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THE INFLUENCE OF PLASMA TEMPERATURE ON WAKE WAVE GENERATION

A. Ts. AMATUNI, S. S. ELBAKIAN and E. V. SEKHPOSSIAN

Yerevan Physics Institute, Alikhanian Brothers Street 2
375036, Yerevan, Armenia

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An equation of motion for a charged relativistic electron fluid in hydrodynamic approximation at nonzero temperature is obtained using the relativistic-invariant form of Vlasov's equation. It is shown that the account of the thermal motion of plasma electrons in the calculation of wake waves generated by a relativistic electron bunch moving in plasma leads to insignificant corrections to the wake wave characteristics for all temperatures practically attainable under laboratory conditions.

1 INTRODUCTION

The equations of motion for a charged relativistic fluid can be obtained using the second moments of Vlasov's relativistic equation for a plasma, or phenomenologically via the energy-momentum tensor expansion of the system. Here one should bear in mind that the four-dimensional acceleration is always perpendicular to the four-velocity, and in virtue of that the four-dimensional force is perpendicular to the four-velocity. The explicit account of this circumstance is particularly essential in the extremely relativistic case. Here we use both above-mentioned methods to obtain the equation of motion for the relativistic plasma electrons in the hydrodynamic approximation at nonzero temperature. The derived equation of motion complemented by the equation of state is used to analyze the influence of temperature effects on wake wave generation in a plasma traversed by a fixed electron bunch with a given velocity. The results obtained are compared with ones reported earlier[1–3] in connection with the problem of plasma wake field acceleration.

2 EULER'S EQUATION OF MOTION FOR THE RELATIVISTIC PLASMA ELECTRONS AT NONZERO TEMPERATURE

In order to derive the equations of motion for the relativistic neutral plasma electrons with ions at rest we shall proceed from the relativistic-invariant form of the Vlasov equation[4]:

$$p^i\partial_i f(x, p) + \frac{e}{c} F^{ik}p_k \frac{\partial}{\partial p^i} f(x, p) = C(x, p),$$

$$p \equiv p^i,\ x \equiv x^i,\ \partial_i \equiv \partial/\partial x^i, \tag{1}$$

where $p^i(p^0 = \varepsilon^0/c, \mathbf{p})$ is the four-momentum of plasma electrons, $i = 0, 1, 2, 3$, $g^{ik}(1, -1, -1, -1)$, F^{ik} is the electromagnetic field tensor satisfying the Maxwell equations with sources, plasma ions and electrons have lab. system densities n_0 and n_e respectively, the external beam has a given length d, density n_b and velocity $V_z = V_0$; $C(p, x)$ is the collisions integral which we put zero thus neglecting dissipative processes due to plasma viscosity and heat conduction. Then we define the energy-momentum tensor of plasma electron[4]

$$T^{ik} = c \int \frac{d^3p}{p^0} p^i p^k f(x, p) \tag{2}$$

and obtain from (1) the equations for the second moments of the distribution function $f(x, p)$, for which we multiply (1) by p^k and integrate over d^3p/p^0. Finally we have

$$\partial_i T^i_k - \frac{1}{c} F_{kl} j^l = 0 \tag{3}$$

where

$$j^l = ecn_e u^l, \quad n_e u^l = \int p^l \frac{d^3p}{p^0} f(x, p).$$

Using the Maxwell equations

$$\frac{\partial F^{lk}}{\partial x^k} = -\frac{4\pi}{c} j^l, \quad \frac{\partial F_{lk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^l} + \frac{\partial F_{il}}{\partial x^k} = 0 \tag{4}$$

and introducing the field energy-momentum tensor

$$J^{ik} = \frac{1}{4\pi}\left(-F^{il}F^k_l + \frac{1}{4} g^{ik} F_{lm} F^{lm}\right) \tag{5}$$

we will obtain from (3) the energy-momentum conservation law of the system:

$$\partial_i(T^i_k + J^i_k) = 0. \tag{6}$$

The equations of motion, as it is known[4,5], are derived from (6) by projecting this relation on the direction perpendicular to the four-velocity $u^i(\gamma, \gamma\mathbf{v}/c)$:

$$\partial_i(T^i_l + J^i_l) - u_l u^k \partial_i(T^i_k + J^i_k) = 0. \tag{7}$$

Further we use the expansion of the plasma electron energy-momentum tensor[4–6]

$$T^{ik} = wu^i u^k - Pg^{ik} \tag{8}$$

where $W = \varepsilon + P$ is the thermal function (enthalpy) of the unit volume, ε is the internal energy, P is the pressure in the local proper reference frame.

