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BEAM LOADING OF AN ACTIVE RF CAVITY

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Electron beam loading of a single-pass active cavity coupled with an external generator is considered in a fixed current approximation. The loading effects can be controlled by proper choice of a generalized scattering matrix describing the feeder-cavity coupler. By varying phase shifts at the coupler and its transparency, one can minimize the backward wave, which is a coherent sum of a reflected wave and a beam radiation field. Thus, the accelerating field and/or the limiting current can be increased. Moreover, the phase shift optimization can sustain efficient acceleration over a wide range of loading current variations. The same approach can be useful for lasertron systems and for autoacceleration regimes.

KEY WORDS: RF cavity, coupler, beam loading

1 INTRODUCTION

Optimization of various resonant linear accelerators loaded with intense electron beams has been considered in many publications (see, for example,[1,2]). Here we shall investigate a particular case of a standing wave structure formed by a chain of electrodynamically independent single-gap cavities with external phasing. The system provides additional control flexibility for variations of an accelerated current and is of special interest for free electron laser drivers, as has been described in[3].

A general approach to single-cavity matching can be formulated as minimization of power reflected from the coupler back to the generator. For ideal coupling of an unloaded cavity, i.e. for a vanishing backward wave amplitude, the load impedance from the feeder side must be real. Besides, an optimal coupling coefficient must be provided to transmit the power being dissipated inside the cavity. Then, the load impedance will be equal to the wave impedance of the feeder.

If the cavity is loaded with an electron beam current of the same frequency, there appears an additional field source, which provides coherent radiation that changes the phases and amplitudes of the backward wave and the total accelerating field. The total accelerating voltage across the gap decreases (beam-loading effect) of course, and ideal matching, meaning a zero amplitude backward wave, is generally impossible. However, the radiative field remains coherent with the reflected wave, so the coupler parameters can be chosen to minimize the total backward wave amplitude and the beam loading effect. In other words, proper variation of independent coupler parameters as functions of the current can sustain the total accelerating field and/or increase the limiting current for which acceleration vanishes. Note that the load impedance of the cavity is not necessarily real in this case, and corresponding phase shifts which can be provided by proper dielectric inserts or by other distributed elements are of importance. A similar approach can be used for radiating systems in lasertrons or choppertrons to maximize the power irradiated by the beam.

2 CAVITY DESCRIPTION

For present purposes, the resonator can be treated as a device that transforms the electric field wave coming from the generator into standing oscillations of the field in the accelerating gap. Without specifying its geometry and assuming only that the dimension of the gap is considerably less than the distance from the gap to the coupler, one can consider the system as a succession of two linear transformations described by the scattering matrixes **S** and **C**. The former refers to the coupler and the latter is related to the finite distance between the coupler and accelerating gap, as schematically shown in Fig.1. Denoting the complex amplitudes of the field on the external and internal sides of the coupler by the indexes 0 and 1, respectively, and the directions of propagation from and to the generator by the upper indexes $\pm$, one can write from the definition of scattering matrix:

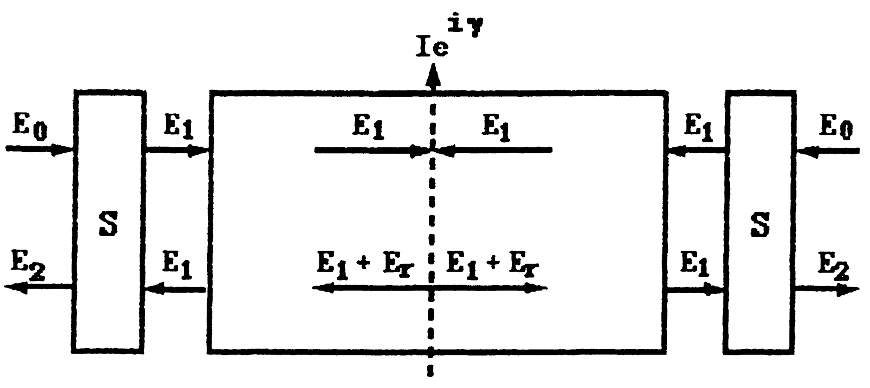

FIGURE 1: A model of an active cavity.

