BEAM DYNAMICS WITH FOUR UNDULATORS ON SUPER-ACO: EXPERIMENTAL AND THEORETICAL RESULTS

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Super-ACO is now running with four undulators. The effects of each insertion have been carefully studied in terms of linear optics, dynamic aperture, fourth order resonances effect, energy acceptance. The situations where two, three or four undulators are closed have also been studied. Experimental results on optics perturbation, chromaticity, vacuum and Touschek lifetimes are presented and compared to simulations and magnetic measurements.

1 INTRODUCTION

Super-ACO is the first operating machine of the third generation of synchrotron radiation sources. This type of machine includes many straight sections for insertions.

In these devices, special arrays of magnets give to the particles a periodic transverse acceleration which leads to a quasi sinusoidal motion and justifies the name of **undulator**.

The BETA code[1] has been used to analyse the effects of insertions and the undulator field is:[2]

$$B_s = -\left(\frac{k}{k_z}\right)\text{Bo}\ \sin(ks)\text{ch}(k_x x)\text{sh}(k_z z)$$

$$B_x = \left(\frac{k_x}{k_z}\right)\text{Bo}\ \cos(ks)\text{sh}(k_x x)\text{sh}(k_z z)$$

$$B_z = \text{Bo}\ \cos\,(ks)\ \text{ch}(k_x x)\text{ch}(k_z z)$$

where $k^2 = k_x^2 + k_z^2 = (2\pi/\lambda)^2$, λ being the undulator period.

The k_x parameter expresses the x- dependence of the real field because of the finite width of the poles in the x-direction. Its value will be chosen to account for experimental effects observed on the beam.

When an undulator is inserted in Super-ACO, the predicted effects are:

for an ideal undulator ($k_x = 0$; $\int B_z\, ds = 0$):

—a vertical focalization which leads to a vertical tune shift,

—a beat of the vertical optical function β_z,

—a drastic effect of fourth order resonances,

—a dynamic aperture reduction due to non linear effects.

for a real undulator ($k_x \neq 0$):

The B_x component of the field is not zero and the finite width of the poles in the x-direction introduces gradients as $\partial B_x/\partial x$ and $\partial B_z/\partial x$.

The finite value of "$\int (\partial B_z/\partial x)ds$" leads to a horizontal tune shift, and the finite value of "$\int (\partial B_x/\partial x)ds$" introduces some additional coupling.

Finally, the finite values of "$\int B_x\, ds$" and "$\int B_z\, ds$" leads to a closed orbit distortion and the x^2-dependence of "$\int B_z\, ds$" can introduce some additional chromaticity if the undulator is located in a straight section where the dispersion function is not zero.

During experiments with the beam, we have measured tune shifts, effects of resonances, lifetime variation (Touschek and vacuum lifetimes), additional chromaticities, and compared the results to simulation ones.

2 THE FOUR UNDULATORS INSTALLED ON SUPER-ACO

Super-ACO is running since March 1987 and is presently equipped with four undulators (two other straight sections are still available). The location of the four undulators installed in the ring is shown in Figure 1.

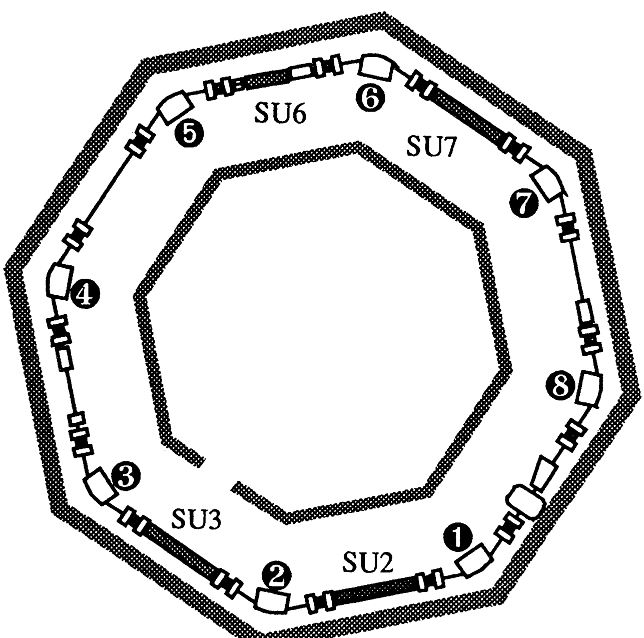

FIGURE 1 Undulators location in Super-ACO.

TABLE 1
Undulators Characteristics.

Name	Description	K	$B(T)$
SU7	2 regular sections: 2 × 10 × 129 mm periods + 1 dispersive section 500 mm	6	0.5
SU6	1 regular section: 14 × 78 mm periods	2.2	0.3
SU3	1 regular section: 24 × 129 mm periods	6	0.5
SU2	Asymmetrical Hybrid Wiggler: 12 × 263 mm periods	11	Max = 1.05 Min = 0.2

Undulators SU6 (regular undulator) and SU2 (asymmetrical hybrid wiggler) are located in non zero dispersion sections, undulators SU3 (regular undulator) and SU7 (optical klystron used for Free Electron Laser operation) are located in zero dispersion sections. Undulators characteristics are summarized in Table 1.

K is the deflection parameter which characterizes the device strength: $K = 0.93 B_0(T) \lambda_0(\text{cm})$ where λ_0 is the undulator period and B_0 the maximum on-axis field.

3 OPTICS PERTURBATION

The optic used presently is the "low emittance" optic (Figure 2) and the positron energy is 800 MeV.

When an undulator is closed, the fourth order symmetry of the machine is destroyed, tunes are shifted and the optical function β_z is modified.

