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HELICAL SNAKES WITH NO ORBIT CORRECTION AND THEIR DISCRETE ANALOGUES

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Possibility of using helical magnetic fields is considered for constructing siberian snakes. Two schemes are described, that do not require additional orbit correction. One of the designs has the radial axis of spin rotation, and the other has axis of about 40 degrees from radial. Analogous discrete snake configurations are also presented.

KEY WORDS: Polarization, Siberian snakes

1 INTRODUCTION

The siberian snake method is applied to cancel the depolarizing effect of numerous spin resonances that occur during acceleration of polarized particles. Ideally, the siberian snake is a device that rotates particle spin around a horizontal axis by 180 degrees and at the same time is optically transparent for particle motion. Therefore, the siberian snake is often considered as a specially arranged sequence of dipole magnets which restores the particle orbit and has the required rotation of the particle spin. However, in many cases helical dipole fields can be more attractive for building a snake. There are two advantages in using the helical dipoles in a snake design. First, its orbit excursion is smaller than that for similar discrete snake configurations, which is very important at low energies. And the second advantage is that smaller total snake field integral allows one to construct a compact helical snake model. It is then less likely to be constrained by the length of straight sections available in any given machine.

Here we investigate two helical snake designs where no additional orbit correction is required outside the helix. We also consider their discrete analogues with similar properties. First, we obtain the expression for the Spin Transfer Matrix (STM) of an arbitrary helix.

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2 STM FOR AN ARBITRARY HELIX

We will use a coordinate system (x, s, z) where s is the direction of the reference particle orbit and z, x are vertical and radial axes, respectively. The Thomas–BMT equation of spin motion[1] written in this coordinate system for the spinor wave function[2] is:

$$\frac{d\psi}{ds} = -\frac{i}{2}\left(\vec{\sigma}\cdot\vec{\Omega}\right)\psi\,, \tag{1}$$

where,

$$\vec{\Omega} = \frac{e}{pc}\left[(1+G)\cdot B_{\parallel} + G\gamma\cdot B_{\perp}\right],$$

G is the anomalous magnetic moment coefficient ($G = 1.79285$ for protons), $B_{\parallel}$ and $B_{\perp}$ are longitudinal and perpendicular components of magnetic field with respect to the particle orbit. Since the ideal magnetic field of an arbitrary helix can be written in the form:

$$B_x = B_0\cos(ks+\alpha);\quad B_z = B_0\sin(ks+\alpha) \tag{2}$$

the equation of spin motion becomes:

$$\begin{aligned}\frac{d\psi}{ds} &= -\frac{i}{2}\cdot\frac{eG\gamma}{pc}B_0\left(\sigma_1\cos(ks+\alpha)+\sigma_3\sin(ks+\alpha)\right)\psi = \qquad (3)\\ &= -\frac{i}{2}\varrho\sigma_1\left(\cos(ks+\alpha) - i\sigma_2\sin(ks+\alpha)\right)\psi = -\frac{i}{2}\varrho\sigma_1 e^{-i\sigma_2(ks+\alpha)}\psi\end{aligned}$$

where $\varrho[\text{rad}] = \frac{eG\gamma}{pc}B_0 = B_0[\text{Tesla}]/1.746$ is the angle of proton spin rotation by one meter of transverse magnetic field B_0. Changing the variable (going into the rotating frame):

$$\varphi(s) = e^{-\frac{i}{2}\sigma_2(ks+\alpha)}\psi(s) \tag{4}$$

the equation of spin motion reduces to:

$$\frac{d\varphi}{ds} = -\frac{i}{2}\left(k\sigma_2 + \varrho\sigma_1\right)\varphi \tag{5}$$

for which the solution is:

$$\varphi(s) = \exp\left\{-\frac{i}{2}ks\left(\sigma_2 + \frac{\varrho}{k}\sigma_1\right)\right\}\varphi(0) \tag{6}$$

Modifying the argument of the exponential in (6) we obtain:

$$\begin{aligned}ks\left(\sigma_2+\frac{\varrho}{k}\sigma_1\right) &= \theta\left(\sigma_2+\sigma_1\tan\delta\right) = \frac{\theta}{\cos\delta}\sigma_2\left(\hat{I}\cos\delta - i\sigma_3\sin\delta\right)\\ &= \theta\lambda\,\sigma_2 e^{-i\sigma_3\delta}\end{aligned}$$

