.4.1 Systems .....	190
11.4.2 Some Results .....	191
11.4.3 Challenges .....	193

11.5 Neutral Atoms .....	194
11.5.1 Trapping Neutral Particles .....	194
11.5.2 Manipulating Neutral Particles .....	195
11.5.3 Gate Operations .....	196
11.6 Interacting Atoms in Optical Lattices .....	197
11.6.1 Interacting Particles in a Periodic Potential: The Hubbard Model .....	197
11.6.2 (Observing) The Mott—Hubbard Transition .....	201
11.6.3 Universal Optical Lattice Quantum Computing? .....	203
<b>12 Solid-State Quantum Computers .....</b>	<b>205</b>
12.1 Solid State NMR/EPR .....	205
12.1.1 Scaling Behavior of NMR Quantum Information Processors .....	205
12.1.2 3113 in Silicon .....	206
12.1.3 Other Proposals .....	208
12.1.4 Single-Spin Readout .....	209
12.2 Superconducting Systems .....	210
12.2.1 Charge Qubits .....	210
12.2.2 Flux Qubits .....	211
12.2.3 Gate Operations .....	212
12.2.4 Readout .....	213
12.3 Semiconductor Qubits .....	214
12.3.1 Materials .....	214
12.3.2 Excitons in Quantum Dots .....	216
12.3.3 Electron Spin Qubits .....	217
<b>13 Photons for Quantum Information .....</b>	<b>219</b>
13.1 "Quantum Only" Tasks .....	219
13.1.1 Quantum Teleportation .....	219
13.1.2 (Super-) Dense Coding .....	221
13.1.3 Quantum Key Distribution .....	222
13.2 A Few Bits of Classical Information Theory .....	225
13.2.1 Measuring Information .....	225
13.2.2 Information Content and Entropy .....	226
13.2.3 Mutual Information and the Data Processing Inequality .....	227
13.2.4 Data Compression and Shannon's Noiseless Channel Coding Theorem .....	228
13.2.5 The Binary Symmetric Channel and Shannon's Noisy Channel Coding Theorem .....	231
13.3 A Few Bits of Quantum Information Theory .....	231
13.3.1 The von Neumann Entropy .....	231
13.3.2 The Accessible Information and Holevo's Bound .....	234
13.3.3 Schumacher's Noiseless Channel Coding Theorem .....	235
13.3.4 Classical Information over Noisy Quantum Channels .....	236

<i>Contents</i>	<i>XI</i>
Appendix	
A Two Spins-1/2: Singlet and Triplet States	237
Bibliography	239
Index	261