SUPPRESSION OF LONGITUDINAL COUPLED-BUNCH INSTABILITIES BY A PASSIVE HIGHER HARMONIC CAVITY

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A radiofrequency system with a passive higher-harmonic cavity is considered for the prevention of coupled-bunch instabilities in an electron storage ring. Expressions are presented for the synchrotron frequency, the synchrotron frequency spread, the bunch length, the onset of the equilibrium phase instability, and the frequency and damping rate of the Robinson instability. These expressions are incorporated into an algorithm to predict parameters for the stable operation of a storage ring. The stability predictions are in good agreement with experimental results at two facilities. The algorithm predicts that a passive third-harmonic cavity can be successfully employed in a current-generation electron storage ring for the production of synchrotron radiation.

KEY WORDS: Collective Effects, instabilities

1 INTRODUCTION

The performance of an electron storage ring may be limited by longitudinal oscillations and emittance growth from coupled-bunch instabilities,[1] as well as particle loss from Coulomb scattering, characterized by the Touschek lifetime.[2] A second radiofrequency (RF) cavity with resonance near a harmonic of the fundamental RF cavity may be used to increase Landau damping of synchrotron oscillations and/or increase the bunch length.[3,4,5] This can prevent coupled-bunch instabilities and increase the Touschek lifetime, providing two important benefits. When this can be achieved with a passive higher-harmonic cavity (a cavity with no external power supply), the difficulty, expense, and time of implementation are greatly reduced.

To counteract the coupled-bunch instability, a higher harmonic cavity must increase Landau damping without causing unwanted side effects — in particular, the equilib-

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rium phase and Robinson instabilities. We consider these instabilities in Section 2, including the effects of beam-loading and finite bunch length. Double-cavity systems that are stable to these instabilities may be usefully employed to increase bunch length and/or Landau damping of synchrotron oscillations. In Section 3, formulas for estimating the Landau damping rate and bunch length in the double-cavity system are presented. An algorithm for using these formulas to evaluate a passive higher-harmonic cavity is presented in Section 4; the algorithm's application to storage rings for synchrotron radiation is presented in Sections 5 and 6. We use the notation of Sands.[6]

2   THE ROBINSON INSTABILITY IN A TWO-CAVITY SYSTEM

Let Cavity 1 be the fundamental RF cavity with resonant frequency ω_1 near ω_g, the generator frequency. Let Q_1 be its quality factor, R_1 its impedance at resonance (one-half of the "accelerator" definition of shunt impedance), and ϕ_1 its tuning angle, defined by $\tan \phi_1 = 2Q_1(\omega_g - \omega_1)/\omega_1$. This tuning angle is the same as that used by Sands[6] and Marchand,[7] and the negative of that used by Wilson.[8] The cavity impedance at ω_g is $R_1 \cos \phi_1 e^{i\phi_1}$. R_1 and Q_1 describe the loaded cavity, i.e., the effective impedance and quality factor of the cavity coupled to the transmitter, which equal $1/(1 + \beta)$ times the unloaded values, where β is the RF coupling coefficient.[6,8]

Cavity 2 is a higher harmonic cavity with resonant frequency ω_2 near $\nu\omega_g$, where ν is its harmonic number. Q_2 is its quality factor, R_2 its resonant impedance, and ϕ_2 its tuning angle, given by $\tan \phi_2 = 2Q_2(\nu\omega_g - \omega_2)/\omega_2$.

Robinson instability is the growth of a rigid-dipole synchrotron oscillation in which all of the bunches are in phase. We consider symmetric oscillating bunches with equilibrium bunch separation τ_o. With symmetric bunches of finite length, we obtain the lowest-order effects of finite bunch length. The current in Cavity 1 from pointlike bunches can be represented by:

$$I(t) = -Ne \sum_{n=-\infty}^{\infty} \delta[t - n\tau_o - A \sin(\Omega n\tau_o)] \; . \tag{1}$$

Here, N is the number of electrons per bunch, e > 0 is the magnitude of the electronic charge, A is the (small) amplitude of the oscillation in units of time, and Ω is the angular frequency. In the frequency domain, we have:

$$I(\omega) = -2\pi \frac{Ne}{\tau_o} \sum_{p=-\infty}^{\infty} \{\delta(\omega - p\omega_o) + \frac{\omega A}{2}[\delta(\omega - p\omega_o + \Omega) - \delta(\omega + p\omega_o - \Omega)]\} \tag{2}$$

where $\omega_o = 2\pi/\tau_o$. The terms proportional to A are sidebands resulting from the oscilllation. Keeping only the dominant terms, the wakefield from the oscillating bunches in Cavity 1 is given by:

$$V_{f1}(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} I(\omega) Z_1(\omega) = -2IR_1\{\cos \phi_1 \cos(\omega_g t - \phi_1)$$

$$+\frac{\omega_g A}{2}[\cos\phi_{1-}\cos(\omega_g t - \Omega t - \phi_{1-}) - \cos\phi_{1+}\cos(\omega_g t + \Omega t - \phi_{1+})]\} , \qquad (3)$$

where $\tan\phi_{1\pm} = 2Q_1(\omega_g \pm \Omega - \omega_1)/\omega_1$, and $I > 0$ is the average beam current magnitude. The wakefield in Cavity 2 is given by the same expression with subscript "1" replaced by "2", and ω_g replaced by $\nu\omega_g$. For symmetric bunches that are not short compared with the RF cavity periods, $V_{f1}(t)$ is modified by the form factor,[8] F_1 where $F_1 = \exp(-\omega_g^2\sigma_t^2/2)$ for a Gaussian bunch with rms bunch length σ_t. $V_{f2}(t)$ is modified by the form factor F_2, which for Gaussian bunches equals $\exp(-\nu^2\omega_g^2\sigma_t^2/2)$.

