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# EVANESCENT WAVE ACCELERATION OF ELECTRONS FROM 0.2c TO 0.95c

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Accelerators for electrons with velocities between 0.2c and 0.95c will be presented. The accelerators use evanescent waves as the accelerating electric field. The source for the evanescent waves will be electromagnetic waves (1 m and 10.5 cm wavelengths) incident at the interface of two materials with different indices of refraction. Continuous phase matching between the electrons and the evanescent wave is possible. Alternatively a non-phase matching accelerator is possible and experimentally easier to build. Accelerator lengths range from 1 to 5 meters for fields at 0.56 MV/m and from 0.2 to 1 meter for fields at 5.6 MV/m.

KEY WORDS: Laser-beam accelerators, wake-field accelerators

## 1 INTRODUCTION

Evanescent wave acceleration of electrons will be shown possible for electrons with velocities in the 0.2c to 0.95c range. Similar accelerators have been proposed for ultrarelativistic electrons.[1,2] The proposed accelerator belongs to the radio-frequency type of linear accelerators, is simple in concept, and needs common materials for construction. Unlike the normal radio-frequency accelerators where the electromagnetic fields and electrons are in the same cavity, the evanescent wave accelerator 'separates' the electromagnetic fields and electrons into two cavities or regions. The accelerating potential in the electron region is from an evanescent wave whose source is an electromagnetic wave in the other region.

The paper is divided into several sections with the first section giving an overview of evanescent waves. This will be followed by a description of electron motion and phase matching. Calculations will be used to show that the concept is valid and workable. The wavelengths considered for the evanescent wave source are 1 meter (0.3 GHz) and 10.5 cm (2.856 GHz). Next, Cherenkov radiation will be briefly discussed. Another technique for producing evanescent waves will then be discussed followed by a conclusion.

## 2 EVANESCENT WAVES

Evanescent waves are known, for example, in visible optics from total internal reflection.[3] Total internal reflection can occur when light in an optical medium with an index of refraction

$n_1$ is incident at the interface to a medium with index of refraction $n_2$, where $n_2 < n_1$. Snell's law, Equation (1), indicates that beyond some critical angle no light passes from the dense medium into the less dense medium. At the critical angle, Equation (2), the light refracted from the dense medium into the less dense medium is traveling parallel to the interface. For angles of incidence greater than the critical angle, all the light is reflected at the interface back into the dense medium. The larger the difference between the indices of refraction the smaller the critical angle.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

$$\theta_{\text{critical}} = \arcsin\frac{n_2}{n_1} \tag{2}$$

Electromagnetic theory boundary conditions require electric fields in all the media at an interface, even for angles of incidence greater than the critical angle. For angles of incidence larger than the critical angle, the fields are evanescent in the less dense medium. If these fields do not couple to another excitation their existence is of no consequence. If these fields do couple to another excitation then the reflected light would show a minimum when coupling occurs.[4]

Evanescent waves decay exponentially away from the interface over several wavelengths. The decay length $\alpha$, Equation (3), depends upon the angle of incidence, $\theta$.[3] $A_0$ is the initial amplitude and $z$ is the distance from the interface for calculating the evanescent wave amplitude for a distance and angle of incidence, Figure 1.

$$\alpha = k_0\left(\frac{n_1^2\sin^2\theta}{n_2^2} - 1\right)^{1/2}, \quad A = A_0 e^{-\alpha z} \tag{3}$$

Polarization of the source determines whether there is a longitudinal component to the evanescent wave.[5] Only TM polarization (P-polarization) has an electric field in the plane of incidence and can have components both parallel and normal to the interface between the media. (The plane of incidence is defined by the surface normal and the wavevector of the incident wave.) The evanescent wave has both longitudinal and transverse components. A ratio between the transverse and longitudinal components shows that the transverse part is larger, Figure 2. Moreover, the components are 90 degrees out of phase.

