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HYBRID NORMALCONDUCTING/SUPERCONDUCTING RF SYSTEM FOR HIGH LUMINOSITY CIRCULAR e^+e^- COLLIDERS

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The new generation of high luminosity e^+e^- colliders require both intense beam and short bunches. This necessitates high power and voltage from the RF system. In this context, using a combination of normal conducting and superconducting RF cavities to achieve a partial or complete separation of functions, powering and longitudinal focusing, seems to be an optimum solution. Within this scheme, it is particularly attractive to operate the superconducting cavities in an entirely idle mode (no external RF source). The steady state and dynamic behaviour of such a system is analyzed and the original Robinson stability criterion extended to the more general case of a double RF system. The B-Meson Factory proposed at CERN (BFI) is taken as an example of one possible application. No major critical issues, as compared to conventional methods, are encountered. On the contrary, the partial or complete separation of functions, characteristic of this system, makes it very flexible. Assuming that the present ideas are confirmed by more detailed studies and experiments with real beam, a large domain of applications could be envisaged.

1 INTRODUCTION

In the feasibility study for a B-Meson Factory using the CERN ISR tunnel and the LEP injection system (BFI)[1], a combination of normal conducting (n.c.) and superconducting (s.c.) RF cavities has been proposed[2,3] as an alternative in the context of upgrading the luminosity from 10^{33} towards 10^{34} $cm^{-2}\,s^{-1}$. In such a hybrid n.c./s.c. system, both systems may contribute to the compensation of the beam energy losses as well as to the longitudinal focusing of the bunches. Another possibility is to limit the role of the s.c. system to a pure focusing task by setting its synchronous phase to zero (no acceleration). In this latter scheme, the s.c. system might be operated in a completely idle mode (i.e., no external RF source). The RF power required for the compensation of the beam energy losses would be entirely provided by the n.c. system. A proper detuning of the s.c. cavities should permit use of the beam induced voltage as the focusing voltage.

The dynamic behaviour of a single RF system, in the presence of beam loading, has been already extensively treated[4–7]. Our purpose is to extend the previous results to the more general case of two RF systems coupled through the beam and quantitatively estimate the effects for the BFI.

2 GENERAL STEADY STATE DESCRIPTION OF THE FUNDAMENTAL INTERACTION BETWEEN THE BEAM AND THE RF SYSTEM

For this study, only the beam-RF system interactions at the fundamental frequency will be considered. The higher order modes (HOM) of the RF cavities are assumed to be sufficiently damped and taken into account only as extra beam energy losses. In addition, we will assume short bunches and $T_b/T_f \ll 1$ (T_b: time between bunch passages, T_f: cavity filling time), two conditions which are fulfilled for the BFI.

The fundamental interaction between the beam and an RF system (see appendix 1) may be then modelled by the equivalent lumped circuit of Figure 1-a and any steady state condition represented by the corresponding phasor diagram of Figure 1-b.

In the presence of two different RF systems, each of them may be described as above† and an equilibrium or steady state condition will be defined by:

$$\Delta U_0/e = V_1 \sin \Phi_{S1} + V_2 \sin \Phi_{S2} \tag{1}$$

$$\sigma_s \propto (h_1 V_1 \cos \Phi_{S1} + h_2 V_2 \cos \Phi_{S2})^{-1/2} \tag{2}$$

In the particular case where the second RF system is idle, the equivalent circuit and phasor diagram are reduced to those of Figures 2-a and b. The additional relations as compared to the general case are:

$$i_{g2} = 0 \tag{3}$$

$$\Phi_{S2} = \Psi_2 - \pi/2 \qquad (<0 \text{ for focusing}) \tag{4}$$

For a s.c. system, $\Psi_2 \simeq \pi/2$ ($\Phi_{S2} \simeq 0$) and the induced voltage is then given by:

$$V_2 = \left(\frac{R}{Q}\right)_2 I_b f_{r2} \,/\, \delta f_2 \tag{5}$$

The beam–cavity interaction corresponds to a power lost by the beam, equal to the cavity dissipation:

$$P_{b2} = V_2 I_b \sin \Phi_{S2} = -P_{d2} = -\frac{1}{2}\frac{V_2^2}{R_2} \qquad (|P_{b2}| \ll P_{b1}) \tag{6}$$

Typical numerical examples are presented in Table I where different solutions are compared for one of the possible $L \simeq 10^{34}$ cm^{-2} s^{-1} versions of the BFI[1–3]:

- n.c. system (when possible);
- s.c. system;
- hybrid n.c./s.c. system, (*) externally powered s.c. cavities;
- hybrid n.c./s.c. system, (**) idle s.c. cavities.