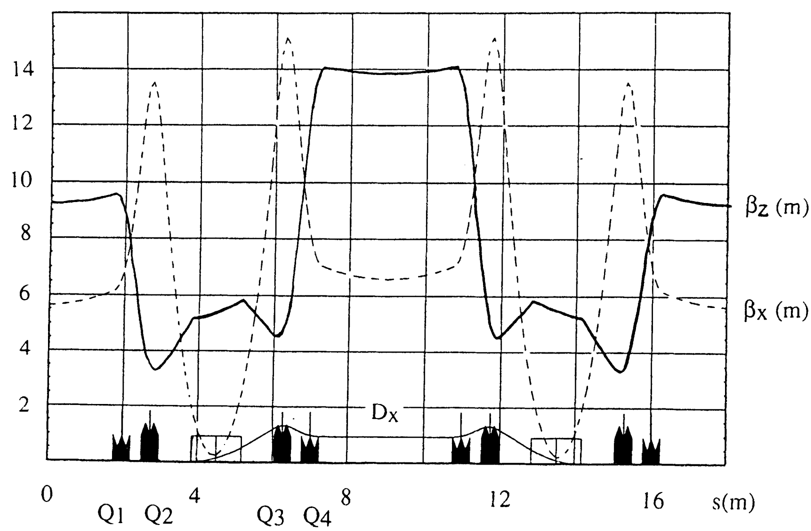

FIGURE 2 Optical functions for one period.

3.1 *Tune Shift*

The rectangular shape of the magnets introduces a wedge focusing which is compensated in the horizontal plan because of the zero field index. In the vertical plan, the total focusing strength is equal to $(\int B_z^2\,ds)/(B\rho)^2$. In our case $\beta_z^* \gg L$ (L is the undulator length) and the predicted tune shift can then be written as follows:

$$\Delta\nu_z = \frac{1}{4\pi}\frac{(\int B_z^2\,ds)}{(B\rho)^2}\,\beta_z^*$$

where β_z^* is the value of β_z at the undulator location ($\beta_z^* = 10$ m) and $(B\rho)$ is the magnetic rigidity (8/3 for Super-ACO).

For each undulator, experimental tune shifts have been measured and compared to predicted ones (Table 2).

The difference between predicted and experimental values shows that each undulator introduces an additional focusing which is due to the finite value of $\int (\partial B_z/\partial x)ds$. In Table 3, we compare the value deduced from additional tune shifts and the one given by magnetic measurements of the undulators field. For SU2, SU6 and the regular part of SU7, the agreement is very good, for SU3 and the dispersive part of SU7, the agreement is less satisfactory.

The tune shifts are compensated by a *global* method using the two adjacent quadrupole families which are Q_1 and Q_2 for SU3 and SU7, and Q_3 and Q_4 for SU6 and SU2.

TABLE 2
Measured Tune Shifts with Undulators.

		SU2	SU3	SU6	SU7	
					Regular part	Dispersive part
Predicted	$\Delta\nu_x$	0	0	0	0	0
	$\Delta\nu_z$	0.07	0.033	0.0073	0.033	0.01
Experimental	$\Delta\nu_x$	0.0053	0.007	0.0007	0	−0.01
	$\Delta\nu_z$	0.0477	0.046	0.003	0.03	0.02

TABLE 3
Undulators Additional Focusing.

$\int (\partial B_z/\partial x)ds$ (G)	SU2	SU3	SU6	SU7	
				Regular part	Dispersive part
Magnetic measurements	−510	260	160	−25	120
Additional focusing	−567	470	144	0	335

For high values of the K-parameter, the strong perturbation of optics can be avoided by using a *local* compensation. It uses only the four quadrupoles which are near the insertion. This type of compensation has been tested with success for the SU2 undulator as we will see later.

3.2 *The β-function Beat*

The insertion of an undulator modifies the β-functions and the beat depends on the undulator strength and location.

We have plotted the variation of the ratio $\Delta\beta_z/\beta_{z0}$ along the machine for several situations. In those simulations, the undulator is replaced by a thin lens which strengths f_x and f_z account for experimental tune variations (this approximation is sufficient to simulate linear effects). In all cases the tune shifts are compensated.

Figure 3 shows that the maximum beat with undulator SU7 is about 25% and is negligible with undulator SU6 because it is weaker.

As the effect of SU3 is the same as SU7, when SU3, SU7 and SU6 are closed together, the beat is reduced to 15% because of the symmetrical position of SU7 and SU3. When the four undulators are closed with a *global* compensation, the beat reaches 70%. As the beam lifetime contribution from elastic scattering on nuclei depends on the maximum value of β_z ($\tau \propto 1/\beta_{z\max}$), we have used a *local* compensation for SU2 to reduce the beat to 20% (Figure 4).

During experiments we have also evaluated the β_z function by measuring the variation of the tune Q_z when the quadrupole gradient is modified. The results are summarized in Table 4 (the accuracy on the β_z values is 1 m).

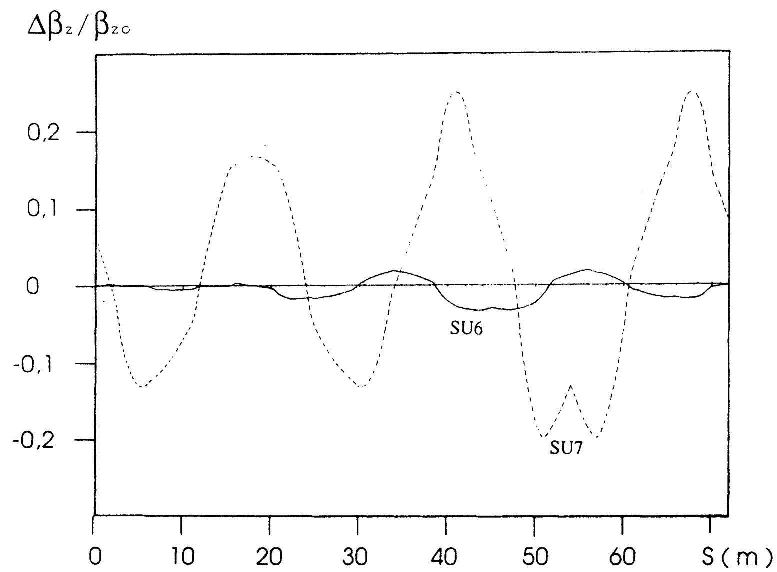

FIGURE 3 Vertical β-function beat with SU6 and SU7 undulators.