There is 'gain' in the evanescent wave electric field amplitude as compared to the incident wave amplitude, Figure 3. This can be used to reduce the magnitude of the electric field needed to produce a certain amount of acceleration. In effect, the acceleration is actually determined by the electric field incident times the 'gain'. One further note is that the shape of the electric field amplitudes varies from the expected $\cos^2\theta$ shape since the fields are evanescent.

Generally, the wavevector in a medium, $K$, is the vacuum wavevector times the index of refraction. For an evanescent wave, the wavevector is the vacuum wavevector, $k_0$, times the index of refraction of the denser medium. The component of the evanescent wavevector parallel to the interface is the evanescent wavevector times the sine of the incidence angle. The evanescent wavevector will be larger than the vacuum wavevector.

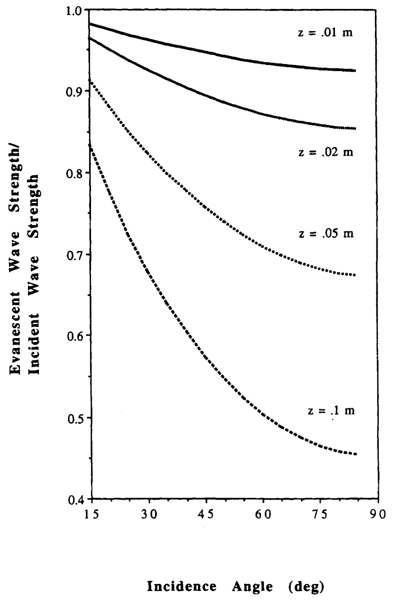


FIGURE 1: Attenuation of the evanescent wave versus angle of incidence for various distances away from the interface. The indices of refraction were (water) 8 and (vacuum) 1. The calculation is for a 1 meter wavelength so by dividing $z$ by one meter one can obtain attenuation for a normalized length and apply the result to other wavelengths.

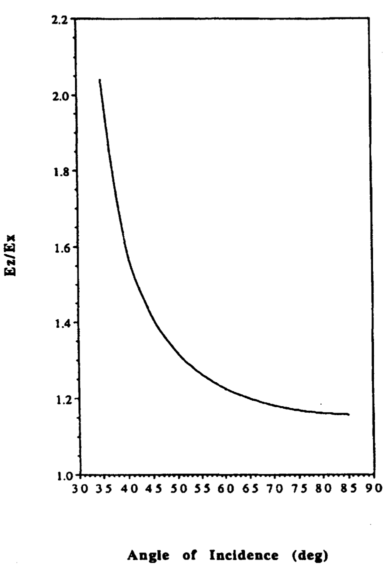


FIGURE 2: Ratio of transverse ($E_z$) to longitudinal ($E_x$) electric field strengths is shown for n=2 and varied angle of incidence. Though not shown, the fields peak at the critical angle, 30 deg.

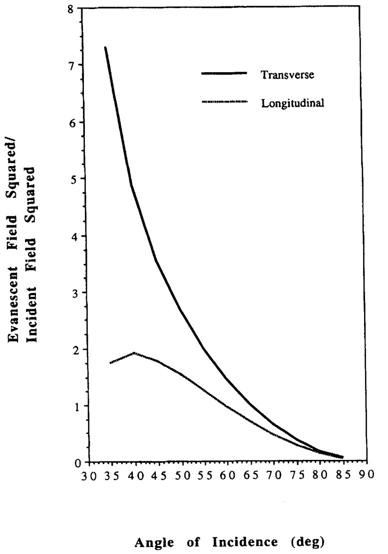


FIGURE 3: Transverse ($E_z$) and longitudinal ($E_x$) fields squared compared to the incident ($E_i$) field squared. Again the index of refraction is 2 and angle of incidence varies. For angles of incidence with electric field ratios greater than one, the evanescent wave has 'gain' as compared to the incident wave.