Substituting (5) and (8) into (7) we finally arrive at the following relativistic equation of motion for the charged electron fluid:

$$wu^k \frac{\partial u_i}{\partial x^k} = \frac{\partial P}{\partial x^i} - u_i u^k \frac{\partial P}{\partial x^k} + \frac{1}{c} F_{il} j^l. \tag{9}$$

Scalar multiplication by u^i gives zero on both sides of Eq. (9), i.e. the total force is perpendicular to the four-velocity, as it should be.

From (9) for the space components $i = 1, 2, 3$ the equation of motion in the three-dimensional form is obtained

$$\frac{w\gamma}{n_e mc^2}\left(\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}\frac{\partial \mathbf{p}}{\partial \mathbf{r}}\right) = -\frac{\gamma^2}{n_e}\frac{\partial P}{\partial \mathbf{r}} - \frac{\gamma^2 \mathbf{v}}{c^2 n_e}\frac{\partial P}{\partial t} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right), \tag{10}$$

where $\mathbf{p} = m\mathbf{v}\gamma$, $\gamma = (1 - v^2/c^2)^{-1/2}$.

Introducing the quantity $p^i = mcu^i$ and passing in (9) to the nonrelativistic limit $^v/_c \ll 1$ we shall obtain the equation for the space components $i = 1, 2, 3$

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}\frac{\partial \mathbf{p}}{\partial \mathbf{r}} = -\frac{1}{n_e}\frac{\partial P}{\partial \mathbf{r}} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right), \tag{11}$$

which sometimes[7,8] is considered as the equation of motion of a relativistic charged fluid. Note that the difference between Eq. (10) and Eq. (11), even if $\mathbf{p}$ in the latter is considered as the relativistic momentum, is particularly significant for $\beta \sim 1$. Eq. (9) in the one-dimensional case coincides with Eq. (9) from Ref. 2.

The antisymmetry of F_{ik} used in the proof leads to the fact that the projection of relation (3) on the direction of the four-velocity gives, just as in the case of a noncharged fluid, zero for the 4-divergence of the entropy flux, which corresponds to the adiabatic character of the motion[5].

As it was mentioned, Eq. (9) can be derived also phenomenologically, i.e. without the use of the moments of the Vlasov equation. In this case one should proceed from the expansion (8) of the energy-momentum tensor of plasma particles and with account of the field induced by them add the energy-momentum tensor (5) of the electromagnetic field[6]. The energy-momentum conservation for the system will be given by Eq. (6) which after projection on the direction perpendicular to the four-velocity[4,5] brings us again to Euler's equation of motion for the relativistic charged fluid (9). Multiplying scalarly (6) by the four-velocity we will obtain in virtue of the field tensor antisymmetry a relation which coincides with the condition of adiabatic motion[5].

The continuity equation $\partial_i(n_e u^i)$ derived as the first moment of Vlasov's equation (1) in the three-dimensional case preserves its form

$$\frac{\partial n_e}{\partial t} + div(n_e \mathbf{v}) = 0 \tag{12}$$

in the relativistic case too, where n_e is the electron density in the lab. system $n_e = n_p\gamma$, n_p is the density in the proper local system, $\mathbf{v}$ is the velocity in the lab. system.

3 THERMAL EFFECTS IN PLASMA WAKE FIELD ACCELERATION

We apply the equation of motion (10), the continuity equation (12) and the Maxwell equations (4) to the problem of wake wave generation in a neutral plasma with ions at rest and an electron bunch moving along the z axis with a constant velocity $v_z = v_0$ with a density of bunch electrons n_b and bunch length d in the lab. system.