Then, in terms of the original spinor wave function ψ the spin transfer matrix for the helix is:

$$\hat{M}_h = e^{\frac{i}{2}\sigma_2(\theta+\alpha)} \cdot \exp\left\{-\frac{i}{2}\theta\lambda\sigma_2 e^{-i\sigma_3\delta}\right\} \cdot e^{-\frac{i}{2}\sigma_2\alpha}\ , \tag{7}$$

where α is the initial field orientation in the helix and $(\theta + \alpha)$ is final. The obtained STM for the helix can be also written in the form of rotation transformation on spin:

$$\hat{M}_h = \hat{I}\left(\cos\frac{\theta\lambda}{2}\cos\frac{\theta}{2} + \sin\frac{\theta\lambda}{2}\cos\delta\sin\frac{\theta}{2}\right) + i\,\vec{\sigma}\cdot\vec{a}_+; \tag{8}$$

$$\vec{a}_+ = \begin{pmatrix} -\sin\frac{\theta\lambda}{2}\ \sin\delta\ \cos\left(\frac{\theta}{2}+\alpha\right); \\ \cos\frac{\theta\lambda}{2}\ \sin\frac{\theta}{2} - \sin\frac{\theta\lambda}{2}\ \cos\delta\ \cos\frac{\theta}{2}; \\ -\sin\frac{\theta\lambda}{2}\ \sin\delta\ \sin\left(\frac{\theta}{2}+\alpha\right) \end{pmatrix}$$

It is also easy to show that if reversing the direction of field rotation in the helix (putting $k_1 = -k$ in (2)) the vector $\vec{a}$ which corresponds to the axis of spin rotation in the helix changes to:

$$\vec{a}_- = \begin{pmatrix} -\sin\frac{\theta\lambda}{2}\sin\delta\cos\left(\frac{\theta}{2}-\alpha\right); \\ \sin\frac{\theta\lambda}{2}\cos\delta\cos\frac{\theta}{2} - \cos\frac{\theta\lambda}{2}\sin\frac{\theta}{2}; \\ \sin\frac{\theta\lambda}{2}\sin\delta\sin\left(\frac{\theta}{2}-\alpha\right) \end{pmatrix} \tag{9}$$

We now consider particle motion in the helix. It is determined by the system of equations:

$$\begin{cases} x'' = \frac{eB_z}{pc} = \frac{eB_0}{pc}\sin(ks+\alpha) \\ z'' = -\frac{eB_x}{pc} = -\frac{eB_0}{pc}\cos(ks+\alpha) \end{cases} \tag{10}$$

The general solution of system (10) is:

$$\begin{cases} x = -r\sin(ks+\alpha) + C_1 s + C_0; \\ z = r\cos(ks+\alpha) + \tilde{C}_1 s + \tilde{C}_0; \end{cases} \tag{11}$$

where $r[\mathrm{m}] = \frac{eB_0}{pc:\,k^2} = \varrho/(G\gamma\ k^2) = 0.914\left(\frac{\varrho}{k}\right)^2/(B_0[\mathrm{T}]\ E[\mathrm{GeV}])$, and $\tilde{C}_{0,1}, C_{0,1}$ are constants determined by initial conditions. Solution(11) contains a spiral trajectory plus a straight line solution. To minimize orbit excursions inside the helix one can use a system of correction magnets to match the incoming beam with the spiral trajectory.[3] However, the particle orbit can also be restored without additional correction magnets. In this article we consider snakes consisting of two helices so that the orbit excursion stored in the first helix is canceled in the second. The orbit excursion in this case is larger than for pure spiral