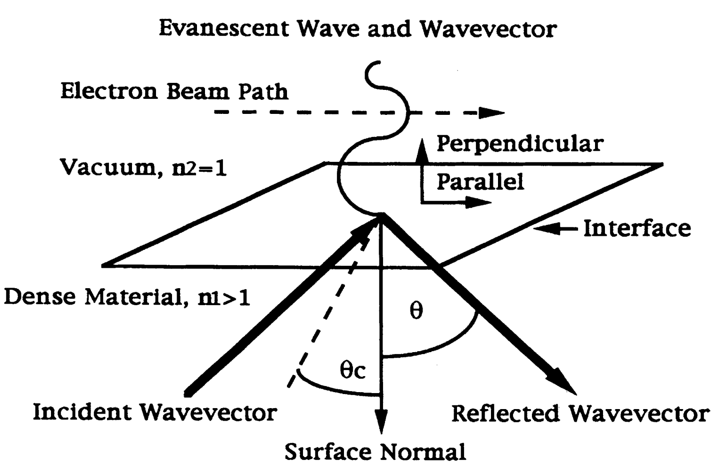


FIGURE 4: Shown are the incident and reflected electromagnetic wavevectors, the evanescent wavevector components, and the electron beam path. The incident and reflected electromagnetic fields are transverse to the respective wavevectors. The interface between the media is the origin of the evanescent wave. The exponential decay of the evanescent wave with distance from the interface is represented by the decreasing radius of the oscillatory pattern above the point of incidence. Not shown is the longitudinal evanescent wave field parallel to the evanescent wavevector component parallel to the interface. When the angle of incidence, $\theta$, is greater than the critical angle, $\theta_c$, evanescent waves will exist in the vacuum.

Finally, though the description given of evanescent waves is taken from visible optics the concept is valid for all electromagnetic wave frequencies. The effects existence is dependent upon a difference in material properties for media at an interface. Figure 4 shows a representation of a basic configuration for accelerating electrons with evanescent waves.

## 3 EQUATIONS OF MOTION AND PHASE MATCHING

The laboratory rest frame will be used throughout for describing the motion of the electrons. Velocity after acceleration is simply related to the acceleration and initial velocity, Equation (4). Distance travelled during acceleration is $d$, Equation (5).

$$v_{\text{final}} = v_{\text{initial}} + at \tag{4}$$

$$d = v_{\text{initial}}t + \frac{1}{2}at^2 \tag{5}$$

Acceleration is determined by the electric field amplitude and the mass is corrected for relativistic effects, Equation (6). Electric charge and electron mass are represented by $q$ and $m$ respectively. The ratio of the electron velocity to the speed of light is $\beta$. For a positive acceleration, the electric field must be negative; $(-q)(-E) = qE$.

$$a = \frac{qE}{m}\left(1-\beta^2\right)^{1/2}, \beta = \frac{v}{c} \tag{6}$$

Phase matching between the electrons and evanescent wave is possible. The phase matching equation is Equation (7).[1] The vacuum wavelength is $\lambda_0$. A specific index of refraction for a specific electron velocity can be found for phase matching to occur. The sine term is for the wavevector component parallel to the interface with $\theta$ being the angle of incidence which can be selected. In a system trying to increase the electron velocity, the phase of the electrons and wave are soon out of synch if the incidence angle and index of refraction remain fixed.

$$K\beta = k_0 = \frac{2\pi}{\lambda_0}, \; K = nk_0 \sin\theta \tag{7}$$

There are several ways to change the wavevector as the velocity increases to maintain phase matching. One way to retain phase matching would be to continuously change the wavevector, $K$, by decreasing the index of refraction as the electron velocity increases. Alternatively, for a fixed index of refraction, the incidence angle could be varied continuously as velocity increased. The incidence angle must remain greater than the critical angle otherwise evanescent waves do not exist. If there is phase matching then it is possible to ignore the transverse electric field component in some situations. By having the electron ride the crest of the longitudinal oscillation the transverse component will be zero since it is out of phase with the longitudinal component by 90 degrees.