We assume that the bunch is infinitely extended in the transverse directions. In the calculation of the plasma electron energy-momentum tensor we adopt the model of a relativistic ideal gas where the momentum distribution is given by the Juttner distribution[4]. Then the state equation will be

$$P = n_p kT = \frac{n_e kT}{\gamma} \tag{13}$$

and the thermal function (enthalpy) calculated for a unit volume

$$\begin{aligned} w = \varepsilon + P &= n_p mc^2 K_3\left(\frac{mc^2}{kT}\right)\Big/ K_2\left(\frac{mc^2}{kT}\right) \\ &= \frac{n_e mc^2}{\gamma}\left[1 + \frac{5}{2}\frac{kT}{mc^2} + \frac{15}{8}\left(\frac{kT}{mc^2}\right)^2 + \cdots\right], \end{aligned} \tag{14}$$

where K_2, K_3 are the modified second-order Bessel functions[4].

Authors of Refs 1–3 have adopted the water-bag model for the plasma electrons (the momentum distribution function is constant in the momentum range $p_+ \div p_-$ and is zero beyond this range). One can readily see that the distinction between the adopted models results only in an insignificant difference in numerical coefficient of temperature power series expansions.

Now we introduce the effective electron mass

$$m^* = mK_3\left(\frac{mc^2}{kT}\right)\Big/ K_2\left(\frac{mc^2}{kT}\right). \tag{15}$$

One can see from the expansions (14) that for $kT \ll mc^2$

$$\frac{m^*}{m} = 1 + \frac{5}{2}\frac{kT}{mc^2} + \frac{15}{8}\left(\frac{kT}{mc^2}\right)^2 + \cdots$$

For $kT = mc^2 (T = 6.10^9\ {}^\circ\mathrm{K})$ $m^*/m = 4{,}3$; however, at such high temperatures one should take into account electron-positron pair creation. This effect may be treated in the same manner as outlined in Ref. 9. However, here the equation of motion for electrons (9) should be complemented with that for positrons the presence of which will in turn affect via Maxwell's equations the electromagnetic (electric in our case) field in the plasma. The resultant set of equations complemented with the continuity equations for electrons and positrons with adequate boundary conditions at $\tilde{z} = d$ differs substantially from the set of equations (9, 12, 4) analyzed here, and is a subject of separate consideration. Therefore, below we will restrict ourselves to the case $kT \ll mc^2 = 0.5$ MeV. Recall that for the flowing discharge $kT = 2$ eV, for solar corona $kT = 200$ eV, for laser thermonuclear fusion $kT \approx 5$ keV.

Now we write out the equation of motion (10) for the one-dimensional case and adopt as usual all the quantities are dependent on $\tilde{z} = z - v_{ph}t$, where $v_{ph} = v_0$ (the stationary case). Let us introduce the momentum $p_z^* = m^* v_z \gamma$ and the dimensionless momentum $\rho_z = p_z^*/m^*c = \beta_z\gamma$. Just as in Refs 1–3 we will consider $\gamma = (1 - v_z^2/c^2)^{-1/2} = (1 - \beta_z^2)^{-1/2}$ assuming that the oriented motion of plasma electrons along the z axis has a velocity much higher than the thermal velocity. Ultimately, Eq. (10) will take the form

$$(\beta_z - \beta_0)\frac{d\rho_z}{d\tilde{z}} = -\frac{1}{m^*c^2 n_e}\frac{dP}{d\tilde{z}} - \frac{\gamma^2\beta_z}{m^*c^2 n_e}(\beta_z - \beta_0)\frac{dP}{d\tilde{z}} - \frac{e}{m^*c^2}E_z, \tag{16}$$

where e is the absolute value of the electron charge, $\beta_0 = v_0/c$.