In addition to the wakefield from the passing beam bunches, the effective generator current [6, 8] of magnitude i_{g1} causes a voltage contribution in Cavity 1. We let θ_{f1} be the angle by which this current lags the sinusoidal beam current component at frequency ω_g; the voltage contribution from the generator current thus lags the wake by the same angle:

$$V_{g1}(t) = -i_{g1}R_1\cos\phi_1\cos(\omega_g t - \theta_{f1} - \phi_1) . \qquad (4)$$

A similar expression holds when Cavity 2 has external power, with subscripts "1" replaced by "2" and ω_g replaced by $\nu\omega_g$. Summing the voltages encountered by an electron which passes through Cavity 1 at time t, we have:

$$V(t) = -R_1\{i_{g1}\cos\phi_1\cos(\omega_g t - \theta_{f1} - \phi_1) + 2F_1 I\cos\phi_1\cos(\omega_g t - \phi_1)$$

$$-\omega_g AF_1 I[\cos\phi_{1+}\cos(\omega_g t + \Omega t - \phi_{1+}) - \cos\phi_{1-}\cos(\omega_g t - \Omega t - \phi_{1-})]\}$$

$$-R_2\{i_{g2}\cos\phi_2\cos(\nu\omega_g t - \theta_{f2} - \phi_2) + 2F_2 I\cos\phi_2\cos(\nu\omega_g t - \phi_2)$$

$$-\nu\omega_g AF_2 I[\cos\phi_{2+}\cos(\nu\omega_g t + \Omega t - \phi_{2+}) - \cos\phi_{2-}\cos(\nu\omega_g t - \Omega t - \phi_{2-})]\}. \qquad (5)$$

For a Robinson oscillation with a growth or damping rate that is small compared to the filling rate of the RF cavities, the above expression will be approximately true when the amplitude A is a function of time.

In a ring far above transition energy, the m-th arrival time, t_m, of an electron in Cavity 1 obeys this equation:[9]

$$\frac{d(t_m - mT_o)}{dm} = \frac{T_o\alpha}{E}\epsilon_m, \qquad (6)$$

where α is the momentum compaction, E is the equilibrium electron energy, T_o is the revolution period, and ϵ_m is the energy deviation of the electron. Differentiating the above equation with respect to m, and averaging over all the electrons in the bunch we have:

$$\frac{d^2 <t_m>}{dm^2} = \frac{T_o\alpha}{E}\frac{d\epsilon_m}{dm} = \frac{T_o\alpha e}{E}[<V(t_m)> -V_s] , \qquad (7)$$

where $V_s > 0$ is the synchronous voltage. We evaluate the left-hand side for the slowly growing or decaying oscillation $< t_m > = mT_o + A(mT_o)\sin(\Omega mT_o)$, evaluate the right-hand side (RHS) using our expression for $V(t)$ and a symmetric bunch shape, and equate them to obtain:

$$V_s = -R_1 F_1 i_{g1} \cos\phi_1 \cos(\theta_{f1} + \phi_1) - 2R_1 F_1^2 I \cos^2\phi_1$$

$$-R_2 F_2 i_{g2} \cos\phi_2 \cos(\theta_{f2} + \phi_2) - 2R_2 F_2^2 I \cos^2\phi_2, \tag{8}$$

$$\Omega^2 = \frac{e\alpha\omega_g}{T_o E}\{R_1 F_1 i_{g1} \cos\phi_1 \sin(\phi_1 + \theta_{f1}) + R_1 F_1^2 I[\sin 2\phi_1 - \frac{1}{2}(\sin 2\phi_{1-} + \sin 2\phi_{1+})]$$

$$+\nu R_2 F_2 i_{g2} \cos\phi_2 \sin(\phi_2 + \theta_{f2}) + \nu R_2 F_2^2 I[\sin 2\phi_2 - \frac{1}{2}(\sin 2\phi_{2-} + \sin 2\phi_{2+})]\}, \tag{9}$$

and

$$\alpha_R = -\frac{1}{A}\frac{dA}{dt} = \frac{e\alpha\omega_g I}{2ET_o\Omega}[F_1^2 R_1(\cos^2\phi_{1-} - \cos^2\phi_{1+}) + F_2^2 \nu R_2(\cos^2\phi_{2-} - \cos^2\phi_{2+})]$$

$$= \frac{4\alpha e I}{ET_o}[F_1^2 R_1 Q_1 \tan\phi_1 \cos^2\phi_{1+} \cos^2\phi_{1-} + F_2^2 R_2 Q_2 \tan\phi_2 \cos^2\phi_{2+} \cos^2\phi_{2-}], \tag{10}$$

where α_R is the Robinson damping rate, positive for a damped case. In the absence of Robinson oscillations ($A = 0$), the total voltage at the bunch center ($t = 0$) given by equation (5) differs from the synchronous voltage, V_s, for bunches of finite duration.[8] For the case of a passive higher harmonic cavity ($i_{g2} = 0$) and short bunches ($F_1 = F_2 = 1$), the above expressions for the Robinson frequency and damping rate agree with a prior calculation.[10] However, for finite length bunches, equations (8) and (9) differ from the prior calculation, because our calculation does not linearize the contribution to $V(t)$ that is independent of A in equation (5).

In order to prevent the equilibrium phase instability (also known as the zero-frequency Robinson instability), we must have $\Omega^2 > 0$. This ensures that there is a restoring force for the coherent phase motion of the bunches.[6] At the instability threshold, the RHS terms multiplied by I are zero in equation (9), so the stability requirement may be written as:

$$F_1 R_1 i_{g1} \cos\phi_1 \sin(\phi_1 + \theta_{f1}) + \nu F_2 R_2 i_{g2} \cos\phi_2 \sin(\phi_2 + \theta_{f2}) > 0. \tag{11}$$

For short bunches with $F_1 = F_2 = 1$, this stability requirement has been given in references 5 and 6. The total voltage in Cavity 1, in the absence of Robinson oscillations, may be written as:

$$V_{T1}\cos(\omega_g t + \psi_1) = -R_1[i_{g1}\cos\phi_1\cos(\omega_g t - \theta_{f1} - \phi_1) + 2IF_1\cos\phi_1\cos(\omega_g t - \phi_1)], \tag{12}$$

where V_{T1} is the magnitude of the voltage in Cavity 1, and ψ_1 is the phase of the bunch centers, which equals zero for a bunch passing through the cavity at the peak positive voltage. For short bunches with $F_1 = 1$, this phase angle equals the synchronous phase angle used by Sands[6] and Wilson,[8] and is the complement of the angle ϕ_s used by some authors, for which zero phase corresponds to no energy gain by the bunch. An analogous equation holds for Cavity 2.

For the case of a passive second cavity, $i_{g2} = 0$, and $\psi_2 = \pi - \phi_2$. The equilibrium phase stability criterion may then be written as:

$$V_{T1} \sin \psi_1 > R_1 F_1 I \sin 2\phi_1 . \tag{13}$$

For $F_1 = 1$, this is identical to the result of Robinson[11] for a single-cavity system. Energy dissipation in a passive higher harmonic cavity affects stability by decreasing the synchronous phase angle, thereby lowering the current threshold for a given voltage and tuning angle in Cavity 1.

