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TRANSVERSE ENERGY DISTRIBUTION OF ASYMMETRIC OR MISMATCHED K-V BEAMS*

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Kapchinsky and Vladmirsky have defined a self-consistent particle distribution function, known as K-V distribution, which satisfies the time-independent Vlasov equation and has linear space-charge forces.[1] We examine the generalized form of the K-V distribution for an asymmetric system with different focusing forces and emittances in the two transverse directions. The transverse energy distribution of a K-V beam is a delta function when the beam is symmetric and matched with equal transverse emittances. We show that, the transverse energy distribution changes to a flat-top shape with a finite width when the two transverse focusing constants or emittances are different and the beam is matched in both directions. We have also determined the transverse energy distribution for a K-V beam undergoing small mismatch oscillations in the emittance dominated regime.

KEY WORDS: Beam transport, particle dynamics

1 INTRODUCTION

The well-known K-V distribution function[1] is a self-consistent solution of the time-independent Vlasov equation. It has the property that, the external forces due to applied fields and the self forces due to the space-charge and the beam current, are both linear functions of the transverse displacements of the particles. In this case, the transverse emittances ϵ_x and ϵ_y (or the normalized emittances when acceleration is involved) of a beam remain constant through a "smooth" focusing channel. The term "smooth" corresponds to slowly or adiabatically varying focusing strength. Due to this property the K-V distribution is a very useful design tool and can serve as an equivalent uniform beam for other, more realistic particle distributions having the same rms properties (beam width, emittance).[2–4]

Thus, it is important to know this distribution function for an ideal K-V beam under different conditions. For a symmetric matched K-V beam passing through a smooth focusing channel the transverse energy distribution is a delta function. However, in many practical cases, such as in synchrotrons or in sheet beam devices, this assumption is not valid as the focusing forces and/or the transverse emittances in the two directions may have different

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values. In this paper, we have determined the transverse energy distribution for such an asymmetric or mismatched K-V beam.

First, we redefine the K-V distribution in Section 2 in a more generalized form, which takes into account different values of transverse emittances ($\epsilon_x \neq \epsilon_y$) and the cross-terms xx' and yy' corresponding to a mismatched beam. In Section 3, we determine the transverse energy distribution for the following three cases: (1) symmetric, matched K-V beam through smooth focusing channel, (2) asymmetric, matched K-V beam through smooth focusing channel, and (3) K-V beam undergoing small mismatch oscillations for an emittance dominated case. Section 4 contains the concluding remarks.

2 FORMULATION OF GENERALIZED K-V DISTRIBUTION

We start with the equations of particle trajectories in a channel where the external forces are linear functions of x and y, the transverse displacements from the beam axis. For the K-V distribution the electric and magnetic self forces are also linear functions of the transverse displacements. In order to have these linear dependences the condition for paraxial motion, i.e. $v_x, v_y, \ll v_z$, must be satisfied. The particle trajectories obey the following second order linear differential equations, known as Mathieu-Hill equations:

$$x'' + \kappa_x(z)x = 0, \tag{1a}$$

$$y'' + \kappa_y(z)y = 0. \tag{1b}$$

The coefficients κ_x and κ_y include both external focusing and self forces and are assumed to be slow functions of the axial distance z. Following Courant and Snyder,[5] an invariant of motion A_x^2 in the XX' plane can be written as,

$$\frac{x^2}{w_x^2} + (w_x x' - w_x' x)^2 = A_x^2 , \tag{2}$$

where $w_x(z)$ is the amplitude of the Floquet function which is the eigenfunction of the Mathieu-Hill equation and satisfies the following differential equation:

$$w_x'' + \kappa_x w_x - \frac{1}{w_x^3} = 0. \tag{3}$$

The parameter A_x, remains constant through the channel. Equation 2 can be written as,

$$\hat{\gamma}_x x^2 + 2\hat{\alpha}_x x x' + \hat{\beta}_x x'^2 = A_x^2 . \tag{4}$$

where the Courant-Snyder parameters $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{\gamma}_x$ satisfy the relations $\hat{\alpha}_x = -w_x w_x'$, $\hat{\beta}_x = w_x^2$, $\hat{\gamma}_x = 1/w_x^2 + w_x'^2$, and hence $\hat{\beta}_x \hat{\gamma}_x - \hat{\alpha}_x^2 = 1$. The notation ($\wedge$) is introduced to differentiate these parameters from the common use of γ as a relativistic factor and β as a velocity normalized to the speed of light. An invariant of motion in the YY' plane is obtained in a similar way, which is given by,

$$\hat{\gamma}_y y^2 + 2\hat{\alpha}_y yy' + \hat{\beta}_y y'^2 = A_y^2 \,. \tag{5}$$

For any particle the values of A_x and A_y remain constant through the channel, but for different particles these values differ based upon their initial conditions.