From the continuity equation (12) under the same assumptions and using the conditions $\rho_z(d) = 0$, $n_e(d) = n_0$ at the bunch front we will have

$$n_e = \frac{n_0\beta_0}{\beta_0 - \beta_z} = \frac{n_0\beta_0\sqrt{1+\rho_z^2}}{\beta_0\sqrt{1+\rho_z^2} - \rho_z}. \tag{17}$$

Using expressions (13) and (17) and having in mind that $\beta_z = \rho_z/\sqrt{1+p_z^2}$ from (16) we can express the field E_z through ρ_z as follows:

$$\frac{eE_z}{m^*c^2} = -\frac{d}{d\tilde{z}}[\sqrt{1+\rho_z^2} - \beta_0\rho_z] - \frac{kT}{m^*c^2}\frac{[\sqrt{1+\rho_z^2} - \beta_0\rho_z]^2}{\sqrt{1+\rho_z^2}[\beta_0\sqrt{1+\rho_z^2} - \rho_z]}\frac{d\rho_z}{d\tilde{z}}. \tag{18}$$

Using then the Poisson equation for the region inside the bunch ($n_b \neq 0$)

$$\frac{dE_z}{d\tilde{z}} = 4\pi e(n_0 - n_b - n_e), \tag{19}$$

multiplying it by E_z and integrating over $\tilde{z}$ using also (17), (18) and the boundary conditions $\rho_z(d) = 0$, $E_z(d) = 0$, at the bunch front we arrive at the following expression for the field E_z inside the bunch ($n_b \neq 0$) which with indices z omitted we denote by E_b:

$$\begin{aligned} E_b = \pm\sqrt{2}\frac{m^*c\omega_p}{e}&\left\{\left[\left(1 - \frac{n_b}{n_0}\right)(1 - \sqrt{1+\rho^2}) - \frac{n_b}{n_0}\beta_0\rho\right]\left(1 - \frac{kT}{m^*c^2}\right) - \frac{kT}{m^*c^2}\right.\\ &\times\left[1 - \frac{\beta_0}{\beta_0\sqrt{1+\rho^2} - \rho} + \left(\gamma_0 - \frac{n_b}{n_0}\frac{1}{\gamma_0}\right)\right.\\ &\left.\left.\times\ln\left|\frac{\beta_0(\sqrt{1+\rho^2} - \beta_0\rho + \sqrt{1-\beta_0^2})}{(1+\sqrt{1-\beta_0^2})(\beta_0\sqrt{1-\rho^2} - \rho)}\right|\right]\right\}^{1/2}, \end{aligned} \tag{20}$$

where

$$\omega_p^* = \sqrt{4\pi e^2 n_0/m^*},\ \gamma_0 = (1 - \beta_0^2)^{-1/2},\ \beta_0 = \frac{v_{ph}}{c} = \frac{v_0}{c}.$$

For the case $T = 0$ the expression for E_b coincides with that obtained earlier[10–13] for the field inside the bunch in a cold relativistic hydrodynamic plasma. As for the nonrelativistic plasma ($\rho \ll 1$), here temperature corrections practically do not change the field.

The range of variation of ρ is determined from the positivity condition of the radicand in (20). At $T = 0$ we have from this condition[10–13].

$$-\rho_0 \leq \rho \leq 0,\ \rho_0 = \frac{2a\beta_0}{1 - a^2\beta_0^2},\ a = \frac{n_b/n_0}{1 - n_b/n_0},\ a\beta \lesssim 1, \tag{21}$$

$\rho = -\rho_0$ is the root of this expression, and the field is zero at $\rho = -\rho_0$.

In the considered case $T \neq 0$, $kT/mc^2 \ll 1$ the range of variation of ρ is determined by some maximum value $\rho_0^* = -\rho_0 + \varepsilon\rho'$, $|\varepsilon\rho'| \ll \rho$, $\varepsilon \equiv kT/m^*c^2 \ll 1$. Substituting this value of ρ into (20) and equating it to zero we will obtain the following correction to $-\rho_0$ up to the first-order term in kT/m^*c^2:

$$\rho' \approx \frac{\sqrt{1-\beta_0^2}}{3}(1-\sqrt{1-\beta_0^2}),\ \rho_0^* = -\frac{\beta_0}{\sqrt{1-\beta_0^2}}\left[1 - \frac{kT}{3\beta_0^2 m^*c^2}(1-\beta_0^2)(1-\sqrt{1-\beta_0^2})\right],$$

$$\rho_0^* \leq \rho \leq 0 \quad \text{for } a = \frac{1}{1+\sqrt{1-\beta_0^2}} \tag{22}$$

and

$$\rho_0^* = -2\frac{n_b}{n_0}\beta_0\left(1 + \frac{2kT}{\beta_0^2 m^*c^2}\right) \quad \text{for} \quad a \ll 1.$$