For an active second cavity, the equilibrium phase stability criterion may be written as

$$F_1 V_{T1} \sin \psi_1 + \nu F_2 V_{T2} \sin \psi_2 > R_1 F_1^2 I \sin 2\phi_1 + \nu R_2 F_2^2 I \sin 2\phi_2. \tag{14}$$

This expression has been previously given for short bunches whose form factors equal one.[7]

Robinson stability is provided by a positive value of α_R:

$$F_1^2 R_1 Q_1 \tan \phi_1 \cos^2 \phi_{1+} \cos^2 \phi_{1-} + F_2^2 R_2 Q_2 \tan \phi_2 \cos^2 \phi_{2+} \cos^2 \phi_{2-} > 0. \tag{15}$$

Typically, $R_2 < R_1$, $Q_2 < Q_1$, and $F_2 < F_1$, helping to provide stability for a higher harmonic cavity with a negative tuning angle. However, if the Robinson frequency Ω is large compared with $\omega_1/2Q_1$, or if ϕ_1 is small, the product $\tan \phi_1 \cos^2 \phi_{1+} \cos^2 \phi_{1-}$ may be small. In this case, the contribution towards Robinson damping from Cavity 1 will be small, so that Robinson instability may easily result from a negative tuning angle in the higher harmonic cavity.

If a passive higher-harmonic cavity is brought on line by slowly adjusting its tuning angle from a large positive or negative value, or if current is accumulated with a fixed tuning angle in the passive cavity, the equilibrium phase and Robinson instabilities may occur in an intermediate state, preventing the cavity from being brought on line. Thus, it is useful to consider the stability properties of the intermediate states as well as the intended final state.

The previous calculations yield two additional quantities. The analogue of equation (12) for Cavity 2 gives the peak voltage in a passive cavity as:

$$V_{T2} = 2IF_2 R_2 \cos \phi_2, \tag{16}$$

while the dissipated power is given by $V_{T2}^2/2R_2$.

3   THE SYNCHROTRON FREQUENCY, SYNCHROTRON FREQUENCY SPREAD, AND BUNCH LENGTH

In the absence of the equilibrium phase and Robinson instabilities, the voltage experienced by an electron from the cavity fields is given by equation (5) with $A = 0$. This can be written as:

$$V(t) = V_{T1} \cos(\omega_g t + \psi_1) + V_{T2} \cos(\nu \omega_g t + \psi_2). \tag{17}$$

Letting τ be the arrival time of an electron relative to the synchronous phase, the synchrotron motion obeys:

$$\frac{d^2\tau}{dt^2} = \frac{\alpha e}{ET_o}[V(\tau + t_s) - V_s], \tag{18}$$

where eV_s is the energy lost per electron while transiting the ring, and t_s is the time difference between the bunch center and the synchronous time, satisfying $V(t_s) = V_s$. The RHS of equation (18) can be written as the derivative of an effective potential $U(\tau)$, which can be expanded in powers of τ:

$$\frac{d^2\tau}{dt^2} = -\frac{dU}{d\tau} = -\frac{d}{d\tau}(a\tau^2 + b\tau^3 + c\tau^4 + \dots). \tag{19}$$

The coefficients obey:

$$\frac{\omega_s^2}{2} = a = \frac{\alpha e \omega_g}{2ET_o}(V_{T1} \sin \psi_{1s} + \nu V_{T2} \sin \psi_{2s}), \tag{20}$$

$$b = \frac{\alpha e \omega_g^2}{6ET_o}(V_{T1} \cos \psi_{1s} + \nu^2 V_{T2} \cos \psi_{2s}), \text{ and} \tag{21}$$

$$c = -\frac{\alpha e \omega_g^3}{24ET_o}(V_{T1} \sin \psi_{1s} + \nu^3 V_{T2} \sin \psi_{2s}), \tag{22}$$

where $\psi_{1s} = \psi_1 + \omega_g t_s$, and $\psi_{2s} = \psi_2 + \nu \omega_g t_s$ are the synchronous phase angles, and ω_s is the synchrotron frequency when a is positive.

We first consider the case where a is positive and the potential well is approximately quadratic in the region occupied by the bunch. Perturbed harmonic motion in the above potential well may be represented by sinusoidal motion about a nonzero equilibrium value of τ, plus higher harmonic terms. Substituting this form into the equation of motion and keeping the lowest order terms gives the frequency as a function of the amplitude τ_s:

$$\omega^2 = \omega_s^2\{1 + \tau_s^2[\frac{3c}{2a} - (\frac{3b}{2a})^2]\}. \tag{23}$$

Thus, for a Gaussian energy distribution,[9] the synchrotron frequency spread obeys:

$$\sigma_{\omega_s} = \omega_s \sigma_t^2 \mid \frac{3c}{2a} - (\frac{3b}{2a})^2 \mid = \frac{\alpha^2 (\sigma_E/E)^2}{\omega_s} \mid \frac{3c}{2a} - (\frac{3b}{2a})^2 \mid . \tag{24}$$

Here, σ_E is the energy deviation of the bunch, and σ_t the rms bunch length. In the absence of a second RF cavity, this reduces to:

$$\sigma_{\omega_s} = \omega_s (\omega_g \sigma_t)^2 (\frac{1}{8} + \frac{1}{4} \cot^2 \psi_{1s}). \tag{25}$$

For the case where V_s is a small fraction of the RF voltage, the $\cot^2 \psi_{1s}$ term can be ignored, giving the "octupolar" approximation for the synchrotron frequency spread.

The above results are accurate when the synchrotron frequency spread is small compared to the synchrotron frequency. In this case, when evaluating the coefficients a, b, and c, to neglect the difference between ψ_{1s} and ψ_1, and the difference between ψ_{2s} and ψ_2 will not cause significant error.