The original K-V distribution function f is defined by,[1]

$$f = f_o \delta(F - F_o) \,, \tag{6}$$

where f_o and F_o are constants and δ represents Dirac-delta function. For a general case, F can be written as,

$$F = A_x^2 + sA_y^2 \,. \tag{7}$$

s takes the value of unity under the assumption that $\epsilon_y = \epsilon_y = \epsilon$. In Ref. 1 all the properties of the K-V distribution are determined for $s = 1$.

First, we define the generalized K-V distribution by treating the transverse emittances ϵ_x and ϵ_y independently as follows:

$$f = f_m \delta(G - 1) \,, \tag{8}$$

where f_m is a constant and G is a normalized function given by,

$$G = \frac{A_x^2}{\epsilon_x} + \frac{A_y^2}{\epsilon_y} = \frac{(\hat{\gamma}_x x^2 + 2\hat{\alpha}_x xx' + \hat{\beta}_x x'^2)}{\epsilon_x} + \frac{(\hat{\gamma}_y y^2 + 2\hat{\alpha}_y yy' + \hat{\beta}_y y'^2)}{\epsilon_y} \,. \tag{9}$$

This definition of the generalized distribution function follows from the earlier definitions of K-V distribution in the literature.[1,6] [1] This definition is equivalent to the original distribution (Eq. (6) and (7)), if we assign $F_o = \epsilon_x$ and $s = \epsilon_x/\epsilon_y$.

First, we will recap some of the important properties of this distribution function. The reader is referred to Ref. 12 for detailed derivation. For this distribution, the representation points of all particles in the beam lie on the surface of the hyperellipsoid in the four-dimensional phase space $XYX'Y'$ given by,

$$\frac{A_x^2}{\epsilon_x} + \frac{A_y^2}{\epsilon_y} = 1. \tag{10}$$

The projection of this hyperellipsoid in any two-dimensional plane is a uniform density ellipse. The projections in the XX' plane and YY' plane are ellipses given by $A_{x_{\max}}^2 = \epsilon_x$ and $A_{y_{\max}}^2 = \epsilon_y$, with the phase-space areas of $\pi\epsilon_x$ and $\pi\epsilon_y$, respectively. The projection in the XY plane is an upright ellipse with semi-axes $x_m = \sqrt{\hat{\beta}_x \epsilon_x}$ and $y_m = \sqrt{\hat{\beta}_y \epsilon_y}$ and having a cross-sectional area of $A_0 = \pi x_m y_m$. The space-charge density ρ_o of the beam can be written in terms of the beam current I and the axial velocity of the particles v_z as,

$$\rho_o = \frac{I}{A_0 v_z} = \frac{I}{\pi v_z \sqrt{\hat{\beta}_x \hat{\beta}_y \epsilon_x \epsilon_y}} \tag{11}$$

The space-charge potential Φ_s, can be written as,

$$\Phi_s = -\frac{\rho_o}{4\varepsilon_0}\left[x^2 + y^2 - \frac{x_m - y_m}{x_m + y_m}(x^2 - y^2)\right] \tag{12}$$

This implies that, the electrostatic and magnetic self-forces for this distribution are linear functions of the transverse displacements x and y. Thus, the generalized distribution satisfies the basic premise of the K-V distribution.

3 TRANSVERSE ENERGY DISTRIBUTION

To obtain the corresponding distribution of transverse energy $H_\perp$ we must evaluate the following integral over the 4-dimensional phase-space volume,

$$f(H_\perp) = f_m \iiiint_V \delta(G-1)\delta(H_\perp - \kappa_x x^2 - x'^2 - \kappa_y y^2 - y'^2)dv\ . \tag{13}$$

We have analyzed this distribution under different conditions.