$$\begin{pmatrix} E_0^- \\ E_1^+ \end{pmatrix} = \mathbf{S} \begin{pmatrix} E_0^+ \\ E_1^- \end{pmatrix}.$$

If losses in the coupler are neglected and the transformation is considered reciprocal, the matrix $\mathbf{S}$ should be unitary and symmetrical, which yields four relations between its complex elements:

$$\begin{gathered} S_{11}S_{11}^* + S_{21}S_{21}^* = 1, \\ S_{12}S_{12}^* + S_{22}S_{22}^* = 1, \\ S_{11}^*S_{12} + S_{22}S_{21}^* = 0, \\ S_{12} - S_{21} = 0 \end{gathered} \tag{1}$$

and leaves free only three real parameters:

$$\mathbf{S} = \begin{pmatrix} -\chi \ \exp(i\alpha) & k \\ k & \chi \ \exp(-i\alpha) \end{pmatrix} \exp(i\mu), \tag{2}$$

$$0 < k < 1.$$

Here, $k = \sqrt{1 - \chi^2}$ is the coupling coefficient, μ the phase shift of the transmitted wave and $\mu - \alpha$ the phase shift of the reflected wave. The transformation matrix of the resonator itself should take into account the phase shift δ during propagation of the wave from the coupler to the gap (dependence of the field on time is chosen $\sim e^{-i\omega t}$), irreversible ohmic damping of amplitude described by the coefficient $q < 1$, and reversible coefficient of amplitude transformation from coupler to gap G. In general form

$$\begin{pmatrix} E_1^- \\ E_2^+ \end{pmatrix} = \mathbf{C} \begin{pmatrix} E_1^+ \\ E_2^- \end{pmatrix}, \tag{3}$$

where

$$\mathbf{C} = q \begin{pmatrix} 0 & G^{-1} \exp(i\delta) \\ G \ \exp(i\delta) & 0 \end{pmatrix}.$$

There still remains to relate the amplitudes of waves $E_2^{\pm}$ leaving and coming to the gap (Fig.1). In the absence of a beam, the assumptions of standing oscillations in the gap would mean automatically that $E_2^+ = E_2^-$, which in the presence of losses could be realized physically for cophasal excitation of the resonator by two couplers symmetrically located relative to the gap. To avoid complicated calculations, we limit ourselves to the case of symmetrical excitation.

A modulated beam passing through the gap in the plane of symmetry creates an additional field of radiation E_r coherent with the other fields and propagating from the gap to the couplers. Therefore,

$$E_2^- = E_2^+ + E_r \tag{4}$$

and E_a, the total accelerating field in the gap will be

$$E_a = E_2^+ + E_2^- = 2E_2^+ + E_r \tag{5}$$

Using the algebraic linear relations (3) - (5), we now can express the amplitude of the wave leaving the resonator E_0^- and the accelerating field E_a in terms of the amplitude of the radiation field E_r and the amplitude of the incident wave E_0^+, which we shall consider real for determining the beginning of phase reckoning. Thus, we obtain

$$E_0^- = Z_0^- E_0^+ + \frac{Z_0^+ E_r}{2G}; \quad E_a = Z_0^+ E_0^+ G + Z_r E_r \tag{6}$$

where

$$Z_0^+ = \frac{2qk\ \exp(i\mu + i\delta)}{1 - \chi q^2 \exp(i\Delta)},$$
$$Z_0^- = \frac{\left[q^2 \exp(i\Delta) - \chi\right] \exp(i\mu + i\alpha)}{1 - \chi q^2 \exp(i\Delta)}, \tag{7}$$
$$Z_r = \frac{1 + \chi q^2 \exp(i\Delta)}{1 - \chi q^2 \exp(i\Delta)},$$
$$\Delta = 2\delta - \alpha + \mu.$$

Returning to what has been discussed above one can see that in the absence of the loading current the standing wave coefficient $Z = (E_0^+ + E_0^-)/(E_0^+ - E_0^-)$ is equal to unity when $Z_0^- = 0$. The latter means that two requirements mentioned above:

$$2\delta + \mu - \alpha = 0; \quad k^2 = 1 - q^4 \tag{8}$$

must be satisfied. The first shows that the cavity is tuned, and the second indicates that the coupling coefficient must be small for small ohmic losses.