The above formulas allow a calculation of the synchrotron frequency — and thereby the bunch length — for a system with a higher harmonic RF cavity. The growth rate and coherent frequency shift from the coupled-bunch instability can also be estimated with the knowledge of the synchrotron frequency and impedance of the RF cavity parasitic modes. The resonant dipole interaction of short bunches with a longitudinal cavity mode of impedance $Z(\omega_{CB})$ at frequency $\omega_{C.B.}$ is:[1]

$$\mid \Delta \omega \mid^{C.B.} = \frac{eI\alpha\omega_{C.B.}Z(\omega_{C.B.})F^2_{\omega_{C.B.}}}{2ET_o\omega_s}, \tag{26}$$

where $F_{\omega_{C.B.}}$ is the bunch form factor at the frequency $\omega_{C.B.}$. Because of the large number of parasitic modes in an RF cavity, resonant or near-resonant dipole interaction may be a probable occurence in a typical storage ring with multiple bunches.[12] In addition, the variation in parasitic mode frequencies with cavity tuning angle and temperature, as well as variations in storage ring parameters, may result in occasional or frequent resonant interactions even when the instantaneous chance of resonant interaction is low. Radiation damping is expected to reduce the resonant frequency shift by the longitudinal radiation damping rate.

The resonant frequency shift may be compared with the calculated synchrotron frequency spread to determine whether Landau damping is sufficient to ensure stability. For example,[13] a growing dipole mode in a Gaussian bunch with rms bunch length shorter than approximately one-sixth the cutoff period will be Landau-damped when the magnitude of the coherent frequency shift is less than $0.78\sigma_{\omega_s}$. Experimental thresholds for coupled-bunch instability are consistent with this instability criterion;[14,15] an experiment in which radiation damping is expected to exceed Landau damping at threshold also supports the validity of equation (26) for estimating coupled-bunch instability growth.[4]

When operating the second RF cavity so that $a = 0$, the potential well occupied by the bunch is not quadratic. This potential well is given by:

$$U(\tau) = -\frac{\alpha e}{ET_o} \int_{t_s}^{t_s+\tau} [V(t') - V_s]dt' \tag{27}$$

and will be occupied to the filling height $U_o = \frac{\alpha^2}{2}(\frac{\sigma_E}{E})^2$. From equation (27), one can estimate the bunch length σ_t in the non-quadratic well. The typical oscillation frequency obeys:

$$\omega_{typical} \approx \frac{\alpha}{\sigma_t}\frac{\sigma_E}{E} = \frac{1}{\sigma_t}\sqrt{2U_o}. \tag{28}$$

Because the confining potential is provided by the $c\tau^4$ term, we use the following results for a biquadratic well:[3]

$$\sigma_t = 0.69(\frac{U_o}{c})^{1/4} \tag{29}$$

and:[16]

$$\omega_{\sigma_t} = 1.17(cU_o)^{1/4}, \tag{30}$$

where ω_{σ_t} is the angular frequency of synchrotron oscillations of amplitude σ_t. In the biquadratic well, the angular frequency of synchrotron oscillations is proportional to their amplitude.

With a biquadratic well, the longitudinal coupled bunch instability is most easily driven at a frequency near $1.7\omega_{\sigma_t}$. Landau damping is overcome[17] when the coherent frequency shift (calculated for perturbed synchrotron oscillations with $\omega_s = 1.7\omega_{\sigma_t}$ in equation 26) has magnitude greater than $0.3\omega_{\sigma_t}$. (Chin[16] obtains a similar criterion of $0.2\omega_{\sigma_t}$.) The formulas for a biquadratic well can be used to estimate the behavior when the potential well is not approximately quadratic. Because the bunch may be quite asymmetric in a nonquadratic well (for $b \neq 0$), the Robinson frequency and damping rates calculated earlier are approximate.

4   AN ALGORITHM FOR ANALYSIS OF A PASSIVE HIGHER-HARMONIC CAVITY

For beam current levels high enough to make the wakefields from the beam comparable to the fields provided by the generator current, a passive higher harmonic cavity is capable of significantly modifying the beam behavior. We consider a ring with a passive higher-harmonic cavity in which the RF voltage of Cavity 1 is maintained at a specified value, and Cavity 1 is operated in the "compensated condition",[6] in which the tuning angle is adjusted so that the generator current is in phase with the voltage. Alternatively, the tuning angle of Cavity 1 may be specified. We neglect the possibility of turbulent bunch-lengthening from the microwave instability and restrict our analysis to the case where the single bunch currents are so small that turbulent bunch-lengthening is not a concern.

As input to the analysis, values for the following variables must be specified:

V_{T1} : the peak RF voltage in Cavity 1 (or effective voltage if there are multiple fundamental cavities or a finite transit time correction).

Q_1^o : the unloaded quality factor of Cavity 1.

R_1^o: the unloaded impedance of Cavity 1 at resonance (or effective impedance if there are multiple fundamental cavities).

β : the RF coupling coefficient for Cavity 1.

α : the momentum compaction.

T_o : the revolution period.

ω_g : the generator angular frequency.

E : the electron energy.

σ_E : the electron energy spread resulting from synchrotron radiation emission.

I : the average beam current magnitude.

V_s: the synchronous voltage.

ν : the harmonic number of Cavity 2.

Q_2 : the quality factor of Cavity 2.

R_2 : the impedance at resonance of Cavity 2.

ϕ_2 : the tuning angle of Cavity 2.

τ_L : the longitudinal radiation damping time.

$Z(\omega_{C.B.})$: the parasitic impedance driving coupled bunch oscillations.

$\omega_{C.B.}$: the frequency of the parasitic mode.

The calculated bunch length depends upon the form factors F_1 and F_2, which are themselves functions of bunch length. We initially let $F_1 = 1$ and $F_2 = 0.1$ and proceed until the bunch length is determined. We then calculate the form factors from the bunch length. If the newly calculated form factors differ significantly from their previous values, we take a weighted average (0.9 times the previous form factor plus 0.1 times the most recently calculated form factor), and repeat the process. In this way, the influence of the higher harmonic cavity on the computed dynamics is gradually increased until the bunch length converges.

Our algorithm proceeds as follows.

Step 1. Calculate ψ_1, the phase angle of the bunch center, which is given by equation (8) for the case of a passive higher harmonic cavity:

$$V_s = F_1 V_{T1} \cos \psi_1 - 2I R_2 F_2^2 \cos^2 \phi_2. \tag{31}$$

If this equation can only be solved with $| \cos \psi_1 | > 1$, then there is no possible equilibrium phase of the bunch in Cavity 1. This occurs when the sum of synchrotron

radiation losses and losses to Cavity 2 exceeds the possible energy gain from Cavity 1. If so, discontinue the calculation.