The functional dependence of A_x^2 and A_y^2 (in Eq. (4) and (5)) can be further simplified for a matched beam. First, we will determine the values of the Courant-Snyder parameters for a perfectly matched K-V beam through a smooth focusing channel. For a K-V beam matched in the XX' plane, the amplitude function w_x is constant, i.e. $w_x'' = w_x' = 0$. Hence, $\hat{\alpha}_x = -w_x w_x' = 0$ and $\hat{\gamma}_x = 1/w_x^2 = 1/\hat{\beta}_x$. Equation (4) can now be written as,

$$A_x^2 = \frac{1}{\hat{\beta}_x}x^2 + \hat{\beta}_x x'^2\ . \tag{14}$$

Notice that this is an upright ellipse in the XX' plane with no xx' cross-terms. Also, from Eq. (3) we get,

$$\kappa_x = \frac{1}{w_x^4} = \frac{1}{\hat{\beta}_x^2}\ . \tag{15}$$

For a matched beam through a constant focusing channel the Hamiltonian $H_x = \kappa_x x^2 + x'^2$ is a constant of motion for each particle. As stated before, κ_x includes both the external focusing and the space-charge forces. For a matched beam through a channel with slowly varying focusing fields the Hamiltonian is not a conserved quantity. However, a distribution function of transverse energies can be determined at a given cross-section of a beam.

Now, we analyze three different situations. The first two cases are for the matched K-V beams and the Eqs. (14) and (15) can be applied to them. Whereas, the third case corresponds to mismatched oscillations and $\hat{\alpha} \neq 0$. We have analyzed this case under a simplifying assumption of small mismatch oscillations in the emittance-dominated regime.

3.1 Symmetric, Matched K-V Beam in a Smooth Focusing Channel

In the symmetric case, we have $\epsilon_x = \epsilon_y = \epsilon$ and $\hat{\beta}_x = \hat{\beta}_y = \hat{\beta}$. Thus,

$$G = \frac{A_x^2}{\epsilon_x} + \frac{A_y^2}{\epsilon_y} = \frac{1}{\epsilon}\left[\left(\frac{1}{\hat{\beta}}x^2 + \hat{\beta}x'^2\right) + \left(\frac{1}{\hat{\beta}}y^2 + \hat{\beta}y'^2\right)\right]$$
$$= \frac{1}{\kappa x_m^2}\left[\kappa(x^2 + y^2) + x'^2 + y'^2\right] = \frac{H_\perp}{H_{\perp 0}}, \quad (16)$$

where $x_m = y_m = \sqrt{\beta\epsilon}$ is the beam radius, $H_\perp$ is the transverse Hamiltonian of a particle, and $H_{\perp o} = \kappa x_m^2$. Thus, the distribution function can be described as a delta function in the transverse energy as,

$$f = f_H \delta(H_\perp - H_{\perp 0}) \quad (17)$$

where $f_H = \kappa x_m^2 f_m$.

3.2 Asymmetric, Matched K-V Beam in a Smooth Focusing Channel

In this section, we will consider a K-V beam matched in both the XX' and the YY' planes but having different transverse emittances $\epsilon_x \neq \epsilon_y$ or different focusing strengths $\kappa_x \neq \kappa_y$ (and hence, $\hat{\beta}_x \neq \hat{\beta}_y$) or both. The normalized function G can then be written as,

$$G = \frac{1}{\epsilon_x}\left(\frac{1}{\hat{\beta}_x}x^2 + \hat{\beta}_x x'^2\right) + \frac{1}{\epsilon_y}\left(\frac{1}{\hat{\beta}_y}y^2 + \hat{\beta}_y y'^2\right)$$
$$= \frac{\kappa_x x^2 + x'^2}{\kappa_x x_m^2} + \frac{\kappa_y y^2 + y'^2}{\kappa_y y_m^2}$$
$$= \frac{H_x}{H_{x0}} + \frac{H_y}{H_{y0}}, \quad (18)$$

where H_{x0} and H_{y0} are the maximum possible values of the Hamiltonians H_x and H_y, respectively. From Eqs. (9) and (18), we can write the net transverse Hamiltonian $H_\perp$ as,

$$H_\perp = H_x + H_y = R_g H_{x0} + (1 - R_g)H_{y0}, \quad (19)$$

where $R_g = H_x/H_{x0}$ is uniformly distributed from zero to unity. This implies that, $H_\perp$ is uniformly distributed from H_{x0} to H_{y0} and we have a flat-top distribution as shown in Figure 1. This can be mathematically expressed as,

$$f = f_m \frac{|u(H_\perp - H_{x0}) - u(H_\perp - H_{y0})|}{|H_{x0} - H_{y0}|}, \quad (20)$$

where u represents a unit-step function.