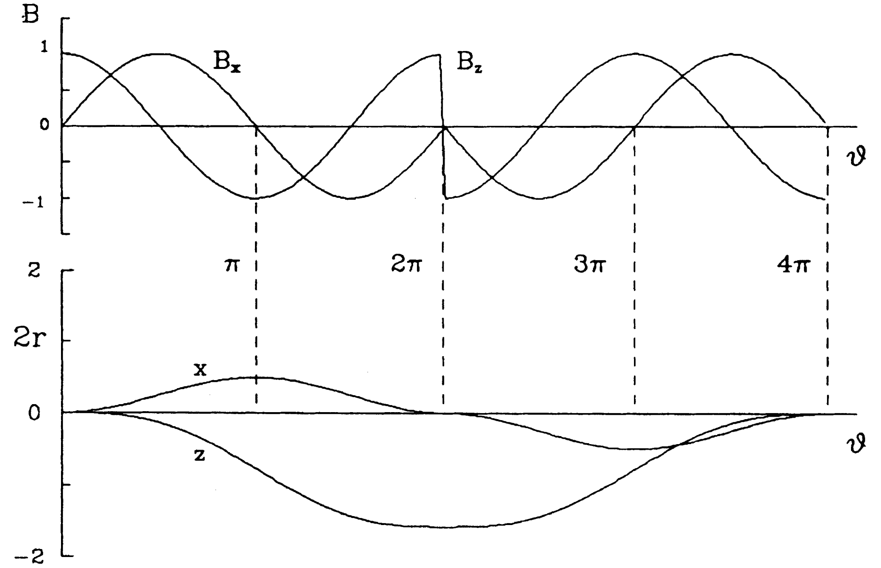

FIGURE 1: Antisymmetric helical snake magnetic field configuration and particle orbit excursion profile.

trajectory, but may be suitable in practice. Such optically transparent system can be made either by reversing the direction of field rotation in the second helix (symmetric scheme) or by reversing the initial field orientation in the second helix (antisymmetric scheme). We shall consider both possibilities.

3 HELICAL SNAKES SCHEMES

When saying antisymmetric snake we understand such snake field configuration that horizontal field projection is symmetric about the center of the snake and vertical projection of magnetic field is antisymmetric. This symmetry applied to the snake field configuration allows one to obtain horizontal snake axis automatically. For the symmetric snake both vertical and horizontal field components are symmetric with respect to the snake center; this configuration of the magnetic field makes possible to obtain the axis of spin rotation with no longitudinal component (i.e. lying in the perpendicular plane). It means that rotating the whole snake around the longitudinal direction one can find a position for which the snake axis is exactly radial. First, we discuss the antisymmetric snake. It consists of two identical full twist helices with 180° phase shift in the field orientation between the first and second helices. The snake field configuration and particle trajectory inside the snake are shown in Figure 1. The spin transfer matrix for this scheme is a product of two parts:

$$\hat{S} = \left(\hat{I} \cdot \cos\frac{\theta\lambda}{2} + i\,\sin\frac{\theta\lambda}{2}\left(\vec{\sigma}\cdot\vec{b}\right)\right)\left(\hat{I} \cdot \cos\frac{\theta\lambda}{2} + i\,\sin\frac{\theta\lambda}{2}\left(\vec{\sigma}\cdot\vec{a}\right)\right) \qquad (12)$$

and since both parts have negative direction of field rotation with initial phases $\alpha_1 = \frac{\pi}{2}$; $\alpha_2 = -\frac{\pi}{2}$ and $\theta = 2\pi$, the axes of spin rotation $\vec{a}$ and $\vec{b}$ for each part are:

$$\vec{a} = (0;\ \cos\delta;\ \ -\sin\delta)$$

$$\vec{b} = (0;\ \cos\delta;\ \sin\delta)$$

Hence, spin transfer matrix (12) can be rewritten as:

$$\hat{S} = \hat{I}\left(1 - 2\sin^2\frac{\theta\lambda}{2}\cos^2\delta\right) + i\vec{\sigma}\cdot\left\{\vec{e}_s\cot\frac{\theta\lambda}{2}\frac{1}{\cos\delta} + \vec{e}_x\tan\delta\right\}2\sin^2\frac{\theta\lambda}{2}\cos^2\delta. \quad (13)$$

The condition for the spin to rotate by 180 degrees in this system will be:

$$\sin^2\left(\pi\sqrt{1+\frac{\varrho^2}{k^2}}\right)\frac{1}{1+\frac{\varrho^2}{k^2}} = \frac{1}{2}. \quad (14)$$

If the above condition is satisfied, the expected orientation of the snake axis is:

$$\tan\varphi_s = \cot\frac{\theta\lambda}{2}\frac{1}{\sin\delta} \quad (15)$$

Before solving equations (14) and (15) for the antisymmetric snake parameters we now consider the symmetric helical snake. The field configuration and particle orbit excursion for the snake are shown in Figure 2. In this scheme two snake helices have different directions of field rotation and the same initial phases $\alpha_1 = \alpha_2 = 0$. Similarly to the antisymmetric scheme the spin transfer matrix for this configuration can be written in the same form (12) but the axes of spin rotation $\vec{a}$ and $\vec{b}$ for each helical part of the snake are different:

$$\vec{a} = (-\sin\delta;\ -\cos\delta;\ 0)$$

$$\vec{b} = (-\sin\delta;\ \ \cos\delta;\ 0)$$

It also changes the expression for the STM of the snake:

$$\hat{S} = \hat{I}\left(1 - 2\sin^2\frac{\theta\lambda}{2}\sin^2\delta\right) - i\vec{\sigma}\cdot\left\{\vec{e}_x\cot\frac{\theta\lambda}{2}\frac{1}{\sin\delta} - \vec{e}_z\cot\delta\right\}2\sin^2\frac{\theta\lambda}{2}\sin^2\delta\ , \quad (16)$$

and the condition of 180 degree spin rotation becomes:

$$\sin^2\left(\pi\sqrt{1+\frac{\varrho^2}{k^2}}\right)\frac{1}{1+\frac{k^2}{\varrho^2}} = \frac{1}{2} \quad (17)$$

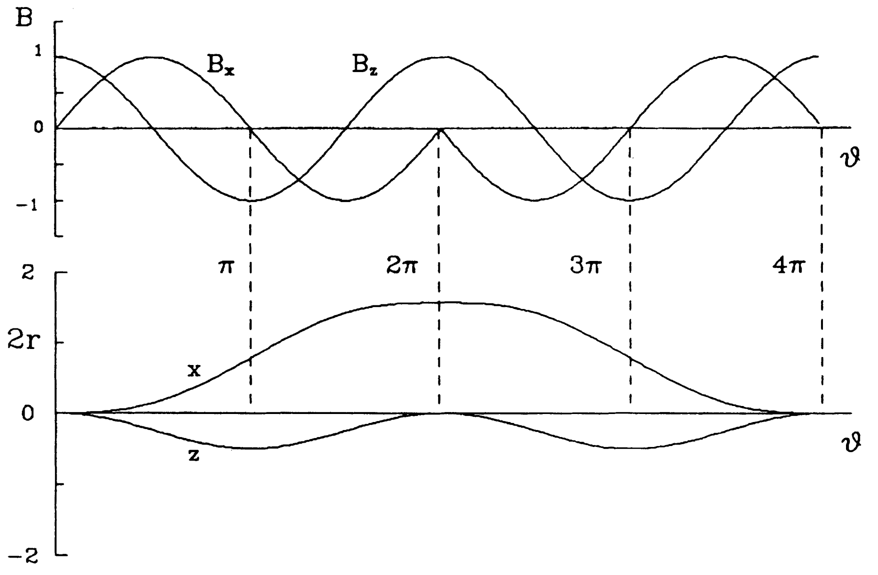

FIGURE 2: Magnetic field configuration and particle orbit excursion profile for the symmetric helical snake.

As it follows from (16) this snake has radial axis of spin rotation if rotated around the longitudinal direction by angle β:

$$\tan\beta = -\tan\frac{\theta\lambda}{2}\cos\delta \tag{18}$$

he obtained result is interesting for the purpose of using two snakes in an accelerator. Helical snake schemes with orbit correction[3,4] have their axes within few degrees from longitudinal direction and when using a pair of snakes in an accelerator their axes should be orthogonal.[5] Thus, the described symmetric snake can be used in pair with the one, whose axis is longitudinal. It may be also important for practical realization that both snake schemes described here can be split in two parts with no effect on the orbit excursion inside the snake. It follows from the fact that the snake configurations restore particle velocity at the snake center in both x and z planes. Hence, the snake can be made as a system of two separate magnets.

In order to obtain parameters of the described snakes we should resolve the conditions of 180 degree spin rotation (14) and (17). Figure 3 shows behavior of the functions $f(\xi)$ and $g(\xi)$ that correspond to the left parts of the conditions for the symmetric and antisymmetric helical snakes:

$$f(\xi) = \sin^2\left(\pi\sqrt{1+\xi^2}\right)\frac{1}{1+\xi^{-2}}; \quad g(\xi) = \sin^2\left(\pi\sqrt{1+\xi^2}\right)\frac{1}{1+\xi^2} \tag{19}$$