Step 2. Calculate the tuning angle of Cavity 1 for operation in the compensated condition:[6]

$$\tan \phi_1 = \frac{2F_1 I R_1}{V_{T1}} \sin \psi_1. \tag{32}$$

Alternatively, this tuning angle can be specified if the compensated condition is not utilized.

Step 3. Calculate the coefficients a, b, and c, neglecting the phase difference between the bunch center and the synchronous phase. For a passive higher harmonic cavity, the values are:

$$a = \frac{\alpha e \omega_g}{2ET_o} (V_{T1} \sin \psi_1 + \nu I F_2 R_2 \sin 2\phi_2), \tag{33}$$

$$b = \frac{\alpha e \omega_g^2}{6ET_o} (V_{T1} \cos \psi_1 - 2\nu^2 I F_2 R_2 \cos^2 \phi_2), \quad \text{and} \tag{34}$$

$$c = -\frac{\alpha e \omega_g^3}{24ET_o} (V_{T1} \sin \psi_1 + \nu^3 I F_2 R_2 \sin 2\phi_2). \tag{35}$$

Step 4. a) If a is positive and $c < 0.45[a/(\alpha \sigma_E/E)]^2$, then the synchrotron confining potential is mostly quadratic. The synchrotron frequency and bunch length obey:

$$\omega_s = \sqrt{2a} \quad \text{and} \tag{36}$$

$$\sigma_t = \frac{\alpha \sigma_E}{E \omega_s}. \tag{37}$$

The synchrotron frequency spread follows from equation (24).
  b) Otherwise, the confining potential is mostly biquadratic so that:

$$\sigma_t = 0.69(\frac{U_o}{c})^{1/4} \quad \text{and} \tag{38}$$

$$\omega_{\sigma_t} = 1.17(cU_o)^{1/4}, \tag{39}$$

where $U_o = \frac{\alpha^2}{2}(\frac{\sigma_E}{E})^2$. The most unstable frequency is $1.72\omega_{\sigma_t}$.
  We have chosen the dividing line between quadratic and nonquadratic potentials so that the bunch lengths determined by equations (37) and (38) are equal, stabilizing the iteration of bunch length.

Step 5. Use the bunch lengths to estimate the form factors: $F_1 = \exp(-\omega_g^2 \sigma_t^2/2)$ and $F_2 = \exp(-\nu^2 \omega_g^2 \sigma_t^2/2)$.

Repeat steps 1–5 if the form factors differ greatly from the previous input values. For new input values, use a weighted average of the two most recent calculations of the form factors.

After several iterations of steps 1–5, we have quantities calculated using form factors that are consistent with the bunch length.

Step 6. Determine if the dipole longitudinal coupled bunch instability is damped. For a mostly quadratic synchrotron potential, the coherent frequency shift is given by equation (26). To account for radiation damping, τ_L^{-1} is subtracted from this frequency shift. If the resulting frequency shift is less than $0.78\sigma_{\omega_s}$, Landau damping will prevent growth. For the case of a nonquadratic synchrotron potential, eq. (26) can be used with the most unstable frequency $1.72\omega_{\sigma_t}$ in place of ω_s. Landau damping is sufficient to prevent the coupled bunch instability growth if the coherent frequency shift is less than $0.3\omega_{\sigma_t}$.

At the chosen dividing line between the quadratic and nonquadratic regimes, the bunch lengths calculated in the two regimes are equal, while the most unstable frequency and coherent frequency shift are equal within 2 percent. However, the calculated Landau damping rates for the quadratic and non-quadratic regimes may be discontinuous across the dividing line. This is a result of the inaccuracy in using estimates for almost-quadratic and purely biquadratic potentials in the two regimes.

Step 7. Determine if the equilibrium phase instability will occur. Stability is assured if:

$$F_1 I < \frac{V_{T1} \sin \psi_1}{R_1 \sin 2\phi_1}. \tag{40}$$

Step 8. If the previous inequality is satisfied, calculate the Robinson frequency:

$$\Omega^2 = \frac{e\alpha\omega_g}{T_o E} \{ F_1 V_{T1} \sin \psi_1 - \frac{R_1 F_1^2 I}{2} (\sin 2\phi_{1-} + \sin 2\phi_{1+})$$

$$+ \nu R_2 F_2^2 I \sin 2\phi_2 - \frac{\nu R_2 F_2^2 I}{2} (\sin 2\phi_{2-} + \sin 2\phi_{2+}) \}. \tag{41}$$

This calculation requires iteration. One can start by evaluating the RHS with zero beam current, and then iterate using a weighted average of the most recently computed value of Ω and the previously computed value.

Step 9. Once the Robinson frequency is known, the Robinson damping rate can be calculated; a positive value gives stability:

$$\alpha_R = \frac{4\alpha e I}{E T_o} [F_1^2 R_1 Q_1 \tan \phi_1 \cos^2 \phi_{1+} \cos^2 \phi_{1-} + F_2^2 R_2 Q_2 \tan \phi_2 \cos^2 \phi_{2+} \cos^2 \phi_{2-}]. \tag{42}$$

The desirable tuning angles ϕ_2 are those which provide stability against the Robinson, equilibrium phase, and coupled bunch instabilities. It may be preferable to have a positive value for the coefficient a in order to avoid a double-peaked bunch shape. We have found it convenient to evaluate the expected behavior of a higher harmonic cavity by performing the above calculation for a sequence of values of ring current and tuning angle. In iterated calculations, the iteration is concluded and a flag is set if convergence does not occur within a reasonable number (~ 500) of iterations.

Generally, there is uncertainty in the input value of the parasitic mode impedance, which changes with tuning angle, beam current, temperature, etc. The uncertainty in this input parameter leads to uncertainty in the calculated threshold for coupled-bunch instability. The calculated bunch lengths in the presence of the coupled-bunch instability do not include any lengthening that may result from the instability.