Notice that, this function reduces to a delta function in the transverse Hamiltonian when $H_{x0} = H_{y0}$, i.e. $\epsilon_x\sqrt{\kappa_x} = \epsilon_y\sqrt{\kappa_y}$. Thus, even when the beam is not symmetric we can still

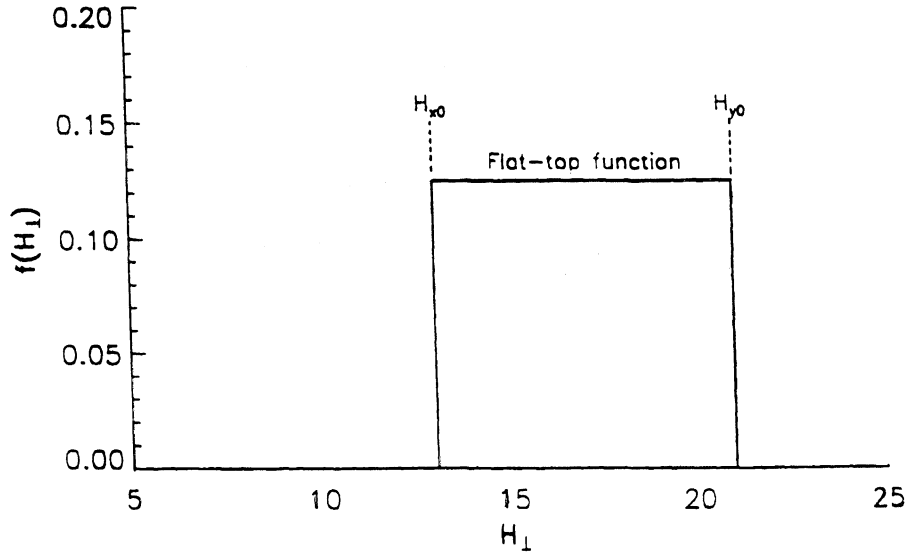

FIGURE 1: Variation of particle density f versus the net transverse energy $H_{\perp}$ for a matched, asymmetric K-V beam through a uniform focusing channel for H_{x0}=13 and H_{y0}=21 in arbitrary energy units.

get a delta function in the transverse Hamiltonian if the beam parameters satisfy the equality $\epsilon_x/x_m = \epsilon_y/y_m$, which can also be expressed as, $\epsilon_x\sqrt{\kappa_x} = \epsilon_y\sqrt{\kappa_y}$.

3.3 K-V Beam Undergoing Small Mismatch Oscillations

In Sections 3.1 and 3.2, we considered the cases in which the beam was perfectly matched. When the beam is not matched, on the other hand, it undergoes oscillations in beam radius. The single-particle Hamiltonian, in general, is not a constant due to coupling between the transverse and longitudinal motion. Here, we will analyze the transverse energy distribution for a K-V beam undergoing small mismatch oscillations through a smooth focusing channel.

First, we will analyze a beam having small mismatch in the X-direction and then we will extend the analysis to two dimensions. We assume that the beam parameters are sufficiently away from the space-charge dominated regime, i.e. we will analyze the emittance dominated case. This implies that the variation in space-charge contribution to κ_x due to small mismatch oscillations is a second order term and can be neglected. Under these assumptions, the x-component of the Hamiltonin H_x remains constant if the external focusing force is constant.