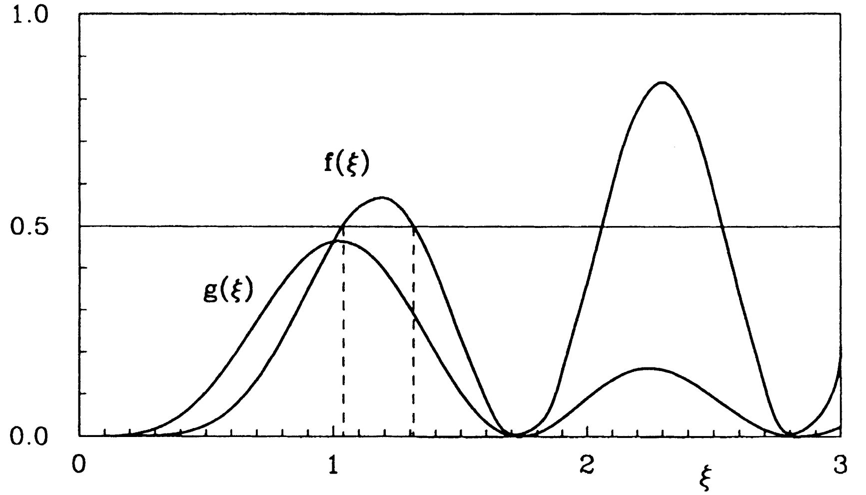

FIGURE 3: Behavior of the functions $f(\xi)$ and $g(\xi)$ for the helical snakes' conditions of 180° spin rotation.

Only for the symmetric scheme there are solutions with 180 degree spin rotation. They correspond to 100% snakes. As for the antisymmetric scheme, there are only partial snake solutions with smaller angle of spin rotation. But as we will see later, there is a way to modify the antisymmetric scheme in order to obtain 100% antisymmetric helical snake. Among the solutions for the symmetric snake the one with the smallest value of $\xi = \varrho/k$ gives minimum snake field integral. The solution was obtained solving equation (17) numerically and Table 1 summarizes the snake scheme parameters.

TABLE 1: Symmetric helical snake parameters.

Root value of $\xi = \varrho/k$	1.037
Length of the helix period (assuming $B = 1.7$ Tesla)	6.69 m
Field integral ($\int B\,dl$)	22.742 T·m
Snake axis	radial
Angle β the snake must be rotated around $\hat{s}$-axis to obtain radial snake axis	74.723 deg
Maximum orbit excursion	$x{=}2\pi r$; $z{=}2r$
	$r{=}0.983/(B_0[\mathrm{T}]\ E[\mathrm{GeV}])$

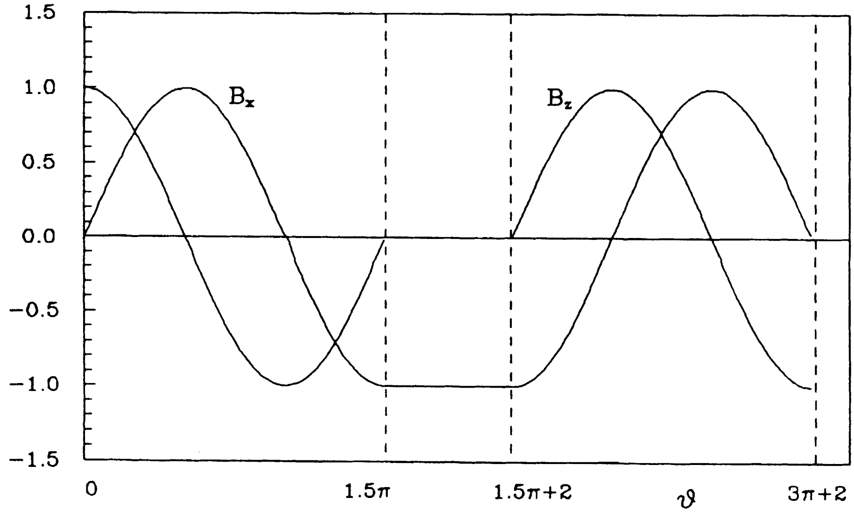

FIGURE 4: Modified antisymmetric helical snake magnetic field configuration.

4 MODIFIED ANTISYMMETRIC HELICAL SNAKE

The antisymmetric helical snake described in the previous section doesn't allow one to obtain 180 degree spin rotation, but it can be modified for this purpose. We substitute the central part of that scheme ($+\pi/2$; $-\pi/2$ around the center) with a horizontal field insert in such a way that the orbit restoration is preserved. Then, the field integral of the insert must be equal to $2B_0/k$ in order to restore particle orbit in the vertical plane. On the other hand, to obtain orbit restoration in the horizontal plane the length between two remaining helical parts must be equal to $2/k$. Therefore, one can make a one magnet antisymmetric helical snake with the field configuration as it is shown in Figure 4, where in the center there is a dipole insert with constant field (not rotated). The spin transfer matrix for this scheme is a product of three parts:

$$\hat{S} = \left(\hat{I}\cdot A - i\,\vec{\sigma}\cdot\vec{b}\right)\left(\hat{I}\cos\frac{\phi}{2} + i\,\sigma_1\sin\frac{\phi}{2}\right)\left(\hat{I}\cdot A - i\,\vec{\sigma}\cdot\vec{a}\right) \tag{20}$$

where:

$$A = \frac{1}{\sqrt{2}}\left(\sin\frac{\theta\lambda}{2}\cos\delta - \cos\frac{\theta\lambda}{2}\right) \tag{21}$$

$$\vec{a} = \frac{1}{\sqrt{2}}\left(\sin\frac{\theta\lambda}{2}\sin\delta;\ \sin\frac{\theta\lambda}{2}\cos\delta + \cos\frac{\theta\lambda}{2};\ -\sin\frac{\theta\lambda}{2}\sin\delta\right)$$

$$\vec{b} = \left(a_x;\ a_y;\ -a_z\right)$$

here ϕ is the angle of spin rotation in the insert, which is equal to $\phi = 2\varrho/k$. After multiplication in (20) the STM of the helical system transforms into:

$$\hat{S} = \hat{I}\left(\sin\frac{\phi}{2}\sin^2\frac{\theta\lambda}{2}\sin 2\delta - \cos\frac{\phi}{2}\sin\theta\lambda\cos\delta\right) - i\vec{\sigma}\cdot\vec{d} \tag{22}$$

$$\vec{d} = \begin{pmatrix} \sin\frac{\phi}{2}\left(1 - 2\sin^2\frac{\theta\lambda}{2}\sin^2\delta\right) + \cos\frac{\phi}{2}\sin\theta\lambda\sin\delta \\ -\cos\frac{\phi}{2}\cos\theta\lambda + \sin\frac{\phi}{2}\sin\theta\lambda\sin\delta \\ 0 \end{pmatrix}$$

The obtained STM of the antisymmetric helical snake gives the constraint of 180 degree spin rotation:

$$\frac{2\xi}{\sqrt{1+\xi^2}}\left(\sin^2\left(\frac{3\pi}{4}\sqrt{1+\xi^2}\right)\sin\xi - \frac{\cos\xi}{\sqrt{1+\xi^2}}\sin\left(\frac{3\pi}{2}\sqrt{1+\xi^2}\right)\right) = 0 \tag{23}$$

Here ξ is substituting ϱ/k. Solving this equation numerically we define other parameters of the snake summarized in Table 2. It is to be noted that the orientation of the snake axis is fixed for this scheme at 39.47° to the radial direction. It solely depends on the value of ϱ/k , which is determined by the condition of 180° spin rotation. However, using two additional dipoles, one on each side of the snake, one can obtain snakes with different snake axis orientations. It would then require changes in the helix parameters in order to keep particle orbit restoration. As an example we refer to the snake described in.[4] It consists of one full twist helix and two correction dipoles. The length of the constant field insert is zero for this example. The snake axis for such scheme is close to the longitudinal direction.

We now proceed to the consideration of discrete analogues for the described helical snakes.

TABLE 2: Modified antisymmetric helical snake parameters.

Root value of $\xi = \varrho/k$	0.8819
Length of the helix period (assuming $B = 1.7$ Tesla)	5.69 m
Field integral ($\int B\,dl$)	17.59 T·m
Snake axis φ_s	39.47° from radial
Maximum orbit excursion	z=1.5$r(\pi+1)$; x=2r
	r=0.711/(B_0[T] E[GeV])

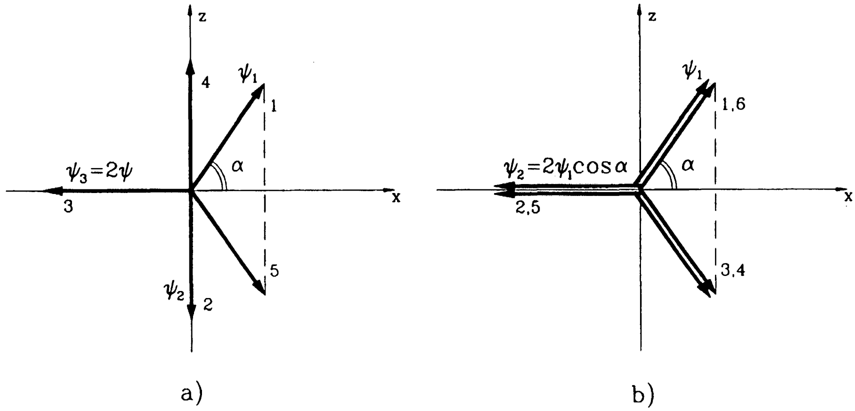

FIGURE 5: Discrete analogues for the helical snakes: (a) five magnet antisymmetric snake; (b) six magnet symmetric snake.