5   APPLICATION TO THE STORAGE RINGS AT MAX-LAB AND BESSY

As an application of our algorithm, we considered experiments with a third-harmonic cavity at the 500-MeV storage ring MAX-lab in Lund, Sweden. With a Cavity 1 effective voltage of ~ 65 kV and $\beta = 3$, the coupled-bunch instability was suppressed in experiments with negative tuning angles (bunch-lengthening) in a passive third-harmonic cavity for currents of 80–190 mA, and with positive tuning angles for smaller currents. Stable operation required that the Cavity 1 tuning angle be positive relative to the compensated condition. With a Cavity 1 voltage of 180 kV and $\beta = 2$, use of the third-harmonic cavity was unsuccessful, and observations suggested that this was a result of the Robinson instability.[14]

We first modeled a worst-case scenario: resonant interaction with a parasitic impedance of 0.2 MΩ and resonant frequency 1.5 times the fundamental frequency, corresponding to an unattenuated parasitic cavity mode.[12,18,19] We found that the third-harmonic cavity could not suppress the coupled-bunch instability in this case. However, the fundamental cavity at MAX-lab has spurious-mode attenuation, which results in order-of-magnitude reduction of about 80% of the parasitic modes. The resonance width of an attenuated mode will also be increased by an order of magnitude, so that coupled-bunch instability driven by an attenuated mode is about 40 times as likely as that from an unattenuated mode. Thus, we next considered interaction with an attenuated impedance of 0.02 MΩ, which is more likely to correspond to the experimental conditions.

In Fig. 1(a), a stability plot is shown for operation of the fundamental cavity at 65 kV, with the Cavity 1 tuning angle 3 degrees positive of compensated condition. The remaining parameters are shown in Table 1. In the absence of the passive cavity, the coupled-bunch instability is present for all currents shown, as indicated by the results for passive cavity tuning angles of $\pm 90°$. For currents of 40–200 mA, stable operation is shown when the tuning angle of the passive cavity is negative. For currents of 70 mA or less, stable operation occurs with a positive tuning angle. For currents exceeding 40 mA, passive cavity tuning angles near $0°$ are not possible because the equilibrium phase is unstable or does not exist.

TABLE 1: Parameters for Figure 1

	Figure 1(a)	Figure 1(b)	Figure 1(c)	Figure 1(d)
storage ring	MAXlab	SRRC	SRRC	SRRC
E	500 MeV	1.3 GeV	1.3 GeV	1.3 GeV
σ_E/E	4×10^{-4}	6.6×10^{-4}	6.6×10^{-4}	6.6×10^{-4}
V_s	5 keV	73 keV	73 keV	73 keV
T_o	1.1×10^{-7} s	4.0×10^{-7} s	4.0×10^{-7} s	4.0×10^{-7} s
α	0.04	6.8×10^{-3}	6.8×10^{-3}	6.8×10^{-3}
ω_g	3.14 GHz	3.14 GHz	3.14 GHz	3.14 GHz
R_1^o(unloaded)	3.5 MΩ	6 MΩ	6 MΩ	6 MΩ
Q_1^o(unloaded)	40,000	37,000	37,000	37,000
β	3	1.24	1.24	1.24
V_{T1}	65 kV	800 kV	800 kV	800 kV
ν	3	3	3	3
R_2	0.6 MΩ	1.2 MΩ	0.6 MΩ	1.2 MΩ
Q_2	10,000	10,000	10,000	10,000
$Z(\omega_{C.B.})$	0.02 MΩ	0.2 MΩ	0.02 MΩ	0.02 MΩ
$\omega_{C.B.}$	4.8 MHz	4.8 MHz	4.8 MHz	4.8 MHz
τ_L	50 ms	8.7 ms	8.7 ms	8.7 ms

Similar results were obtained when the Cavity 1 tuning angle was 0–10° positive of the compensated condition. When the Cavity 1 tuning angle is more than 1° negative of the compensated condition, the equilibrium phase instability prevents stable operation for currents exceeding 100 mA. These results indicate that the compensated condition is marginally stable; the Cavity 1 tuning angle must be positive relative to the compensated condition to ensure stable operation at the higher currents utilized.

We also considered a Cavity 1 voltage of 180 kV and β of 2. For currents less than 200 mA, the Robinson instability onsets as the cavity tuning angle is increased from $-90°$, before the coupled bunch instability is stabilized. This may prevent tuning in the cavity to the small region of stable negative tuning angles indicated for currents exceeding 150 mA, precluding successful use at MAX-lab.

Comparing the results of the analysis with the experimental observations at MAX-lab, we note excellent qualitative and good quantitative agreement when we assume

that the coupled-bunch instability results from a suppressed parasitic RF cavity mode
with an impedance of 0.02 MΩ. The analysis predicts the approximate conditions
where the passive cavity can be successfully employed.

We then considered parameters corresponding to an experiment with a passive
eighth-harmonic cavity at BESSY.[4] In this experiment, a passive cavity was success-
ful up to a current of 220 mA. In modeling this case, coupled-bunch modes driven by
the higher harmonic cavity appeared the most difficult to suppress. Thus, we assumed
that the coupled bunch instability was driven by an attenuated impedance of 0.02 MΩ
at one and one half times the higher harmonic resonant frequency. The analysis al-
gorithm predicted stable operation in the "bunch lengthening" mode with negative
tuning angle for currents of 0–160 mA. With a parasitic impedance that is 20% lower
(0.016 MΩ), stable operation is predicted for currents of 0–220 mA, identical to the
experimental results. This comparison with BESSY gives excellent qualitative agree-
ment, and demonstrates that precise quantitative agreement is dependent upon the
assumed value of the parasitic impedance driving the instability. Because precise val-
ues of this quantity are usually not available, one expects uncertainty in the predicted
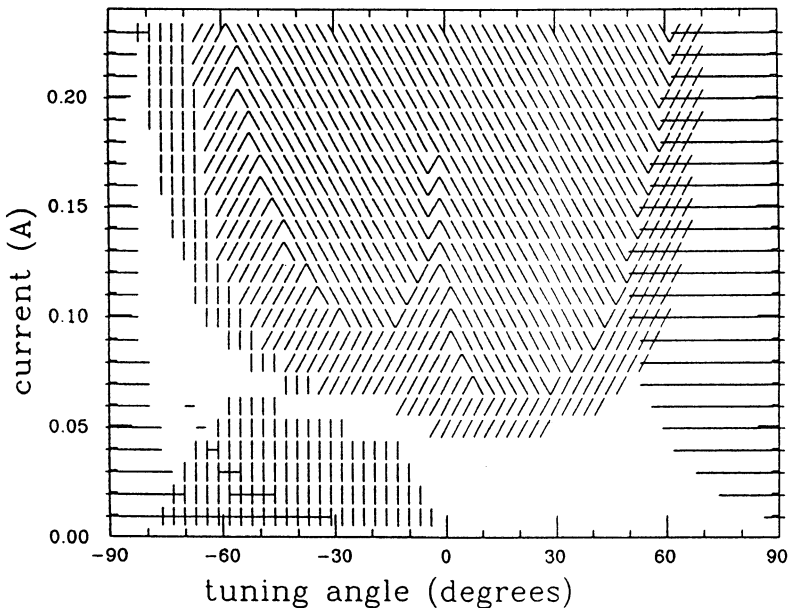range of stable operation.