The variation of beam width x_m along the channel length is shown in Figure 2(a). We consider cross-sections in the transverse plane at the maximum, minimum, and average values of x_m. For the sections at the maximum and minimum values of x_m, we have $w_x' = 0$ and $w_x'' \neq 0$. These sections are represented by points A and C, respectively, in Figure 2(a). Thus, for both sections we have $\hat{\gamma}_x = 1/\hat{\beta}_x = 1/w_x^2$ and $\hat{\alpha}_x = 0$. The corresponding projections of the distribution function in the XX' plane are upright ellipses as shown in

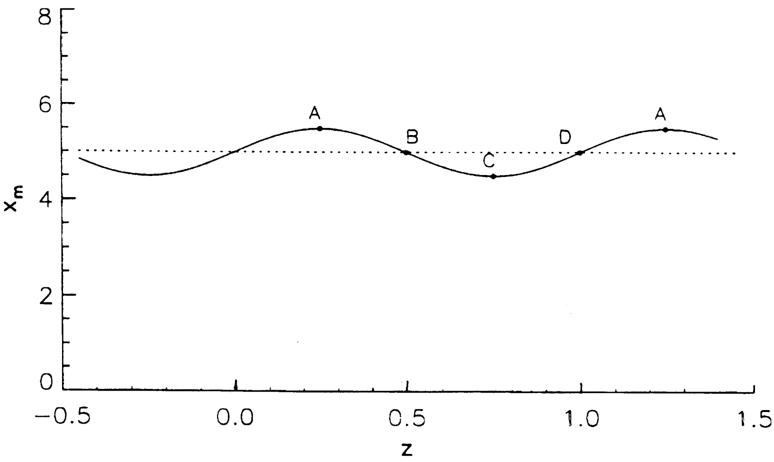

FIGURE 2a:

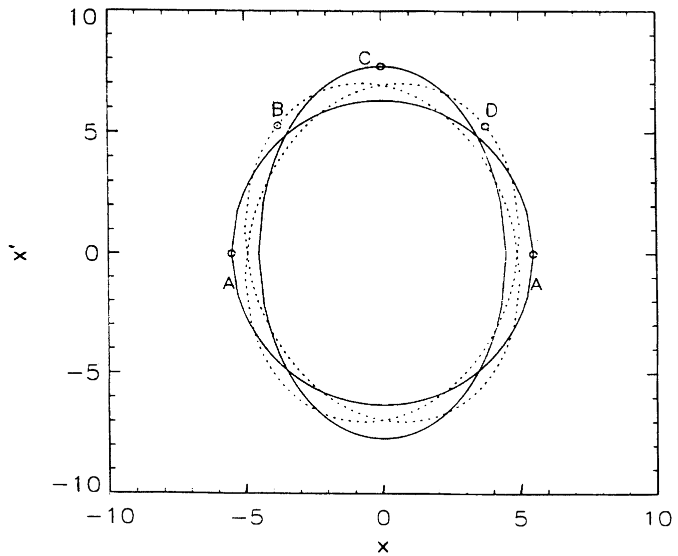

FIGURE 2b:

FIGURE 2: (a) Variation of beam width along the x-direction x_m, versus the axial distance z showing mismatch oscillations or $\Delta\kappa_x/\kappa_x$=0.4. The figure shows the positions of A, B, C, D, and A cross-sections. A and C sections are at the maximum and the minimum of x_m and the sections B and D are at its average value.
(b) Emittance ellipses in the XX' phase space at A, B, C, D, and A cross-sections. The sections A and C have upright ellipses and are shown by the solid curves. Whereas, the sections B and D have tilted ellipses and are shown by the dotted curves. A particle at the extreme location at the section A is traced by an open circle at each section in the XX' phase space.

Fig. 2(b). From Eq. (3) we can also get,

$$w_x'' = -\sqrt{\hat{\beta}_x}\left(\kappa_x - \frac{1}{\hat{\beta}_x^2}\right) = -\sqrt{\hat{\beta}_x}(\Delta\kappa_x) \, . \tag{21}$$

By small mismatch, we have assumed that $\Delta\kappa_x = (\kappa_x - 1/\hat{\beta}_x^2) \ll \kappa_x$. The beam-width x_m at the two sections (A and C) is given by,

$$x_{m_{A,C}} = \sqrt{\hat{\beta}_x\epsilon_x} = \left(1 + \frac{\Delta\kappa_x}{4\kappa_x}\right)\sqrt{\frac{\epsilon_x}{\kappa_x^{1/2}}} \, . \tag{22}$$

For sections A and C the magnitude of $\Delta\kappa_x$ is the same, but there is a difference in sign. Depending upon the sign of $\Delta\kappa_x$, the value of x_m would either be at the maximum or minimum. The variation Δx_m in the beam width can be written as,

$$\frac{\Delta x_m}{x_{m0}} = \frac{1}{2}\frac{\Delta\kappa_x}{\kappa_x} \, , \tag{23}$$

where, $x_{m0} = \sqrt{\epsilon_x/\kappa_x^{1/2}}$ is the average beam width.