Now we turn to the determination of the wake fields behind the bunch. To do that one should put $n_b = 0$ in (19) and integrate the set of equations (18) and (19) over $\tilde{z}$ with respect to boundary conditions on the rear boundary of the bunch ($\tilde{z} = 0$). Here the values of $E(0)$ and $\rho(0)$ are determined by the bunch length d and are assumed continuous on the boundary.

Let $E(0) = 0$, $\rho(0) = \rho_0^*$. Then after integration we have the following expression for the wake field:

$$\begin{aligned} E = \pm \frac{m^*c\omega_p^*}{e}\sqrt{2}\,\Bigg\{&(\sqrt{1+\rho_0^{*2}} - \sqrt{1+\rho^2})\left(1 - \frac{kT}{m^*c^2}\right) \\ &- \frac{kT}{m^*c^2}\left[\frac{\beta_0}{\beta_0\sqrt{1+\rho_0^{*2}} - \rho_0^*} - \frac{\beta_0}{\beta_0\sqrt{1+\rho^2} - \rho}\right. \\ &\left.+ \gamma_0 \ln\left|\frac{(\beta_0\sqrt{1+\rho_0^{*2}} - \rho_0^*)(\sqrt{1+\rho^2} - \beta_0\rho + \sqrt{1-\beta_0^2})}{(\sqrt{1+\rho_0^{*2}} - \beta_0\rho_0^* + \sqrt{1-\beta_0^2})(\beta_0\sqrt{1+\rho^2} - \rho)}\right|\right]\Bigg\}^{1/2}. \end{aligned} \tag{23}$$

For $T = 0$ this expression coincides with the corresponding expression for the wake field without account of thermal oscillations of the plasma as obtained in Refs 10–13 and its maximum value differs insignificantly from the maximum value of the field (23) under the conditions $kT/mc^2 \ll 1$ and (22).

The expression for $d(\rho)$ is given by the second integral of the set (18–19). However the values of d for different ρ and $kT/mc^2 \ll 1$ will again differ insignificantly from the results obtained in Ref. 10.

We also present the expression for the wake field for arbitrary length d of the bunch, i.e. for arbitrary given (versus d) values of $E_b(0)$ and $\rho(0)$ on the rear boundary of the bunch under continuity condition:

$$E = \pm \frac{m^* c\omega_p^*}{e} \sqrt{2} \left\{ \left[\left(1 - \frac{n_b}{n_0}\right) - \frac{n_b}{n_0} (\beta_0 \rho(0) - \sqrt{1+\rho^2(0)}) - \sqrt{1+\rho^2} \right] \left(1 - \frac{kT}{m^* c^2}\right) \right.$$
$$- \frac{kT}{m^* c^2} \left[1 - \frac{\beta_0}{\beta_0 \sqrt{1+\rho^2} - \rho} - \frac{n_b}{n_0 \gamma_0} \ln \left| \frac{\beta_0(\sqrt{1+\rho^2(0)} - \beta_0 \rho(0) + \sqrt{1-\beta_0^2})}{(1+\sqrt{1-\beta_0^2})(\beta_0\sqrt{1+\rho^2(0)} - \rho(0))} \right| \right.$$
$$\left. \left. + \gamma_0 \ln \left| \frac{\beta_0(\sqrt{1+\rho^2} - \beta_0 \rho + \sqrt{1-\beta_0^2})}{(1+\sqrt{1-\beta_0^2})(\beta_0\sqrt{1+\rho^2} - \rho)} \right| \right] \right\}^{1/2}.$$

Here $\rho(0)$ is determined from the second integral of the set (18–19) for given d, and $E(\rho(0))$ - in accordance with the continuity condition for the field on the rear boundary of the bunch is equal to $E_b(\rho)$ (Eq. (20)) at $\rho = \rho(0)$.