FIGURE 1(a): Stability plot for MAX-lab with the fundamental cavity 3° positive of the compensated
condition. Instabilities are predicted for a range of storage ring currents and tuning angles of a third
harmonic cavity. – : coupled-bunch instability. | : Robinson instability. / : equilibrium phase instability. \ :
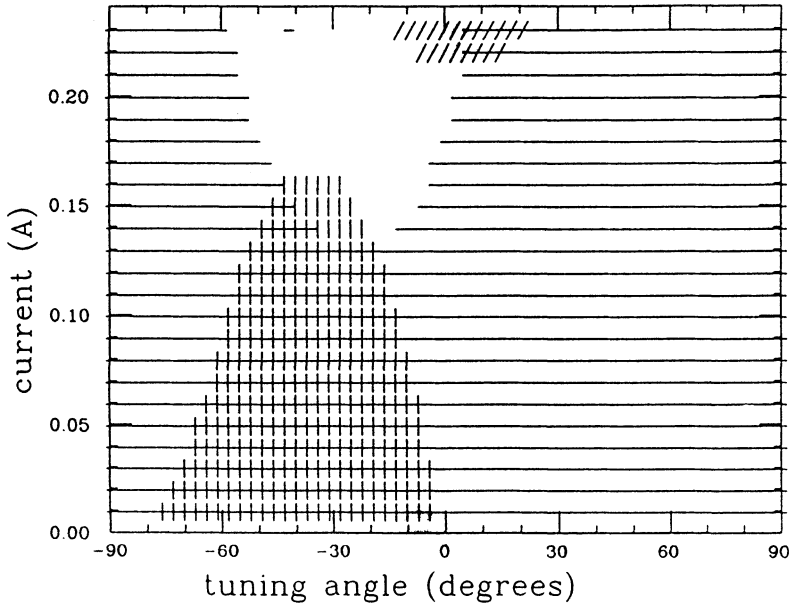there is no equilibrium phase. Stable operation is indicated by the absence of these symbols.

FIGURE 1(b): Stability plot for SRRC with an unattenuated parasitic mode and two third-harmonic cavities. See Fig. 1(a) for an explanation of symbols.
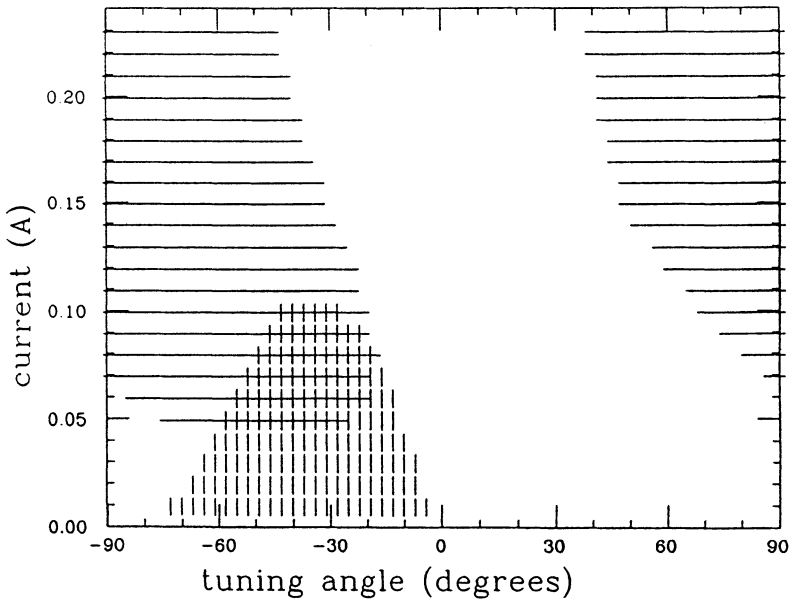

FIGURE 1(c): Stability plot for SRRC with an attenuated parasitic mode and one third-harmonic cavity. See Fig. 1(a) for an explanation of symbols.
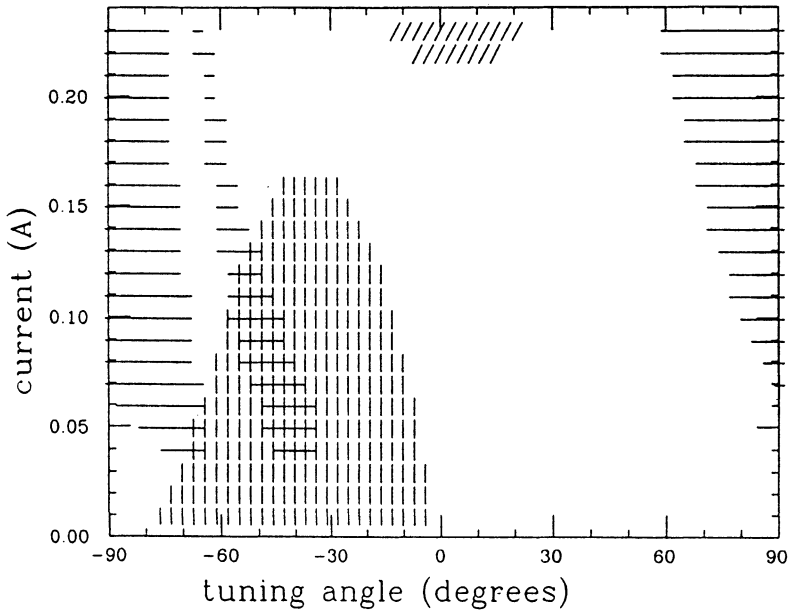
FIGURE 1(d): Stability plot for SRRC with an attenuated parasitic mode and two third-harmonic cavities. See Fig. 1(a) for an explanation of symbols.