The coefficient of ohmic damping q introduced above is closely related, of course, to the inherent Q-factor of the resonator. To elucidate this relationship, we note that the square of the modulus of the resonance denominator in the expressions for impedances (7) can be written in the form

$$|1 - \chi q^2 \exp(i\Delta)|^2 = (1 - \chi q^2)^2 + 4\chi q^2 \sin^2(\Delta/2) \tag{9}$$

and becomes small for $\chi q^2 \to 1$, $\Delta \to 0$. The total phase shift Δ is, of course, a function of the frequency ω and becomes zero for $\omega = \omega_0$, where ω_0 is the fundamental frequency of the operating mode. In the neighborhood of this resonance value,

$$|1 - \chi q^2 \exp(i\Delta)|^2 \approx \frac{D^2}{4}\left[4\left(\frac{\omega - \omega_0}{\omega_0}\right)^2 + Q_0^{-2}\right] \tag{10}$$

where

$$D = \omega_0 \left| \frac{\partial \Delta}{\partial \omega} \right|_{\omega_0} .$$

The quantity Q_0

$$Q_0 = \frac{D}{2(\chi^{-1/2}q^{-1} - \chi^{1/2}q)} \tag{11}$$

can be identified as a quality factor of the resonator, determined by the width of the resonance curve. In the limit, $k \to 0$; $q \to 1$

$$Q_0^{-1} = D^{-1}[k^2 + (1 - q^4)] \tag{12}$$

where the two r.h.s. terms determine the Q-factors connected with extraction of power and with ohmic losses, respectively. The coefficient of proportionality D^{-1}, the value of which is of the order of unity, depends, of course, on the concrete geometry of the resonator.

3 OPTIMIZATION OF THE ACCELERATING FIELD

To determine the power transmitted to the beam, one has to know a relation between the current $Ie^{i\gamma}$ and the radiation field E_r. For a narrow accelerating gap, one can suppose that $E_r = -\rho I e^{i\gamma}$, i.e., the field is out of phase with the current, exciting a TEM radiation wave in the line with wave impedance ρ. Then, the power is

$$W = Re(E_a I e^{-i\gamma}) = I\ Re(GZ_0^+ E_0^+ e^{-i\gamma} - Z_r \rho I). \tag{13}$$

One can see that an effective accelerating voltage $V_0 = W/IGE_0^+$ (in units of E_0^+) can be optimized with variation of the injection phase γ to match it with the phase of the effective accelerating field. Then

$$V_0 = Z_0^+ - Re(Z_r)J = 2qk\left[(1-\chi q^2)^2 + 4\chi q^2 \sin^2(\Delta/2)\right]^{-1/2} - \tag{14}$$
$$-J(1-\chi^2 q^4)\left[(1-\chi q^2)^2 + 4\chi q^2 \sin^2(\Delta/2)\right]^{-1}.$$

where $J = \rho I/GE_0^+$ (remember that E_0^+ is supposed to be real and positive, defining relative phase shifts).

Eq.(14) describes a two-parameter family of straight lines. Each shows a linear decrease in the accelerating voltage with current starting from a "cool" cavity value and vanishing at a certain limiting current. The value at $J = 0$ is maximized with $\Delta = 0$, and with the optimal coupling coefficient $k = \sqrt{1 - q^4} = Q^{-1/2}$:

$$V_{max} = \frac{2q}{\sqrt{1 - q^4}}. \tag{15}$$

(Of course $q \to 1$ means an infinitely large Q-value of the cavity and hence makes equation (15) not valid). For larger coupling, the field amplitude decreases because of radiation through the coupler; for smaller ones - because of ohmic losses. Note that $k = Q^{-1/2}$ provides at the same time a maximal sensitivity to loading , i.e., this choice minimizes the limiting current to the value $J = 2q/(1+q^4)$.