Achieving a continuously varied angle of incidence is straightforward. The high index material can be thought of as an optical lens. A flat face of the lens is toward the beam line.

All the other faces of the lens may be any shape, and thereby one can fashion the shape of a face to be a diverging lens face. The diverging face will cause the incident electric field to reach the flat face with a continuously varied angle of incidence.

More realistic is to not have phase matching but to have acceleration when the evanescent field oscillation amplitude is negative. Since the electromagnetic wave cannot be rectified, a method has to be found to block the evanescent wave from coupling to the electrons when the wave would act as a decelerating force. The simplest method would be to have drift sections in which the electrons are shielded from the evanescent wave. This is analogous to drift tubes employed in drift linacs.[6]

For a drift tube evanescent wave linac, acceleration is modified to be dependent upon position, time, and phase, Equation (8). The variables $t$ and $x$ are total elapsed time (sum over time increments) and total distance (sum over $d$) the electrons have traveled. Initial phase between the electrons and the sine wave is $\delta$. Initial phase is where in the sine wave the electrons enter the accelerating field. Ideally the electrons enter at the leading edge of the accelerating half of the sine wave. Vacuum wavevector and angular frequency are $k_0$ and $\omega$ respectively.

$$a = \frac{qE}{m}\left(1-\beta^2\right)^{1/2} \sin(nk_0 \sin(\theta)x - \omega t + \delta) \tag{8}$$

Since the electron will not travel at the speed of the evanescent wave, the electron will experience acceleration for half the sine wave and deceleration for the other half of the sine wave. The decelerating part of the sine wave needs to blocked. A drift tube length can be determined by Equation (9) for isolating the electrons from the evanescent field. The electron velocity when deceleration begins is $v_e$.

$$\text{drift} = v_e \left(\frac{\lambda_0}{c - n\,\sin(\theta) v_e}\right) \tag{9}$$

There is real transverse acceleration in the non-phase matching case. However, since the two electric field components are out of phase by 90 degrees it is possible to nullify the transverse acceleration. The electron is subject to acceleration by the longitudinal component for approximately half the sine wave. The transverse component will accelerate the electrons first in one direction and then in the opposite direction for this portion of the sine wave. If the electron is accelerated in the longitudinal direction for a portion of the sine wave symmetric about the maximum acceleration, the transverse acceleration will net no deflection transverse to the direction of travel. This presumes that the distance traveled transversely is small compared to the exponential decay of the evanescent wave amplitude with distance from the interface.

An alternative method to cancel transverse acceleration would be to use opposing evanescent wave fields. If the two evanescent waves were opposed to one another and orthogonal to the beam line, then with careful control of the electric field polarization rotation the opposing evanescent transverse components would cancel but the longitudinal component would be twice as large. One could increase the number of fields to four in the simple case with the fields orthogonal to one another and the beam line.

## 4 CALCULATIONS

Calculating the change in index of refraction or angle of incidence needed for continuous phase matching is straightforward. Phase matching between the incoming electron and evanescent wave occurs if at 0.2c the index of refraction is 8 and the incidence angle is 38.68 degrees. It is presumed that the electrons are riding the maximum evanescent field amplitude and thus the transverse acceleration is zero.

Equations (4–6) are used to first find an acceleration and then the velocity and distance traveled. For electrons, the velocities in question have a noticeable effect on mass. Hence, the acceleration must be recalculated for each time interval. Equation (7) is used to find a changing index of refraction, $n$, for a fixed angle of incidence (38.68 degrees) and for finding a changing angle of incidence, q, for a fixed index of refraction (n=8). Calculations stop when the electrons reach a velocity of 0.95c.