† The parameters will be generally defined as in appendix 1. The index 1 and 2 will be used for the n.c. and s.c. system, respectively.

TABLE I

Possible RF Systems for the $L = 10^{34}\ \mathrm{cm^{-2}\,s^{-1}}$ "Asymmetric Flat Beam Version" of the BFI ($n_{\mathrm{bunch}} = 320$, $\sigma_s = 4.8$ mm, $E = 3.5 * 8$ GeV, $I_{\mathrm{beam}} = 2.56 * 1.12$ A).

	n.c. 3.5 GeV	s.c. 3.5 GeV	s.c. 8 GeV	n.c./s.c.* 8 GeV	n.c./s.c.** 8 GeV
n_{cav}	40	8	64	12/40	32/40
V_{acc}[MV]	20.0	20.0	119.0	5.5/113.5	10.0/110.0
E_{acc}[MV/m]	1.7	8.5	6.3	1.5/9.4	1.0/9.1
$P_{\mathrm{dis}}^{1\mathrm{cav}}$[kW]	37.0	—	—	30.0/—	14.0/—
$P_{\mathrm{HOM}}^{1\mathrm{cav}}$[kW]	35.0	35.0	6.5	6.5/6.5	6.5/6.5
P_{beam}[MW]	2.5	1.5	6.8	2.3/4.4	6.8/0.0
$P_{\mathrm{input}}^{1\mathrm{cav}}$[kW]	100.0	180.0	110.0	220.0/110.0	230.0/0.0
$P_{\mathrm{tot}}^{\mathrm{RF}}$[MW]	4.0	1.5	6.8	2.6/4.4	7.3/0.0
n_{klyst}	5	2	8	3/5	8/0

* Powered s.c. cavities; **idle s.c. cavities.

In all cases, a matched condition is assumed at full beam current (i.e., compensation of the reactive power by detuning and appropriate coupling). The number of cavities is adjusted for practical values of the accelerating gradient, input coupler power, HOM and fundamental dissipation, as well as for easy power splitting (the number of cavities supplied by one klystron is a power of two). Instability thresholds have also been taken into account. For the hybrid n.c./s.c. versions, the frequency of both systems is 500 MHz ($h_1 = h_2$).

The use of s.c. cavities is quasi unavoidable in this version of the BFI because of the space constraints. A hybrid n.c./s.c. system allows a reduction of the number of s.c. cavities for an equivalent total number of cavities and power requirements. Moreover, this solution opens the possibility of operating the s.c. system in an idle mode (no external RF source). Together with the reduction of the number of s.c. cavities, the absence of input coupling ports may represent a serious advantage, especially from the point of view of the parasitic impedances which constitute one of the most critical issue in this machine.

3 RESPONSE TO SMALL MODULATIONS (STABILITY)

It is obvious, on the phasor diagram of Figure 1–b, that any modulation of Φ_S induces modulations of $\tilde{V}$ (amplitude and phase) which, transmitted back through the beam, result in a modification of the synchrotron motion. Depending on the steady state operating condition, this process may become unstable.

"Fast" external loops for controlling the tuning, the amplitude or the phase of the RF parameters might completely alter the behaviour of the global system[6,7]. A situation without (or with "slow") control loops will be assumed in this section.

In the presence of a single RF system, the stability condition is defined by the well known Robinson criterion[5] which, in our notations, can be expressed as:

$$0 < \sin 2\Psi < 2 \cos \Phi_S / Y, \qquad \text{where } Y = i_b/I_0 \tag{7}$$

Extended to the more general case of two combined RF systems, the upper or "high current" stability limit, obtained by a numerical method (see appendix 2), becomes:

$$\sin 2\Psi_1 < \frac{2\cos\Phi_{S1}}{Y_1} + \frac{h_2 V_2}{h_1 V_1 Y_1}\,(2\cos\Phi_{S2} - Y_2 \sin 2\Psi_2) \tag{8}$$

If the second RF system is idle, the second term of the right hand side cancels ($\Phi_{S2} = \Psi_2 - \pi/2$) and we once again obtain the classical Robinson limit for the first RF system, as if it was alone:

$$\sin 2\Psi_1 < 2\cos\Phi_{S1}/Y_1 \tag{9}$$

The main conclusion of this section is that, in the presence of two RF systems, if one of them is idle, the choice for the other of a stable steady state operating condition is sufficient for automatically getting the global system stable; in the general case where both RF systems are externally powered, the stability limit is given by (8). As already mentioned, the situation might be completely modified by the action of "fast" control loops.