5 ANALOGOUS DISCRETE MAGNET SNAKE CONFIGURATIONS

Both symmetric and antisymmetric helical snakes described above have discrete magnet analogues. They can be easily obtained by approximating parts of the helix with conventional dipoles.[7]. Figure 5 shows two discrete magnet snake configurations which can be considered as the analogues for the symmetric and antisymmetric helical snakes. Each vector on these diagrams corresponds to one magnet and shows both the direction of magnetic field and angle of spin rotation in the magnet. It is convenient to consider STM for both discrete schemes as a product of two parts dividing the snakes in the center. Due to the symmetry in the snake field configurations the STM can be written as a product:

$$\hat{S} = \left(\hat{I} \cdot A - i\,\vec{\sigma} \cdot \vec{b}\right)\left(\hat{I} \cdot A - i\,\vec{\sigma} \cdot \vec{a}\right) \tag{24}$$

Then the condition for the spin to rotate by 180 degrees can be written for both snakes in the form:

$$A^2 - \left(\vec{a} \cdot \vec{b}\right) = 0 \tag{25}$$

For the symmetric snake shown in Figure 5:

$$A = \cos\frac{\psi_2}{2}\left(1 - 2\sin^2\frac{\psi_1}{2}\cos^2\alpha\right) + \sin\frac{\psi_2}{2}\sin\psi_1\sin\alpha$$

$$\vec{a} = \left(\cos\frac{\psi_2}{2}\sin\psi_1\cos\alpha - \sin\frac{\psi_2}{2}\cos\psi_1;\right.$$

$$\left.\cos\frac{\psi_2}{2}\sin^2\frac{\psi_1}{2}\sin 2\alpha - \sin\frac{\psi_2}{2}\sin\psi_1\sin\alpha;\ 0\right)$$

$$\vec{b} = (a_x;\ -a_s;\ 0) \tag{26}$$

TABLE 3: Discrete six magnet analogue for the symmetric helical snake.

Angle of spin rotation in the first magnet	ψ_1=125.82 deg
Angle of spin rotation in the second magnet	ψ_2=135.172 deg
Orientation angle of the first magnet	α=57.51 deg
Angle the snake must be rotated around $\hat{s}$-axis to obtain radial snake axis	β=30.455 deg
Field integral ($\int B\,dl$)	23.574 T·m
Snake axis φ_s	radial

where ψ_1, ψ_2 are angles of spin rotation in the first and second snake magnets and α is the first magnet orientation angle.

The axis of spin rotation in the discrete system doesn't contain longitudinal component as well as the symmetric helical snake. Hence, if the whole snake is properly rotated around longitudinal direction the snake axis is exactly radial. Similarly to the helical snake the discrete analogue can be split in two parts since each half restores particle velocity. On the other hand, the third and the forth magnets are identical; therefore, they can be made as one long magnet. This snake scheme has one free parameter which can be varied in order to obtain minimum field integral. The numerically obtained sample solution which gives close to minimum field integral is given in Table 3.

As an analogue of the antisymmetric helical snake we consider the five magnet system[6] shown in Figure 5 that approximates helical field of the snake. It has the vertical projected field antisymmetric and the horizontal projected field symmetric as well as the helical snake. Therefore, angle of spin rotation in the third snake magnet is equal:

$$\psi_3 = 2\psi = 2\psi_1 \cos\alpha \tag{27}$$

where ψ_1 is angle of spin rotation in the first magnet and α is the first magnet orientation angle.

The snake STM also can be written in form (24) where for this scheme:

$$A = \cos\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} + \sin\alpha\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) + \cos\alpha\sin\frac{\psi}{2}\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2}$$

$$\vec{a} = \begin{pmatrix} \cos\alpha\cos\frac{\psi}{2}\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \sin\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} + \sin\alpha\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) \\ \sin\frac{\psi}{2}\left(\sin\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) - \cos\alpha\cos\frac{\psi}{2}\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \\ \cos\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2} - \sin\alpha\cos\frac{\psi_2}{2}\sin\frac{\psi_1}{2}\right) - \cos\alpha\sin\frac{\psi}{2}\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \end{pmatrix} \tag{28}$$

TABLE 4: Five magnet snake examples (l_{gap}=0.4 m, B = 1.7 T).