The length of such an accelerator is on the order of 0.8 m for an evanescent electric field of 0.56 MV/m. Results for a sample calculation are shown in Figure 5. The change in $n$ and $\theta$ are very pronounced at lower velocities. Changing the angle of incidence would probably be easier experimentally than changing the index of refraction. If the magnitude of the field could also be altered along the electron path the rapid change of $n$ and $\theta$ at low velocities could be decreased by reducing the acceleration.

Water at 1 m has an index of refraction of about 8 and low absorption.[7] The high index of refraction is needed so that the incident electron can be in phase with the electric fields at a low initial velocity. At 10.5 cm, water has very significant absorption rendering the concept unworkable. This type of accelerator depends upon material properties which vary with wavelength.

For the evanescent wave drift tube linac, calculations are also straightforward. Transverse acceleration is again ignored. Equation (8) is used to determine the acceleration and Equations (4) and (5) are used to determine the distance traveled and velocity. For the distance and velocity calculations time, $t$, is an incremental time while for the acceleration calculation, $t$, is the total elapsed time. Total distance traveled (sum over d) is the $x$ variable of the acceleration equation.

In the following calculations $x$ starts at zero, elapsed time is zero, time increment is $10^{-12}$ or $10^{-13}$ $s$, phase is 175 degrees, and angular frequency follows from the wavelength of interest. The index of refraction is chosen to be 2 and the incidence angle is 31 degrees which is larger than the critical angle of 30 degrees. Evanescent electric field strength is either 0.56 MV/m or 5.6 MV/m.

Acceleration is calculated and used in Equations (4) and (5) to find the velocity and displacement. The new velocity and elapsed distance are then used to recalculate the acceleration. When the acceleration becomes a deceleration the drift tube length is calculated. This iterative approach is used until a preselected final velocity is reached, 0.95c. Since the total distance traveled is known the accelerator length is also known.

Variations in electric field strength and in wavelength are used to establish a rough understanding of feasibility. For a 1 m wave, the total length of the accelerator at 0.56 MV/m field is 4.12 m and at 5.6 MV/m field the length is 0.12 m. At the lower field strength there is one drift section of length 2.6 m, Figure 6. Since the wavelength is smaller than this, it should be possible to reduce the size of the accelerator by using two sections for

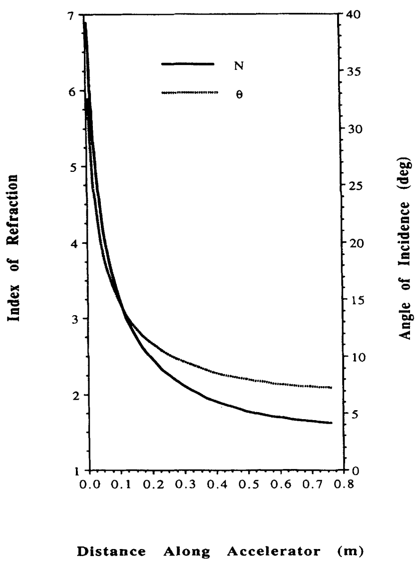


FIGURE 5: For phase matching over the entire length of the accelerator the index of refraction or the angle of incidence must change with distance along the accelerator as shown. Either the index of refraction is fixed at 8 or the angle of incidence is fixed at 38.68 degrees for the calculations. The entrance velocity is 0.2c while the exit velocity is 0.95c.