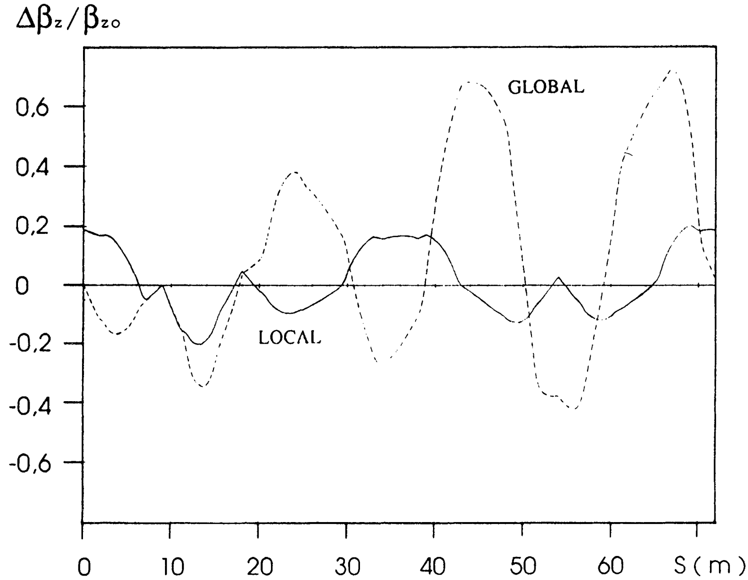

FIGURE 4 Vertical β-function beat with four undulators.

TABLE 4
Measured β_z-functions with Undulators.

	Quadrupole Q20 (Q4 type) Straight section no. 6		Quadrupole Q25 (Q1 type) Straight section no. 7	
	Measurements	Beat	Measurements	Beat
Undulators open	8 m	—	8.7 m	—
SU6, SU7 and SU3 closed	8.4 m	5%	10.5 m	21%
4 undulators Global compensation	11 m	37%	6.4 m	−26%
Local compensation	9 m	12%	10 m	15%

Measurements confirm the predicted β-values and the local compensation leads to a symmetrical machine. This kind of compensation is now used on Super-ACO. The injection rate remains constant (3 A/h) from 0 to 400 mA (stored in 24 bunches). The beam lifetime has been improved from 3.5 h to 4.5 h for 400 mA that is to say 30%.

3.3 *Additional Chromaticity*

In order to avoid instability at high currents, it is necessary to identify and compensate the chromaticity introduced by each insertion. So measurements have

TABLE 5
Chromacity Variation with Undulators.

Insertion	$\Delta\xi_x$	$\Delta\xi_z$
SU7	0	0
SU6	0.7	−1.5
SU3	0.2	0.4
SU2	−1.8	2.1

been done with undulators open and closed, to evaluate the sextupolar residual field due to the x^2-dependence of the finite value $\int B_z\, ds$.

Table 5 gives the chromaticity variations $\Delta\xi_x$ and $\Delta\xi_z$ for each insertion.

As they are located in zero dispersion sections, undulators SU3 and SU7 do not introduce chromaticity (the small variation produced by SU3 is only due to the variation of the optical functions in the non zero dispersion sections).

For SU6 and SU2 the variation is rather large and shows a residual sextupolar strength S which is defined by:

$$\int B_z\, ds = B\rho.S.x^2$$

(where S is expressed in m^{-2}).

The value of S can be deduced from additional chromaticities:

$$\Delta\xi_x = \frac{1}{2\pi}\beta_x^* D_x S$$

$$\Delta\xi_z = \frac{1}{2\pi}\beta_z^* D_x (-S)$$

(a) The SU6 Undulator

The measured optical functions in straight section no. 6 are : $\beta_x = 7$ m, $\beta_z = 11$ m, $D_x = 1.55$ m.

We obtain $S = 0.43\,m^{-2}$, $(-S) = -0.58\,m^{-2}$, so a representative value is $S = 0.5\,m^{-2}$.

Magnetic measurements of the field give the variation of $\int B_z.ds$ versus x. In Figure 5, we have compared the equivalent sextupole and the real field. The agreement is very good in the range $-20\,mm < x < 20\,mm$ where the beam is stored.

(b) The SU2 Undulator

The optical functions in straight section no. 2 are the same as in straight section no. 6. This leads to $S = -1.04\,m^{-2}$ and $(-S) = 0.77\,m^{-2}$, so a reasonable value seems to be $S = -0.9\,m^{-2}$ and the agreement with magnetic measurements is very good as shown in Figure 6.

4 FOURTH ORDER RESONANCES EFFECTS

The non linear part of the undulator field excites fourth order resonances which react on dynamic aperture and as a consequence on beam lifetime.

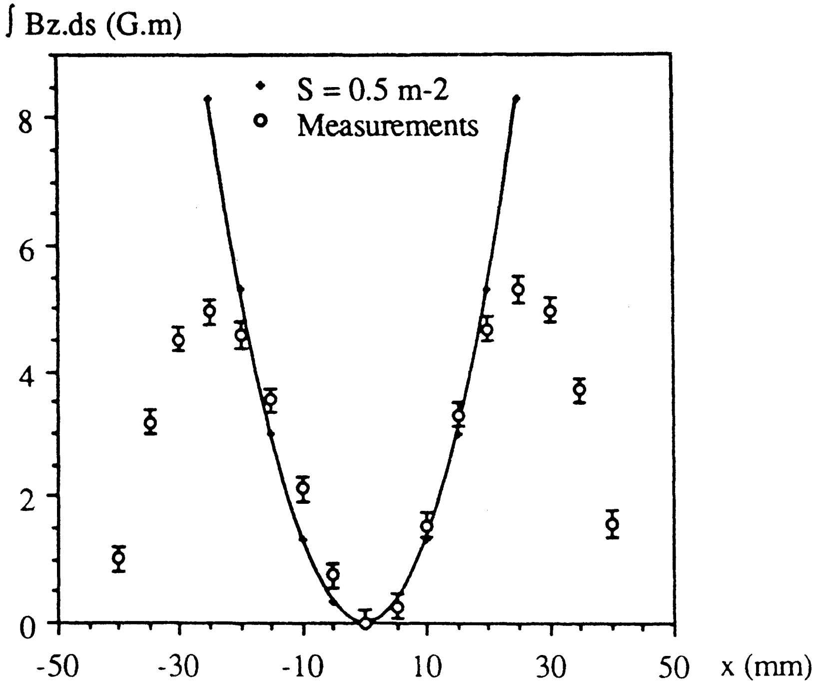

FIGURE 5 Variation of the SU6 vertical field with the horizontal position x.