For the sections A and C we also have,

$$\begin{aligned} R_g &= \frac{A_x^2}{\epsilon_x} = \frac{\hat{\beta}_x}{\epsilon_x}\left(\frac{1}{\hat{\beta}_x^2}x^2 + x'^2\right) \\ &= \frac{1}{\left(1 - \frac{\Delta\kappa_x}{2\kappa_x}\right)\kappa_x x_m^2}\left[(\kappa_x - \Delta\kappa_x)x^2 + x'^2\right] \, . \end{aligned} \tag{24}$$

We can now substitute, $H_x = \kappa_x x^2 + x'^2$ and $H_{x0} = \kappa_x x_m^2$ in Eq. (24) to get H_x as a function of a position x as follows,

$$H_x(x) = R_g H_{x0}\left(1 - \frac{\Delta\kappa_x}{2\kappa_x}\right) + \Delta\kappa_x x^2 \, , \tag{25}$$

where, x^2 varies from zero to $x_1^2 = R_g x_m^2$. Now, we define $\Delta H_x = H_x - R_g H_{x0}$. Then,

$$\Delta H_x(x) = \Delta\kappa_x x_1^2\left(\frac{x^2}{x_1^2} - \frac{1}{2}\right) \, . \tag{26}$$

For any value of R_g (between 0 and 1) the particles lie uniformly along the circumference of an upright ellipse in the XX' plane. By taking the projection on the X-axis, we get the particle density $\rho_x(x)$ to be proportional to $\sqrt{(x_1^2 - x^2)}$. Thus, the energy distribution of the particles for a fixed value of R_g can be written as,

$$\rho_H(H_x) = \frac{\rho_x(x)dx}{dH_x} = \rho_{H0}\frac{x}{x_1}\sqrt{1-\frac{x^2}{x_1^2}}$$

$$= \rho_{H0}\sqrt{-\left(\frac{\Delta H_x}{\Delta\kappa_x x_1^2}+\frac{1}{2}\right)\left(\frac{\Delta H_x}{\Delta\kappa_x x_1^2}-\frac{1}{2}\right)} \qquad (27)$$

The above equation implies that, the energy distribution is independent of the sign of $\Delta\kappa_x$ and hence, it is the same at sections A and C. This is consistent with the fact that under small space-charge conditions H_x is only a function of mismatch parameter and matched focusing parameters and independent of maxima or minima in beam radius or any axial location in between. For a constant focusing channel the energy distribution remains unchanged through the channel. The particle number density as a function of H_x is plotted in Figure 3. It has a dome-shaped distribution centered at $R_g H_{x0}$ and has width proportional to mismatch coefficient $\Delta\kappa_x$.

Although, the energy distribution at sections B and D is the same as shown in Figure 3, the corresponding ellipses in the XX' plane are tilted (Figure 2(b)). The beam width x_m is at its average value x_{m0}. Also, $w_x' \neq 0$ and $w_x'' = 0$, therefore $\hat{\alpha} \neq 0$ and $\hat{\beta}_x^2 = 1/\kappa_x$. The positions of a particle with the highest transverse energy $H_x = R_g H_{x0} + \frac{1}{2}\Delta\kappa_x x_1^2$ in the XX' phase space are shown in Figure 2(b) at the four cross-sections, A, B, C, and D. The particle position undergoes half the rotation in one complete cycle of the envelope oscillation. The beam envelope oscillates with a frequency that is twice as fast as the single-particle oscillation frequency, which is consistent with the earlier analysis presented in Ref. 13.