According to the above considerations, one can optimize the loading characteristic in Fig.(2), changing Δ from 0 at $J = q/(1+q^4)$ to π at $J = q/\sqrt{1-q^4}$. At the left boundary of this interval, the optimized voltage is $V_0 = q/\sqrt{1-q^4}$, at the right side it is equal to $q/(1+q^4)$. (Note that all curves in Fig.(2) are symmetric with respect to the bisectrix $V_0 = J$). Within the optimization interval

$$V_0 J = \frac{q^2}{(1+q^4)\sqrt{1-q^4}}, \tag{16}$$

so the transmitted power and acceleration efficiency are independent of the loading current. For larger currents, i.e., for $q/\sqrt{1-q^4} < J < 2q/\sqrt{1-q^4}$, the voltage is low and again decreases linearly with current when $\Delta = \pi$. Note that for an arbitrary coupling coefficient the phase optimization is available in the interval

$$\frac{qk}{1+\chi q^2} < J < \frac{qk}{1-\chi q^2} \tag{17}$$

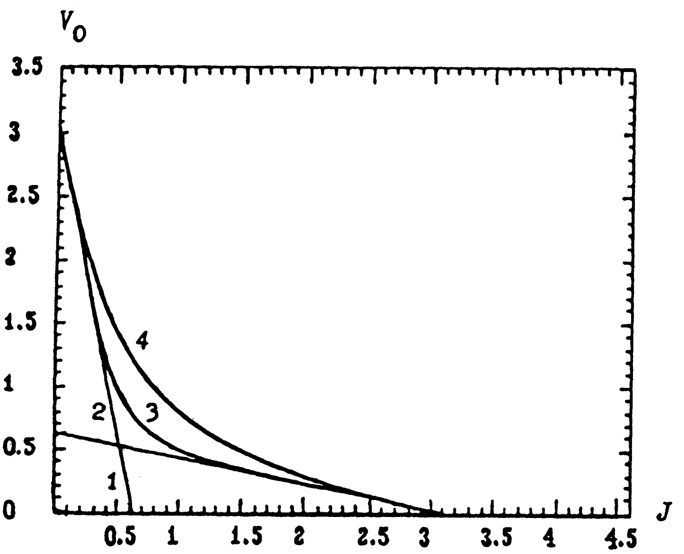

FIGURE 2: Accelerating voltage vs beam current (q=0.9):
1.– Δ=0,2.– Δ=π,3.– Δ=Δ_{opt},4.– k=k_{opt}.

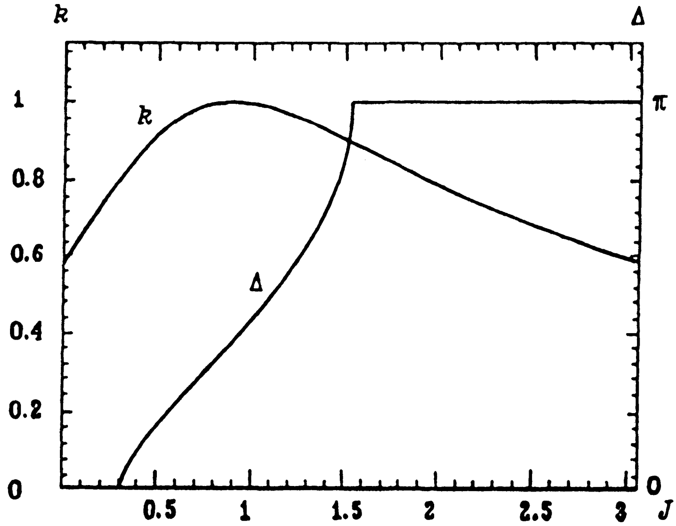

FIGURE 3: Optimal coupling coeffcient k and detuning Δ vs beam current (q=0.9).