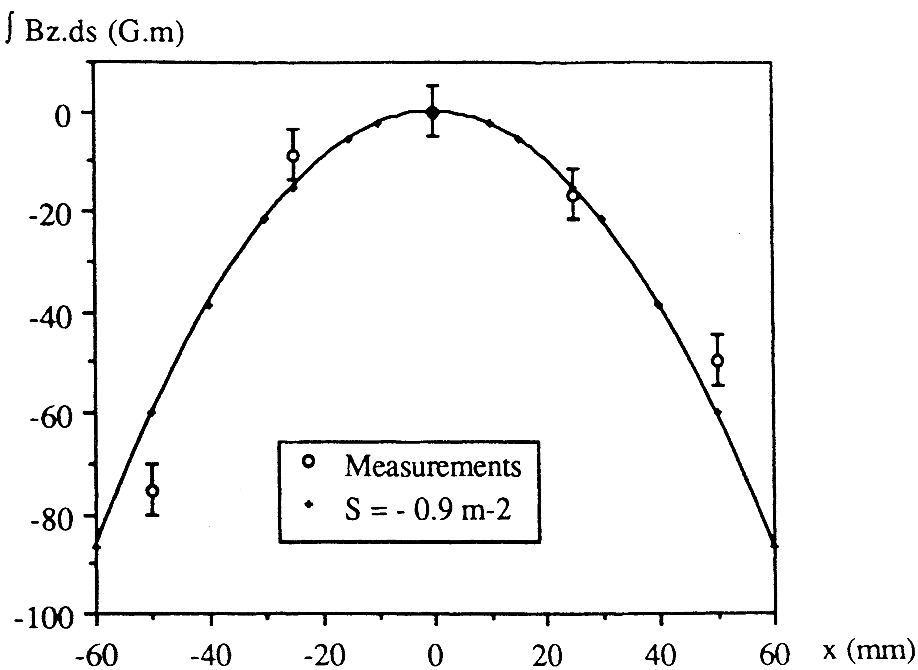

FIGURE 6 Variation of the SU2 vertical field with the horizontal position x.

Using the hamiltonian formalism, the perturbation due to an undulator can be written as:[3]

$$H(x, z, s) = \frac{1}{2(k\rho)^2}\left\{\frac{1}{2}(k_x^2x^2 - k_z^2z^2) - \frac{1}{24}(k_x^4x^4 + k_z^4z^4 - 6k_x^2k_z^2x^2z^2)\right\}$$

This leads to fourth order non linear resonances which are:

$$4Q_x = n$$

$$4Q_z = p$$

$$2Q_x + 2Q_z = q$$

$$2Q_x - 2Q_z = r$$

where n, p, q and r are integer.

To test the effect of these resonances, we have simulated the dynamic aperture for many working points.

To compute the dynamic aperture, BETA uses a tracking method where simple elements are represented by their first order matrix and sextupoles by a second order matrix. Particles are sent with derivatives $x' = z' = 0$ and an initial x position. The limit of the dynamic aperture is the maximum stable vertical position z, for each position x, after a given number of turns. Several tests have shown that 50 twins were sufficient to compare the stability of the different working points. The machine was considered as perfect, without any closed orbit or field errors.

First, we verified in some cases that the dynamic aperture was almost the same with and without longitudinal motion. So it was not introduced in this systematic study and the synchrotron damping was not included.

A large value of $(k_x/k_z)^2 = -0.5$ was chosen to account for the strong effects of resonances observed on the beam.

As an example, when the undulator SU7 is closed, the variation of the vertical aperture as a function of tune Q_x is shown on Figure 7 (for $Q_z = 1.705$). The $x = -1.5$ cm value was chosen to account for large amplitudes during injection.

The drops of aperture are shifted because the large amplitudes produce tune shifts. The same phenomenon is also observed for $Q_z = 1.78$ as shown in Figure 8.

Note: the theoretical locations of the resonances are:

$$Q_x = 4.75 \text{ for } 4Q_x = 19$$

$$Q_x = 4.80 \text{ for } 2Q_x + 2Q_z = 13 \ (Q_z = 1.705)$$

$$Q_x = 4.72 \text{ for } 2Q_x + 2Q_z = 13 \ (Q_z = 1.78).$$

To identify the resonances, we have plotted for several points (Q_x, Q_z), the diagrams (x, x'), (z, z') for non zero initial amplitudes x_0 and z_0. The tracking method is the same as for dynamic aperture. Initial amplitudes were chosen near the stability limit and the number of turns was increased up to 300.

The $4Q_x = 19$ resonance is shown in Figure 9 with $Q_x = 4.74$, $Q_z = 1.78$, $x_0 = 15$ mm, $z_0 = 4$ mm.