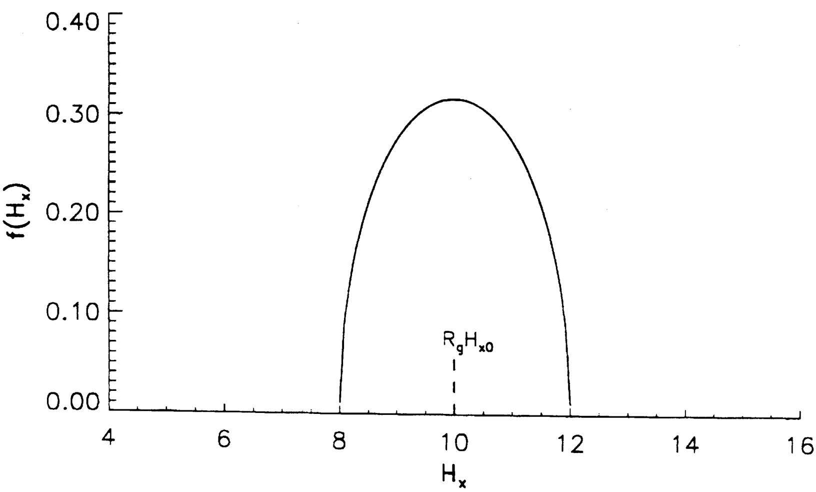

FIGURE 3: Variation of particle density f at a fixed value of constant R_g versus the x-component of the Hamiltonian H_x for $\Delta\kappa_x/\kappa_x$=0.4.

In order to get the net transverse energy $H_\perp$, we need to add the y-component of the Hamiltonian H_y. In a matched beam (zero-th order approximation) H_y is given by,

$$H_y = (1 - R_g)H_{y0} = (1 - R_g)\kappa_y y_m^2 \ . \tag{28}$$

To get the particle distribution as a function of $H_\perp$, we have to combine Eq. (27) with the equivalent function for H_y and then integrate with respect to R_g going from zero to unity. The results for four possible scenarios are numerically evaluated and presented in Fig. 4, viz., (a) a symmetric beam with small radial mismatch oscillations, (b) a zero-th order symmetric beam with small mismatch in x direction, (c) an asymmetric beam with small mismatch oscillations in both x and y directions, and (d) an asymmetric beam with small mismatch only in x direction. Note that, for symmetric radial oscillations the transverse energy distribution (Fig. 4(a)) is somewhat similar to that for a single component, with a fixed R_g shown in Fig. 3 and the width of the distribution spectrum is proportional to $\Delta\kappa_x = \Delta\kappa_y = \Delta\kappa$. Figs. 4(a) and 4(b) reduce to a delta function, given by Eq. (13), as the mismatch parameter tends to zero. Correspondingly, Figs. 4(c) and 4(d) reduce to a flat-top distribution given by Eq. (20).

This analysis can be extended to include the space-charge effects on the beam mismatch oscillations. In this case, the space-charge contribution to κ_x varies along the axial distance as a function of the beam width x_m. Even for a constant external focusing H_x is no longer a constant of motion for each particle. Some qualitative remarks can be made about he transverse energy distribution under these conditions. Although the net transverse energy distribution repeats itself after every beam oscillation the individual particles do not have the same energies and there is a continuous regrouping of the transverse energy between the particles.

4 CONCLUSIONS

Starting from a generalized form of the K-V distribution taking into account different values of the focusing coefficients κ_x and κ_y and transverse emittances, ϵ_x and ϵ_y and small beam mismatches, we have determined the transverse energy distribution of the particles. We have shown that the transverse energy distribution is a delta function for a symmetric, matched beam and a flat-top function for an asymmetric, matched beam through a uniform focusing channel. We have presented a detailed analysis of a beam undergoing small mismatch oscillations in the emittance-dominated regime. A symmetric beam undergoing radial mismatch oscillations has a dome-shaped transverse energy distribution, having a spectral width proportional to the mismatch parameter $\Delta\kappa$. An asymmetric beam undergoing mismatch oscillations has a trapezoidal transverse energy distribution.