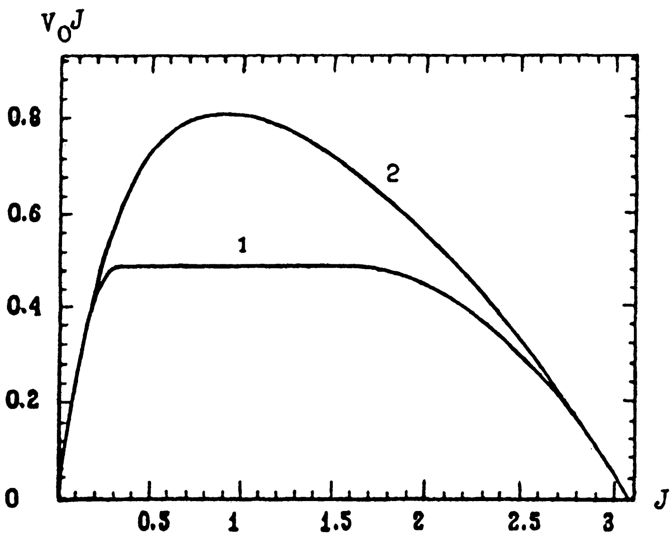

FIGURE 4: Transmitted power vs beam current ($q = 0.9$): 1.- Δ=Δ_{opt}, 2.- k=k_{opt}.

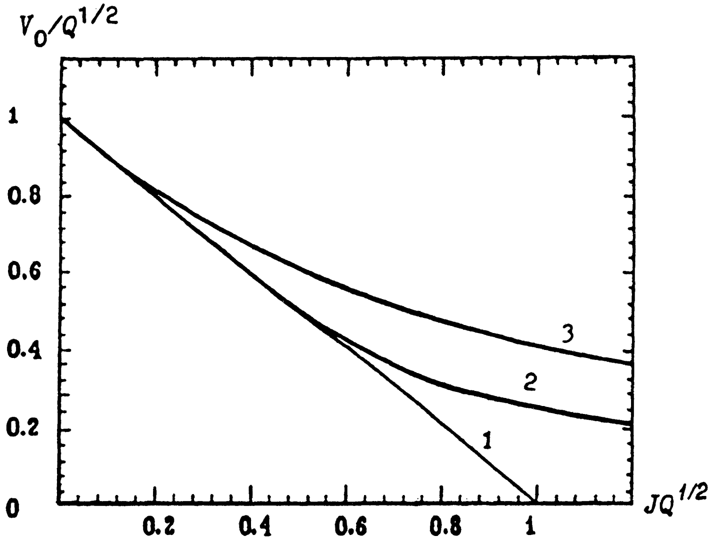

FIGURE 5: Reflected power vs beam current (q=0.9):
1.- Δ=0, 2.- $\Delta=\pi$, 3.- $\Delta=\Delta_{opt}$, 4.- $k=k_{opt}$.

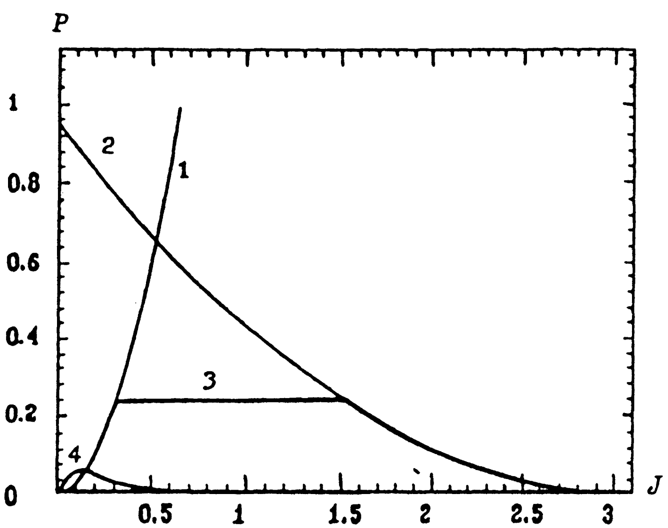

FIGURE 6: Accelerating voltage vs beam current for $Q>>1$:
1.- Δ=0, 2.- $\Delta=\Delta_{opt}$, 3.- $k=k_{opt}$.

and yields the voltage

$$V_0 = \frac{q^2 k^2}{J(1-\chi^2 q^4)}, \tag{18}$$

which has an upper limit $V_0 = q$ for $k = 1$, when the interval shrinks to a point $J = q$.