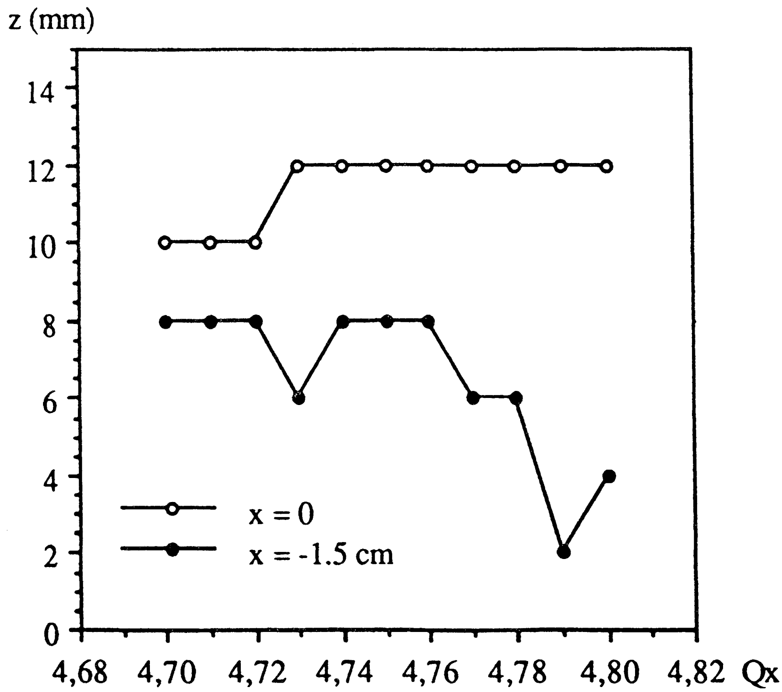

FIGURE 7 Variation of the vertical aperture with Q_x; $Q_z = 1.705$.

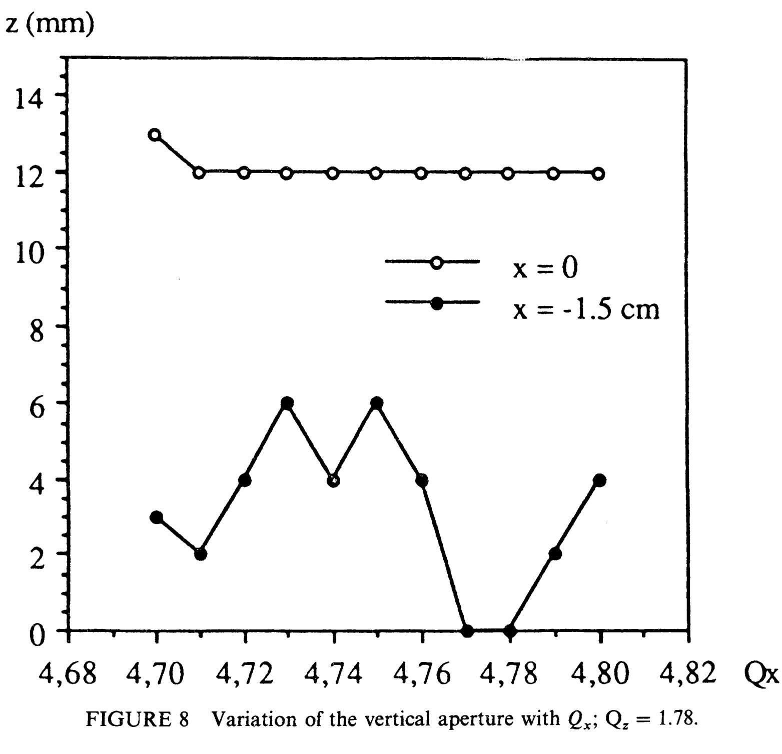

FIGURE 8 Variation of the vertical aperture with Q_x; $Q_z = 1.78$.

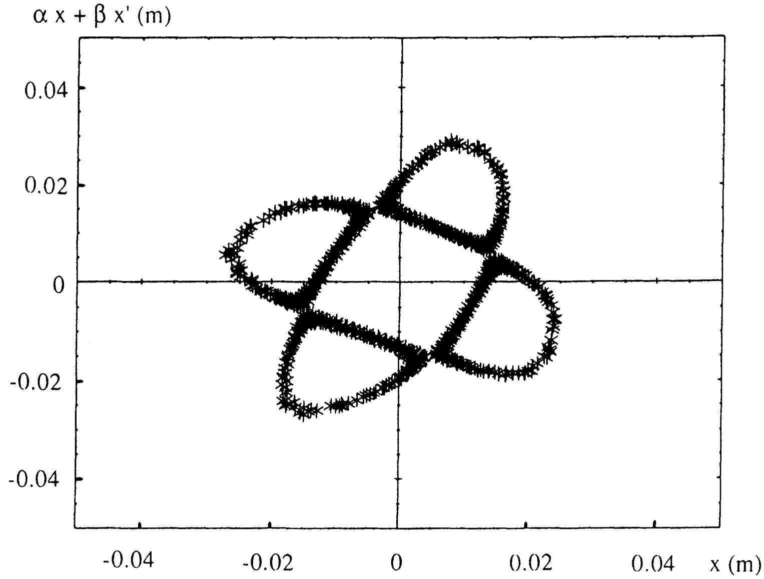

FIGURE 9 Horizontal phases diagram for $4Q_x = 19$.

The $4Q_z = 7$ resonance is identified in Figure 10 with $Q_x = 4.70$, $Q_z = 1.75$, $x_0 = -10$ mm, $z_0 = 1$ and 2 mm.

The coupling resonance $2Q_x + 2Q_z = 13$ is identified using the relation between the horizontal and vertical invariants: $2E_z - 2E_x =$ constant ($E_x = \beta_x \varepsilon_x$). An example is given in Figure 11 for $Q_x = 4.73$, $Q_z = 1.78$, $x_0 = -15$ mm and $z_0 = 8$ mm.