6 APPLICATION TO THE SRRC STORAGE RING

In the 1.3 GeV storage ring under construction at the Synchrotron Radiation Research Center (SRRC), the coupled bunch instability is expected.[12,18,19] It is thus desirable to determine if a passive higher harmonic cavity is capable of preventing this instability. Because the beam pipe aperture and RF frequency of SRRC are nearly the same as those of MAX-lab, we expect that a third harmonic cavity at SRRC would have properties similar to those of MAX-lab; therefore we used the same values of impedance and quality factor. The parameters used are shown in Table 1. We considered a Cavity 1 voltage of 800 keV, with Cavity 1 operated in the compensated condition so that its voltage is in phase with the generator current. The loading of the fundamental is provided by an RF coupling coefficient, β, equalling 1.24, which is near the optimum for full current (200 mA) operation.[8] We first considered a worst-case scenario: resonant interaction with an unattenuated cavity mode whose impedance is 0.2 MΩ[12]. With a single third-harmonic cavity in the ring, the coupled bunch instability could not be suppressed. We then considered two such cavities operated with the same tuning angle, effectively doubling the impedance of the passive cavity. The results of our stability analysis for two third harmonic cavities are shown in Fig. 1(b). Stable operation is possible at currents comparable to the desired maximum current of 200 mA.

At lower current levels, the wakefields in the passive cavity are too small to suppress coupled-bunch instabilities. However, operation of the passive cavity with a negative tuning angle can still cause the Robinson instability, which may be a problem if electrons are accumulated with the passive cavity on line. If the Cavity 1 voltage is reduced from 800 kV to 600 kV, and β increased to 1.43 (its optimum value for 600 kV and a 200 mA ring current), stability can be achieved for a larger current range of 100-200 mA.

Next, we considered a parasitic impedance of 0.02 MΩ, corresponding to a mode in the fundamental cavity which is attenuated by an order of magnitude. Based upon our modeling of the MAX-lab and BESSY experiments, we expect that this will better represent instability caused by a fundamental cavity with spurious mode attenuation. Results are shown in Fig. 1(c) for the case of a single passive cavity. For a ring current of 60 mA or less, the combination of radiation and Landau damping is sufficient to prevent the coupled bunch instability in the absence of a higher harmonic cavity, as shown by the results for passive cavity tuning angles of $\pm 90°$. With a single third-harmonic cavity, the coupled-bunch instability can be suppressed at all currents up to the desired maximum current of 200 mA.

Two third-harmonic cavities with the same tuning angle were also predicted to be effective, as shown in Fig. 1(d). As the passive cavity tuning angle is increased from $-90°$, the coupled bunch instability is expected to disappear, reappear, and disappear again. The reappearance occurs when $(3b/2a)^2$ becomes larger than $3c/2a$; where these two quantities are nearly equal, the Landau damping rate is small according to equation (24). The SRRC instability behavior was little changed for Cavity 1 tuning angles slightly negative or positive of the compensated condition, unlike the MAX-lab results. Thus, we expect that one or two passive higher-harmonic cavities may be utilized at SRRC while Cavity 1 is operated in the compensated condition.

In Figure 2, we consider the same parameters as Figs. 1(c) and 1(d), and show the bunch length as a function of passive cavity tuning angle, for a ring current of 200 mA. The calculated bunch lengths in the presence of the coupled-bunch instability do not include any increase that may result from the instability. For stable operation, the bunch length can be modified in the range of 21–35 picoseconds with a single passive cavity, and from 18–60 picoseconds with two passive cavities. We estimate that the calculated Touschek lifetime[2] is proportional to the bunch length within about 10%.

With a current of 200 mA and an attenuated parasitic mode, stabilization of the coupled-bunch instability with a single passive cavity requires a tuning angle magnitude less than about 40°. According to equation (16), the effective RF voltage of the passive cavity will then exceed 175 kV while the power dissipation exceeds 25 kW. If this power cannot be effectively dissipated, one may use two passive cavities with tuning angles of $-60°$ and effective RF voltages of 110 kV for stable operation, reducing the power dissipation to about 10 kW per cavity. If a tuning angle of $-42°$ is employed to maximize the bunch length with two passive cavities, each passive cavity will have an effective RF voltage of 150 kV and 20 kW power dissipation. By using two passive cavities, the Touschek lifetime may be substantially increased, and stable operation is expected in the worst-case scenario where coupled-bunch interaction results from an unattenuated parasitic cavity mode.
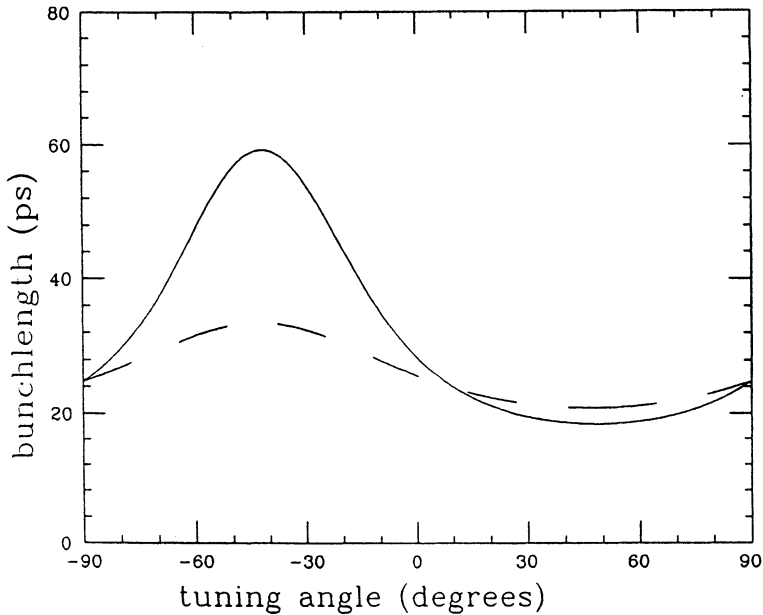
FIGURE 2: Bunch length versus tuning angle for a 200 mA ring current at SRRC. The dashed line shows the results of a single third-harmonic cavity, while the solid line describes two third-harmonic cavities.

7 SUMMARY

Formulas to determine the presence or absence of the equilibrium phase, Robinson, and coupled-bunch instabilities were presented. These results were incorporated into an algorithm to analyze the behavior of a storage ring with a passive higher harmonic cavity. Comparisons with the results of experiments at MAX-lab and BESSY suggest that our model gives quite good predictions of stability.

For the electron storage ring at SRRC, our results support the feasibility of using one or two passive third harmonic cavities to suppress coupled-bunch instabilities. Two cavities may be used to substantially increase the bunch length and Touschek lifetime. The results suggest that passive higher harmonic cavities may find application in current generation electron storage rings. The algorithm described above may be used to evaluate the performance of such cavities.

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