Thus is follows from the above-stated results that the account of thermal motion of plasma electrons leads in all cases to inessential corrections to wake wave characteristics for temperatures practically attainable under laboratory conditions. It is worth to note that this main conclusion of the present paper differs from that of Refs 1–3. In particular this is a result of an incorrect transition from Eq. (22) to Eq. (23) in Ref. 2.

In order to elucidate the question whether the plasma electrons are trapped by the wake wave, we will use the Hamiltonian formalism[14,15]. The Hamiltonian for the plasma electrons with account to temperature effects is

$$H(\rho, \tilde{z}) = [\sqrt{1+\rho^2} - \beta_0 \rho]\left(1 - \frac{kT}{m^* c^2}\right) - \frac{e\phi(\tilde{z})}{m^* c^2}$$
$$+ \frac{kT}{m^* c^2} \sqrt{1-\beta_0^2} \ln \left| \frac{\beta_0(\sqrt{1+\rho^2} - \beta_0 \rho + \sqrt{1-\beta_0^2})}{(1+\sqrt{1-\beta_0^2})(\beta_0 \sqrt{1+\rho^2} - \rho)} \right| \tag{25}$$

being an invariant quantity which is equal to $H = 1 - kT/m^* c^2$; $\phi(\tilde{z}) = \varphi(\tilde{z}) - \varphi_0$, where φ_0 is the value of the potential at points with $\rho = 0$. If $H > H_s$ where H_s is the value of the Hamiltonian on the separatrix, then the plasma electrons are not trapped and are trapped for $H \leq H_s$. To calculate H_s, one should find out the unstable equilibrium points from conditions $\partial H/\partial \rho = 0$, $-\partial H/\partial \tilde{z} = 0$. These points correspond to the values of momentum $\rho_s = \beta_0 - \beta_T/\sqrt{1-\beta_0^2}\sqrt{1-\beta_T^2}$ where $\beta_T^2 \equiv kT/m^* c^2$, and of coordinate $\tilde{z}_s$ at which the potential energy $-e\phi(\tilde{z})/m^* c^2$ achieves a maximum. The allowed limiting value of momentum at the point $\tilde{z}_s$ that follows from the positivity requirement for (17) and expression (22) is

$$\rho_0^* = \frac{\beta_0}{\sqrt{1-\beta_0^2}} - \frac{\beta_T^2}{3\beta_0} \sqrt{1-\beta_0^2}(1 - \sqrt{1-\beta_0^2})$$

being attained for bunch density $n_b = n_0/(2 + \sqrt{1 - \beta_0^2})$. Calculating the value of H_s at these points, one can see that always $H \gg H_s$, i.e. the trapping condition is not satisfied and the plasma electrons cannot be trapped by the wake wave in the considered case.

4 DISCUSSION OF THE RESULTS AND CONCLUSION

The idea using longitudinal wake fields excited by electrons or electron bunches moving through plasma to accelerate charged particles were set up by B. M. Bolotovsky[16] and Ya. B. Fainberg[17] in the early fifties. The initial studies of nonlinear effects relevant to this process were made at nearly the same time by A. I. Akhiezer and R. V. Polovin who formulated and exactly solved the equations for free nonlinear longitudinal waves in an infinite relativistic plasma[18] (see also the work of A. Cavaliere[19]). These results were used by R. J. Noble[20,21] in a study of some quantities relevant to the plasma beat-wave accelerator. In the works of the Yerevan group[22,23,10–13] more or less complete consideration of the nonlinear effects in generation of wake waves excited by extended relativistic electron bunches was carried out. In particular, it was shown, that if $n_b \lesssim n_0/(2 + 1/\gamma_0)$ the maximum value of the wake field (wave breaking limit) $E_{\max} \simeq \sqrt{2}m\omega_p c\gamma_0^{1/2}/e$, where γ_0 is the Lorents factor of the relativistic electron bunch exciting the wake. The Dawson[24] wave breaking limit is equal to $E_{\max}$ when $\gamma_0 \approx 1$. In the linear case $n_b/n_0 \ll 1$ $E_{\max} \simeq 2(m\omega_p c/e)(n_b\beta_0/n_0)$ for arbitrary γ_0. Close results have been obtained by P. Chen, J. M. Dawson, R. W. Huff and T. Katsouleas[25], R. D. Ruth, A. W. Chao, P. L. Morton, P. B. Wilson[26], and later by J. B. Rosenzweig[27,28] where the idea of plasma wake field acceleration was renewed independently of[16,17] as an alternative to the plasma beat wave acceleration scheme.