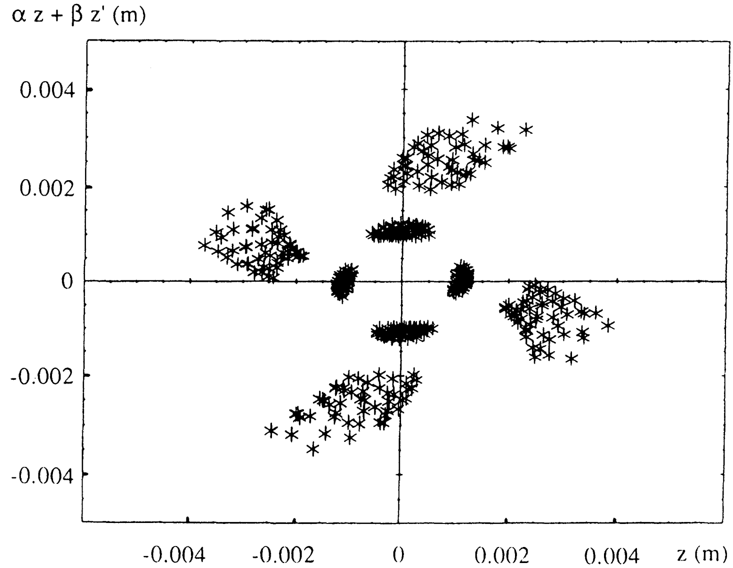

FIGURE 10 Vertical phases diagram for $4Q_z = 7$.

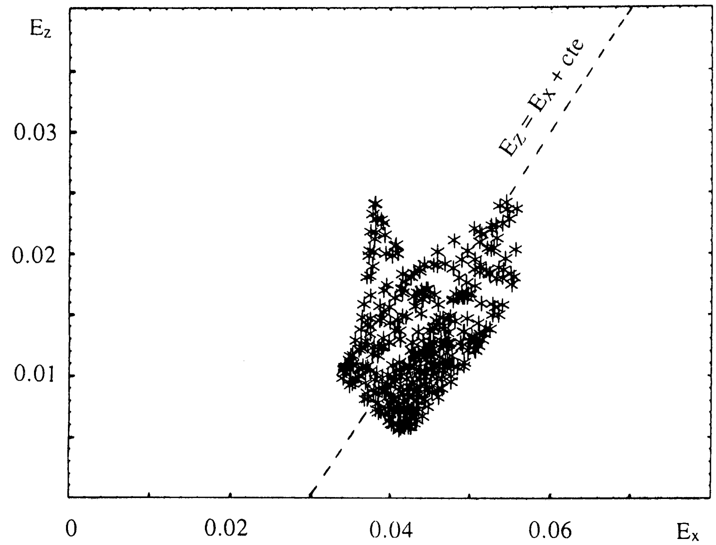

FIGURE 11 (E_x, E_z) diagram for $2Q_x + 2Q_z = 13$.

Similar studies have been performed with the other undulators and the same results were obtained.

To test the effects of fourth order resonances on the lifetime, a beam was stored ($I = 100$ mA) at the working point ($Q_x = 4.72$, $Q_z = 1.70$). Keeping $Q_x = 4.72$, we scanned Q_z from 1.70 to 1.78 and measured the beam lifetime at each step. This experiment has been performed in several cases which are:

—undulator SU6 is closed (Figure 12),

—undulators SU6 and SU7 are closed (Figure 13),

—undulator SU3 is closed (Figure 14),

—undulators SU6, SU7 and SU3 are closed (Figure 15).

In all situations, the resonances $4Q_z = 7$ ($Q_z = 1.75$) and $2Q_x + 2Q_z = 13$ ($Q_z = 1.78$) are dangerous. Two other fourth order resonances: $3Q_x + Q_z = 16$ ($Q_z = 1.79$) and $Q_x + 3Q_z = 10$ ($Q_z = 1.76$) are the most dangerous.

We have of course checked that these resonances were not excited when undulators are open.

The excitation of the $3Q_x + Q_z = n$ resonance has also been observed on SPEAR[4] with an undulator. These resonances could be excited by a skew octupole field but their effect is too large to be related to the possible (but small) roll of the undulator. So, this suggests that the analytical field model is not fully satisfactory as these resonances were not predicted by theory. But it is difficult to modify the undulator field model because this part of the field cannot be measured easily during magnetic measurements.

It is also important to notice that in a situation where SU3 is closed alone, the resonance $Q_x + 3Q_z = 10$ is not dangerous. The main difference is that the skew quadrupole term of the field (which gives coupling) has been minimized during the construction.

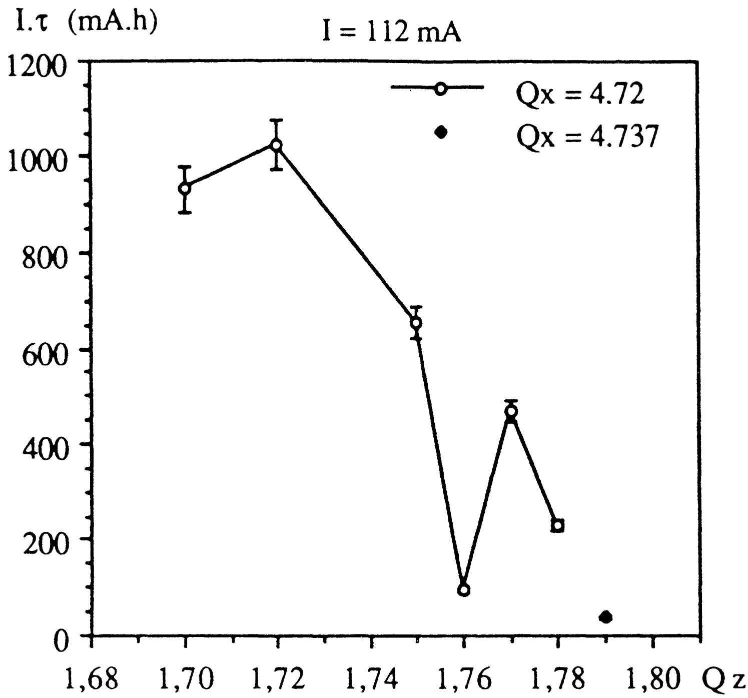

FIGURE 12 Beam lifetime variation with Q_z when SU6 is closed.