The existence of the upper limit indicates that one can optimize V_0 with k. Its value has to drop from $\sqrt{1-q^4}$ at $J = 0$ down to zero at $J = q$ and then to increase with $\Delta = \pi$ up to $\sqrt{1-q^4}$ when the current approaches the maximal limiting value $J = 2q/\sqrt{1-q^4}$. The corresponding envelope equation can be written in a parametric form:

$$J = \frac{q^2 - \chi}{q\sqrt{1-\chi^2}};$$
$$V_0 = \frac{q^2(1-\chi^2) + \chi(1-q^4)}{q(1-\chi q^2)\sqrt{1-\chi^2}}; \tag{19}$$
$$0 < \chi < q^2.$$

Thus, optimization with phase shift and with coupling coefficient can increase the accelerating voltage and/or the limiting current. Moreover, the phase optimization allows constant efficiency while the current varies over wide limits. The corresponding dependencies are shown in Figs.(3), (4),(5). Note, however, that to use effectively the resonant properties of the system and to keep $V_0 >> 1$ one has to deal with $Q >> 1$ and $J << 1$.
The optimization in this limit gives:

$$V_0 = Q^{1/2}(1 - JQ^{1/2}) \qquad txtfor \quad \Delta = 0,\ \chi = q^2;$$

$$V_0 = 1/4J \qquad \text{for} \quad J > Q^{-1/2},\ \chi = q^2,\ \Delta = \Delta(J); \tag{20}$$

$$V_0 = \frac{Q^{1/2}}{JQ^{1/2} + (J^2 Q + 1)^{1/2}} \qquad \text{for} \quad \Delta = 0,\ \chi = \chi(J).$$

The corresponding universal curves are shown in Fig. (6).

4 RADIATION REGIMES

The above considerations can also be applied to systems where the modulated beam-cavity interaction is used to obtain RF power. In these cases, the backward wave power P and the beam radiation losses are the objects of optimization. The optimal choice of the injection phase γ gives:

$$P = \left|E_0^-\right|^2 = E_0^{+2}\left[\left|Z_0^-\right| + \frac{|Z_0^+|\,J}{2}\right]^2. \tag{21}$$

Taking into account that according to (7)

$$\left|Z_0^-\right|^2 = 1 - \left|Z_0^+\right|^2 \frac{1-q^4}{4q^2} \tag{22}$$

one can easily get the maximal value

$$\frac{P_{max}}{E_0^{+2}} = 1 + \frac{q^2 J^2}{(1-q^4)}, \tag{23}$$

which can be reached with a certain optimizing value (J, Δ). In the limit $E_0^+ = 0$ when the second term in (23) dominates $P_{max} = q^2 I^2 / G^2 (1 - q^4)$. Thus introducing an external wave for additional deceleration of the beam, one cannot get back more than the injected power plus the "natural" radiation power.

In the case of an autoacceleration regime, or two-beam acceleration scheme, the internal field amplitude E_a is to be maximized irregardless of the exciting beam phase. According to (7) the maximum of Z_r can be reached with $\Delta = 0$ and $\chi = 1$ and

$$\frac{|E_a|}{I\rho} = \frac{1+q^2}{1-q^2}. \tag{24}$$

Note, however, that all calculations above are relevant mainly to relativistic electron beams because in a fixed current approximation particle energy variations in the gap should be small enough not to change essentially a particle velocity.

5 CONCLUSIONS

The model considered above indicates that an accelerating voltage across a cavity gap can be optimized by proper choice of coupler phase shift Δ and coupling coefficient k, depending on the accelerated current. Phase shift optimization can provide constant efficiency of acceleration over a wide range of beam current variations. The maximized RF power which can be obtained from modulated beams in lasertrons or choppertrons cannot be increased by external deceleration.

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REFERENCES

1. P. Lapostolle, A. Septier. Linear accelerators. North-Holland Publishing Company. Amsterdam. 1970.
2. G. Dôme. RF system: Wave guides and cavities, AIP Conference Proceedings **153** Physics of Particle Accelerators, SLAC Summer School (1985).
3. A free electron laser at P.N. Lebedev Institute. BEAMS-92 Proceedings. Washington, USA. 1992.