As a numerical example for the value of the attainable wake field in the nonlinear case let us take $n_b \simeq 10^{11}\ \mathrm{cm}^{-3}$, $n_0 \simeq 2.10^{11}\ \mathrm{cm}^{-3}$ and the energy of the driving electron bunch 30 MeV. Then $E_{\max} = 4,6 \times 10^8\, V/m$ and the acceleration gradient is 460 MeV/m.

In the works of T. Katsouleas and W. B. Mori[2] and J. B. Rosenzweig[3] an attempt have been made to take into account the thermal effects and plasma electron trapping by the wave wake in generation wake fields. The results of the present work show that the plasma electrons cannot be trapped by the wake wave in the considered case and hence cannot change the amplitude of the wake wave. Thermal effects (see (23), (24)) are proportional to kT/mc^2 and the region of validity of the performed calculation are small.

We calculate from (23) the maximum value of the wake electric field which corresponds to $\rho = 0$ and is equal to

$$E_{\max}(T) \simeq \frac{\sqrt{2}mc\omega_p}{e}(\gamma_0 - 1)^{1/2}\left(1 + \frac{3}{4}\frac{kT}{mc^2} + \cdots\right) \tag{26}$$

TABLE 1
Numerical examples for the ratio $E_{max}(T)/E_{max}(0)$

KT (eV)	2	20	200	5×10^3	5×10^4
T°K	2.3×10^4	2.3×10^5	2.3×10^6	5.8×10^7	5.8×10^8
Nonlinear case					
$\frac{E_{max}(T)}{E_{max}(0)} - 1, \frac{n_b}{n_0} = \frac{1}{2 + 1/\gamma_0}$	3×10^{-6}	3×10^{-5}	3×10^{-4}	7.5×10^{-3}	7.5×10^{-2}
Linear case					
$\frac{E_{max}(T)}{E_{max}(0)} - 1, \frac{n_b}{n_0} \ll 1$	1.1×10^{-5}	1.1×10^{-4}	1.1×10^{-3}	2.75×10^{-2}	1.75×10^{-1}

for the nonlinear case $n_b/n_0 \simeq 1/(2 + 1/\gamma_0)$, $d \simeq 4v_0/\omega_p\gamma_0$ and

$$E_{max}(T) = \frac{2mc\omega_p}{e} \frac{n_b}{n_0} \beta_0\left(1 + \frac{11}{4} \frac{kT}{mc^2} + \cdots\right) \tag{27}$$

for the linear case $n_b/n_0 \ll 1$, $d = \pi v_0/\omega_p$. It is interesting to note that combination of the different thermal effects results a net increase of the maximum value of the wake field with the temperature (up to the first order of magnitude of the small quantity kT/mc^2). This is due to the increase of the effective plasma electron mass m^* (see (15)) and consequently ω_p^* with the temperature in spite of the decreasing role of the factors in Eqs. (23), (24) depending on temperature.

In Table 1 we present the numerical results for the ratio $E_{max}(T)/E_{max}(0)$ as a function of kT (and T) for the nonlinear and linear cases. It is evident that for a wide range of practically attainable values of the temperature the thermal effect is small. Therefore, in planning new experiments on generation of wake waves for charged particles acceleration and in the discussion of the results of already performed ones[29–31], it is impossible to expect more or less drastic deviations from the theoretical predictions obtained for cold (linear or nonlinear) plasma due to thermal and trapping effects.

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