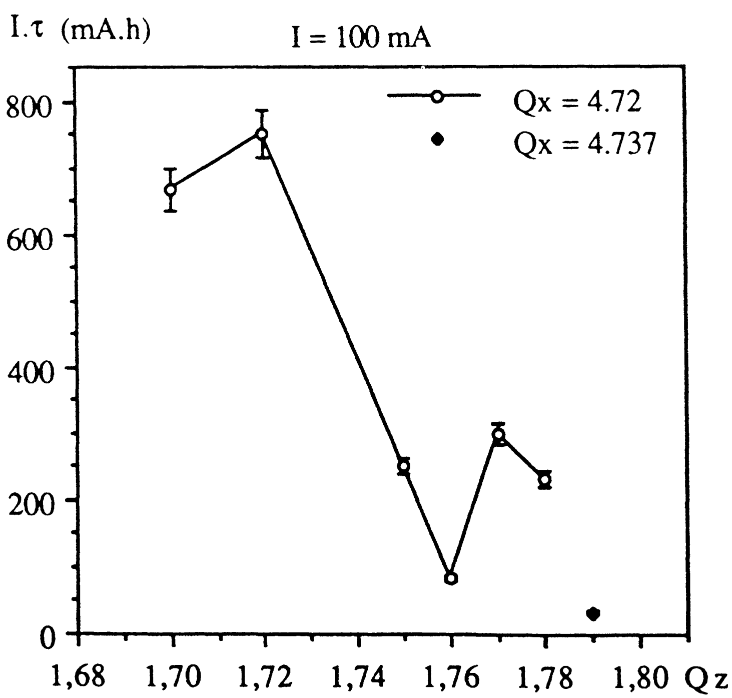

FIGURE 13 Beam lifetime variation with Q_z when SU6 and SU7 are closed.

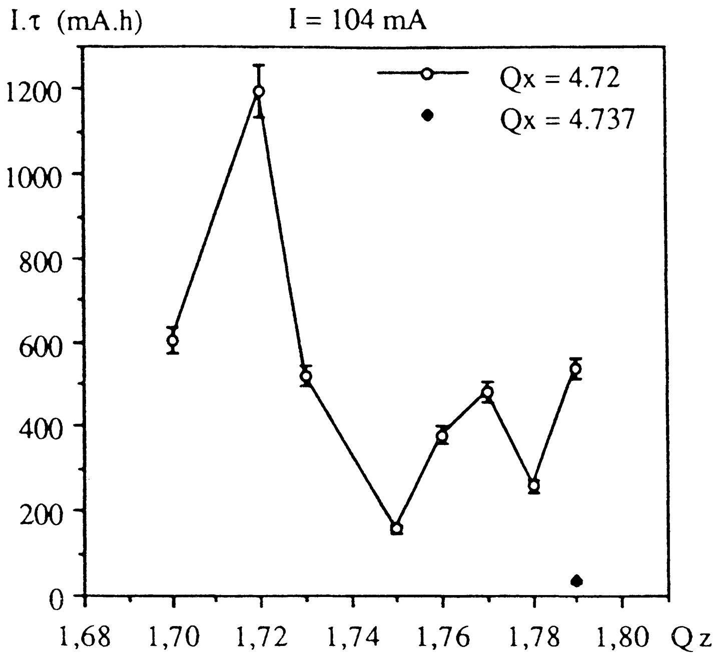

FIGURE 14 Beam lifetime variation with Q_z when SU3 is closed.

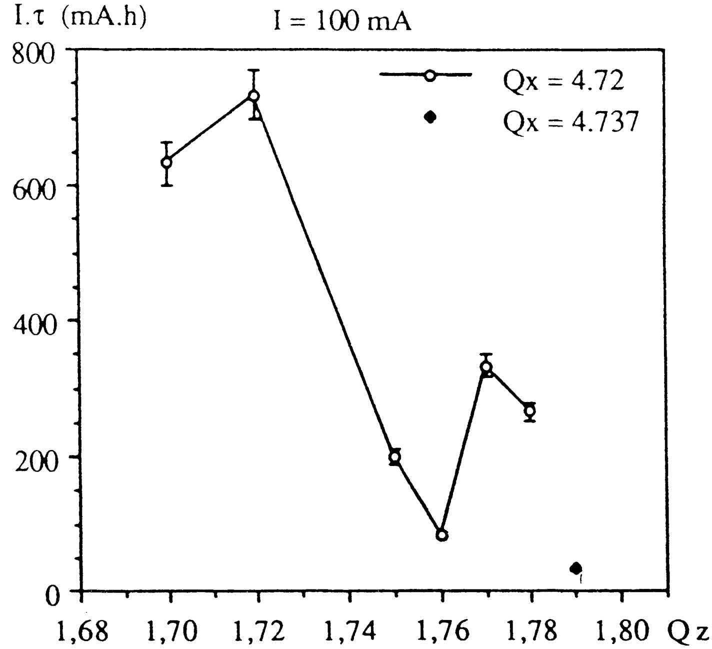

FIGURE 15 Beam lifetime variation with Q_z when SU6, SU7 and SU3 are closed.

5 BEAM LIFETIME REDUCTION

On Super-ACO, the two main contributions to the lifetime are the beam scattering on nuclei and the Touschek lifetime.

5.1 Beam-gas Lifetime

The vacuum contribution depends on the optical functions (β_z essentially) and on the vertical aperture h:

$$\frac{1}{\tau} \propto \frac{\overline{\beta_z}\beta_z^{\max}}{h^2}$$

So the linear perturbation introduced by an undulator (β_z-beat) or a modification of the dynamic aperture can reduce the beam lifetime.

During experiments in July 1990, beam lifetimes have been measured in several situations, with a 400 mA stored current, just near the coupling resonance where the vertical beam size is $\sigma_z = 300\ \mu$m.

The results are summarized in Table 6.

TABLE 6
Beam Lifetime Measurements with Undulators.

Undulators closed	$I\tau$ (mA.h)	Tune correction
SU6 + SU7	2000	Global
SU6 + SU7 + SU3	1700	Global
SU6 + SU7 + SU2	1500	Global
SU6 + SU7 + SU3 + SU2	1300	Global
	1700	Local

These values show the strong effect of undulators especially the SU2 undulator when it is corrected with a global compensation.