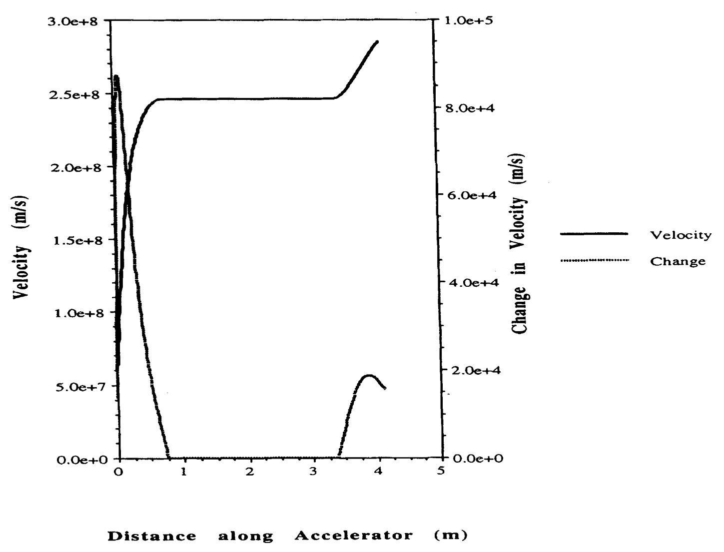


FIGURE 6: Results for a non-phase matching accelerator are shown. While the electron is in the drift tube there is no acceleration and this is seen as the region where velocity is constant versus distance. Also plotted is the incremental change in the velocity. Clear to be seen is the oscillatory change in the velocity with distance along the accelerator.

accelerating the electrons instead of one section with a drift tube. The higher field accelerator has no drift section.

A 10.5 cm wave at 0.56 MV/m field needs a length of 2.3 m to accelerate electrons. At 5.6 MV/m the length is 0.86 m with one drift section of 0.66 m. The problem with 10.5 cm waves at low field strength is that there are many drift sections some of which are small, on the order of 1 cm. Nominally for drift sections longer than the wavelength one could try to use multiple accelerating sections to eliminate the drift sections and shorten the accelerator.

## 5 EMISSION OF ABSORBED ENERGY

There is the possibility of the electrons emitting the acquired energy. Cherenkov radiation occurs when a particle is traveling in a medium faster than the velocity of light can travel in that medium, ($v > c/n$).[8] This is not possible in vacuum since the maximum velocity that a particle can have is $c$ which is the velocity that the emitted energy must travel at. If the electric field of the electrons extends into the dense medium it is possible to have the electron velocity greater than the velocity at which the emitted radiation would travel, $c/n$. For this case, reradiation of energy gained cannot be excluded. Analysis of this problem is beyond the scope of this paper.

## 6 PRODUCING EVANESCENT WAVES WITH A GRATING

There are other ways of producing evanescent waves. Palmer, for example, used a grating to produce the evanescent waves needed for his ultrarelativistic electron accelerator.[1] Grating produced evanescent waves will be difficult at meter long wavelengths. The wavevector of the evanescent wave above a grating is; $K = k_0 + m2p/S$. $S$ is the grating ruling spacing and $m$ is the mode number for the wavevector, usually unity.

For velocities between 0.2c and 0.95c, the grating ruling spacing needed for phase matching varies from 0.25 to 19.0 meters at 1 m wavelength and 0.026 to 1.99 meters at 10.5 cm (using Equation (7)). However, at the electric field strengths discussed earlier the electron is significantly accelerated in short distances, 0.05 meters. Hence, phase matching is again a problem. A more serious concern is the possibility of strongly localized fields at the grating rulings.

## 7 CONCLUSIONS

Accelerating electrons in the velocity range of 0.2c to 0.95c with an evanescent wave accelerator is possible. Two methods for producing evanescent waves exist; by total internal reflection and with a grating. Continuous phase matching between the electrons and the evanescent wave is possible as the electron velocity increases. A non-phase matching system is also possible and would be experimentally easier to build. The non-phase matching system relies on accelerating the electron during the appropriate portion of the periodic electric field. When the field would act as a decelerating force, drift tubes would be used to shield the electron from the field. For both the phase matching and non-phase matching accelerators, there are possibilities for ignoring the transverse component of the evanescent wave. The choice of starting and ending velocities is not rigid for the evanescent wave drift tube linac. For the phase matching linac, the minimum starting velocity is dependent upon the index of refraction of the medium and the incidence angle. The relatively large size make the proposed accelerators useful as an experimental testbed for any form of evanescent wave accelerator.

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