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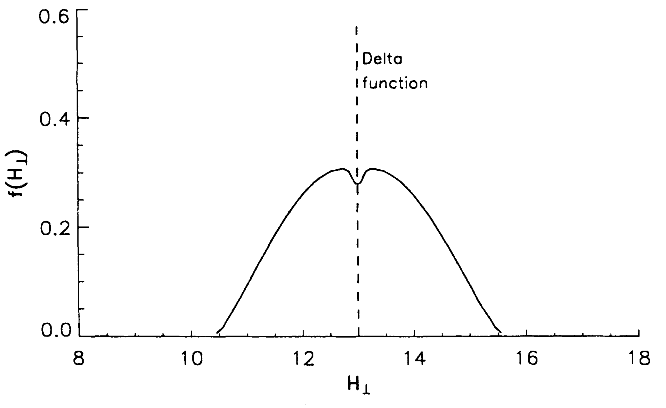

FIGURE 4a:

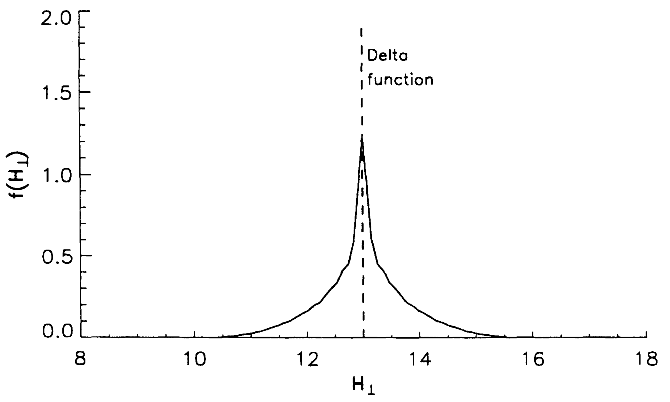

FIGURE 4b:

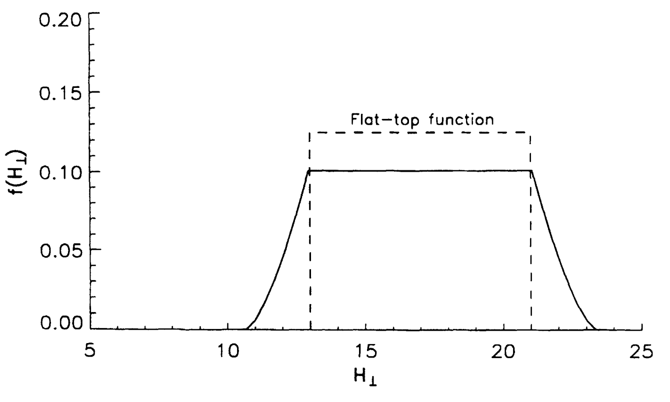

FIGURE 4c:

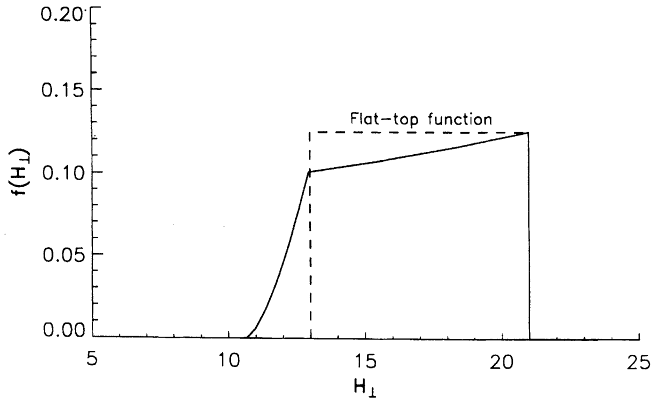

FIGURE 4d:

FIGURE 4: Variation of particle density f versus the net transverse energy $H_\perp$ for the following situations: (a) a symmetric K-V beam with the radial mismatch oscillations, $H_{x0}=H_{y0}=13$ and $\Delta\kappa_x/\kappa_x=\Delta\kappa_y/\kappa_y=0.4$, (b) a zero-th order symmetric K-V beam with the mismatch oscillations only in the x direction, $H_{x0}=H_{y0}=13$, $\Delta\kappa_x/\kappa_x=0.4$, and $\Delta\kappa_y//\kappa_y=0$, (c) an asymmetric K-V beam with the mismatch oscillations in both x and y directions, $H_{x0}=13$, $H_{y0}=21$, and $\Delta\kappa_x=\Delta\kappa_y=0.4\ \kappa_x$, and (d) an asymmetric K-V beam with the mismatch oscilations only in the x direction, $H_{x0}=13$, $H_{y0}=21$, $\Delta\kappa_x/\kappa_x=0.4$, and $\Delta\kappa_y/\kappa_y=0$.

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