α (deg)	ψ_1 (deg)	ψ_2 (deg)	ψ (deg)	φ_s (deg)	$\int B\, dl$ (T·m)
49.82	101.7	148.3	65.61	75.28	19.24
47.97	102.6	144.3	68.69	71.23	19.23
45.79	104.4	140.3	72.8	66.02	19.35
43.13	108	136.9	78.82	58.48	19.73
41.38	111.6	135.7	83.74	52.23	20.17
40.12	115.2	135.8	88.1	46.49	20.66
39.17	118.8	136.7	92.1	41	21.18
38.46	122.4	138.2	95.84	35.62	21.72
37.94	126	140.2	99.38	30.27	22.28

and since $\vec{b} = (a_x;\ a_y;\ -a_z)$ the resulting snake axis lies in the horizontal plane. The five magnet snake has one free parameter which can be varied in order to obtain different snake axis orientations. This is an advantage over the antisymmetric helical snake, however the total field integral of the helical snake is smaller. Table 4 contains solutions for the five magnet snake obtained by varying the angle of spin rotation in the first magnet as a free parameter and assuming for the snake 0.4 m gap between magnets and 1.7 Tesla field strength in all magnets.

6 CONCLUSION

Helical magnetic fields are commonly used for producing circularly polarized synchrotron radiation. The magnet period in such helical undulators lies in the range 20–200 mm. In this paper we have considered another application of the helical system with the magnet period of several meters for constructing spin rotators. We discussed the ideal case when the spin is rotated by the field on the axis of the helix. However, in practice one should take into account off-axis fields in the helix and also the effect of particle orbit excursion. Field quality in the helix is proportional to the factor $(r2\pi/L)^2$, where L is the magnet period and r is off-axis distance. Therefore, even at 10 cm orbit excursion the off-axis fields are small for the helix with 6 m period. The particle orbit excursion itself causes small variations of the snake spin rotation angle and the snake axis orientation, that can be neglected in the first approach. It is also to be noted that the siberian snakes have small focusing effect on orbital motion in the accelerator.[8] This cause excitation of the lattice structure functions and may require local quadrupole correction.

Finishing our consideration we emphasize features of the described snake schemes and their advantages and disadvantages. Both helical snake configurations doesn't require external orbit correction. The symmetric helical snake can be split in two separate magnets while antisymmetric is a one unit magnet. Practical application of the helical snakes may be constrained by the maximum orbit excursion in the snakes, which is at 20 GeV 18 cm for the symmetric snake and 13 cm for the antisymmetric snake assuming 1.7 Tesla field in the helix. The symmetric helical scheme has radial snake axis and for the antisymmetric scheme the snake axis orientation is 39.47° from the radial direction. Both helical snakes have discrete magnet analogues with the same properties but larger orbit excursion and field integral. These discrete schemes can be used in accelerators with higher injection energies.

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REFERENCES

1. V. Bargman, L. Michel and V.L. Telegdi, *Phys. Rev. Lett.*, **2**, 435 (1959).
2. B.W. Montague, "Spinor algebra for polarized beams", *Part. Accel.*, **11**(4), 219 (1981).
3. E.D. Courant, "*Helical Siberian Snakes*", The 8th International Symposium on High Energy Spin Physics, Mineapolis, AIP Conf. Proc. No. 187, 1085 (1988).
4. A.M. Kondratenko,"*Polarized particle beams in accelerators and storage rings*", Dissertation, Novosibirsk 1982. A.M. Kondratenko, *Energy spread, snakes, rotators and wigglers*, in Proc. of the Workshop, Polarized electron acceleration and storage, Hamburg, DESY, L1–L3 (1982).
5. B.W. Montague, *Physics Reports*, **113** No.1, 1 (1984).
6. V.A. Anferov and Ya.S. Derbenev, "*Siberian snakes with small number of magnets for high energy accelerators*", UM HE 93-7, 1993 (submitted to the Part. Accel.).
7. U. Wienands, "*Helical snakes using dipole magnets*", TRIUMF 90–65, 1990.
8. Ya.S. Derbenev, in Bonn 1990 Conf. Proc., High Energy Spin Physics, Vol. 2, 132.