The improvement of the lifetime when a local correction is used, is roughly explained by the reduction of the $\beta_{z\max}$ value:

$$\frac{(I\tau)\,\text{local}}{(I\tau)\,\text{global}} = 1.3 \quad \text{while} \quad \frac{(\beta_z^{\max})\,\text{global}}{(\beta_z^{\max})\,\text{local}} = \frac{22}{13} = 1.7$$

(the β_z values being theoretical values).

In May 1991, new measurements of the beam lifetime with a 400 mA stored current and $\sigma_z = 300\ \mu$m gave when the four undulators are closed (with a local correction for SU2): $I\tau = 2060$ mA.h. This value shows the effect of the conditioning of the vacuum chamber which improves the vacuum lifetime in a very significant way.

5.2 Touschek Lifetime Variation

On Super-ACO, 50% of the beam time is dedicated to the 2 bunch mode operation: 2×120 mA are stored. Then the lifetime is dominated by the Touschek effect.

A direct measurement of the Touschek lifetime was performed using two unequal bunches with respective intensities 5 and 22 mA. The lifetime τ of each bunch was measured and we have:

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{vacuum}}} + \frac{1}{\tau_{\text{Touschek}}}$$

where

$$\frac{1}{\tau_{\text{vacuum}}} = P_0 + \lambda I_{\text{Total}}$$

$$\frac{1}{\tau_{\text{Touschek}}} = \alpha \frac{I_{\text{bunch}}}{\sigma_{\text{bunch}}} \quad (\sigma \text{ being the bunch length})$$

and so

$$\alpha = \left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right) \bigg/ \left(\frac{I_1}{\sigma_1} - \frac{I_2}{\sigma_2}\right)$$

where $I_1 = 22$ mA, $I_2 = 5$ mA, $\sigma_1 = 155$ ps, $\sigma_2 = 92$ ps (σ_1 and σ_2 being experimental values measured for $V_{RF} = 150$ kV).

The two lifetimes τ_1 and τ_2 have been measured when undulators are open and when SU6 and SU7 are closed (Table 7). The beam sizes were $\sigma_x = 200\,\mu$m, $\sigma_z = 130\,\mu$m. It shows a strong reduction of the Touschek lifetime (−43%) when undulators are closed.

TABLE 7
Touschek Lifetime Measurements with Undulators.

	Undulators open	Undulators SU6 and SU7 closed
τ_1 (h)	5.9	3.1
τ_2 (h)	16.8	7.6
$(I\tau)_{\text{Touschek}}$ (mA.h) ($I = 22$ mA)	123	71

New measurements performed in July 1991 have shown that the value of $[(I\tau)_{\text{Touschek}}/\sigma_l]$ is reduced from 1.7 (mA.h/ps) when undulators are open to 1.1 (mA.h/ps) when the four undulators are closed ($\sigma_x = 200\,\mu$m, $\sigma_z = 340\,\mu$m). This means that for Super-ACO the reduction of the Touschek lifetime is the same when two or four undulators are closed.

A numerical study of the Touschek lifetime has shown that the modification of the optical functions, when undulators are closed, reduces the Touschek lifetime by a factor less than 10%.

As a consequence, the strong reduction of the Touschek lifetime when undulators are closed is probably due to the *energy acceptance variation.*

To simulate the Touschek effect,[5] we have studied a particle which receives an energy kick ($\Delta p/p$) after a collision with another particle in the same bunch. If the energy kick is located in a section where the dispersion function D_x is not zero, the betatron oscillation amplitude is increased by a quantity equal to the closed orbit displacement $x_{of} = D_x.(\Delta p/p)$.

So we have tested the transverse stability of the perturbed off-momentum motion to evaluate the maximum stable value of the energy kick $(\Delta p/p)$ and compare it to the RF acceptance in the situations where undulators are open and closed.

For several values of the initial horizontal amplitude x_0, we have deduced the Touschek lifetime variation from the maximum stable values of $\Delta p/p$ (Table 8), the Touschek lifetime being proportional to $(\Delta p/p)^{5/2}$.

When undulators are open, the limitation of $(\Delta p/p)$ comes from the longitudinal motion and the energy acceptance is then given by the RF system: $\varepsilon_e = 1.6\%$.

When two undulators are closed, the maximum stable values of $(\Delta p/p)$ become less than the RF acceptance and the Touschek lifetime is reduced, in agreement with experimental results.

TABLE 8
Maximum Stable Value of $(\Delta p/p)$ for Several Initial Amplitudes x_0.

	$(\Delta p/p)_{max}(\%)$		
x_0 (mm)	Undulators open	SU6 and SU7 closed	τ closed / τ open
1	1.6	1.5	0.85
3	1.6	1.3	0.6
5	1.6	0.6	0.1
10	1.6	0.4	0.03

6 CONCLUSION

For each one of the four undulators installed on Super-ACO, theoretical results and experimental measurements have been compared.

Concerning the optics perturbation, the agreement between magnetic measurements and measurements performed on the beam is very good.

Concerning the non linear effects, simulations with the BETA code have predicted most of the effects observed on the beam but a large value of $(k_x/k_z)^2$ is necessary to account for resonances effects on beam lifetime and two very dangerous fourth order resonances not predicted by theory are excited, showing that the sinusoidal field model is not fully satisfactory.

The strong reduction of the Touschek lifetime observed when undulators are closed, shows a drastic reduction of the energy acceptance and reacts on the total beam lifetime when two strong bunches are stored. A simple method which consists in studying the transverse stability of the perturbed motion after a Touschek collision has confirmed this effect.

Concerning the machine operation, the use of a local compensation for the strongest insertion (SU2) provides a very good beam lifetime and a regular injection rate.

As Super-ACO is running at a low energy (800 MeV), with a large value of the vertical optical function β_z (12 m) in insertion sections and with strong undulators, these good results are very encouraging for new storage rings which will operate with similar insertions but at higher energy and with smaller optical functions.

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