4 TRANSIENT BEHAVIOUR (RESPONSE TO A STEP CHANGE, ΔI_b)

Until now, the ring was assumed to be in its storage mode (i.e., at constant or "slowly" variable beam current, $I_b \simeq I_{b\,\max}$). During the accumulation, each injection pulse will be "seen" by the RF system like a step increase, ΔI_b of I_b. These sudden changes of beam loading, in the absence of "fast" compensation, will temporarily destroy the steady state equilibrium and result in transient oscillations of the RF voltage amplitude and phase.

Due to the slow filling rate of the BFI (>5 minutes), the transient effects are negligible ($\Delta I_b/I_0 < 5\%$) for all the proposed schemes, except—a priori—for the "idle s.c. system version" where $\Delta I_b \gg I_{02}$. For this reason, the study will be now concentrated on this particular case.

The transient build up of the RF voltage, in response to a step change ΔI_b, is described in appendix 3. The resulting effect is a modulation of the energy exchanged between the beam and the RF system. If the s.c. system could be considered individually (as a "free oscillator"), the frequency of the modulation would be δf_2 and its amplitude, expressed in terms of instantaneous power,

$$\Delta\hat{P}_2 = \Delta V_2 I_b, \qquad \text{with } \Delta V_2 = \left(\frac{R}{Q}\right)_2 \Delta I_b f_{r2}\,/\,\delta f_2 \tag{10}$$

Its damping rate would correspond to the natural s.c. cavity filling time ($T_{f2} \simeq 600$ ms). However, the s.c. system cannot be treated independently since it is coupled to the n.c. system through the beam. A complete analysis of the problem requires considering the global system response (in particular, its slowest damping rate) which will depend on the steady state operating conditions of both systems.

5 POSSIBLE OPERATING CONDITIONS FOR THE HYBRID N.C./IDLE S.C. SYSTEM

The steady state properties of the hybrid n.c./idle s.c. system may be summarized by the following relations:

$$P_{b1} \simeq P_b = V_1 I_b \sin \Phi_{S1} \qquad \left(-P_{b2} = \frac{1}{2}\frac{V_2^2}{R_2} \ll P_{b1}\right) \tag{11}$$

$$V_2 = \left(\frac{R}{Q}\right)_2 I_b f_{r2} / \delta f_2 \tag{12}$$

$$\sigma_s \propto (h_1 V_1 \cos \Phi_{S1} + h_2 V_2)^{-1/2} \qquad (h_2 V_2 \gg h_1 V_1 \cos \Phi_{s1}) \tag{13}$$

5.1 N.c. System

Classical methods of operation can be applied for the n.c. system:

- compensation of the reactive power by a proper detuning, δf_1;
- fixed coupling, adjusted such that matching is realized at maximum beam current:

$$\beta_1 = 1 + \left(\frac{P_{b1}}{P_{d1}}\right)_{I_{b\max}} \tag{14}$$

With this mode of operation, the stability criterion given by (9) is automatically fulfilled.

In the storage regime ($I_b \simeq I_{b\,\max}$), the role of the n.c. system uniquely consists in providing the power for the compensation of the beam energy losses. The focusing voltage is assumed to be achieved by the s.c. system. This "separation of functions" offers a degree of freedom in the choice of the working point (Φ_{S1}, V_1). Although it is theoretically possible to operate close to $\Phi_{S1} = 90°$ (minimum voltage and therefore, power requirements), it is practically more realistic to choose $\Phi_{S1} \ll 90°$. This ensures a larger stability margin at the expense of a relatively low increase in power consumption. Table II gives numerical values of the main RF parameters, at

TABLE II

Typical n.c. System Operating Conditions.

Φ_{S1} [°]	V_1 [MV]	P_{d1} [kW]	P^{RF}_{total} [MW]	β_1	δf_1 [kHz]
25.0	14.4	920.0	7.7	8.4	80.0
30.0	12.0	660.0	7.5	11.3	94.0
35.0	10.6	500.0	7.3	14.6	97.0
40.0	9.5	400.0	7.2	18.0	100.0
45.0	8.6	330.0	7.1	21.6	103.0

$P_{b1} = 6.8$ MW; $I_{b\max} = 1.12$ A; $P_{\text{available}} = 8$ MW; $Q_1 = 50\ 10^3$; $(R/Q)_1 = 73$ Ω/cav; $n_{\text{cav}1} = 32$; $T_{f1} = T_{f01}/(1 + \beta_1)$ and T_{f01} (unloaded) $= 32$ μs.

TABLE III
Bucket Width for Different Working Points (500 MHz).

Φ_{S1} [°]	w_1 [cm]	P^{RF}_{total} [MW]	Φ'_{S1}** [°]	w'_1** [cm]
40.0	25.8	7.2	28.0	32.7
35.0	28.5	7.3	26.0	33.8
30.0	31.5	7.5	23.0	35.7
25.0	34.5	7.7	19.0	38.3
*3.0	52.0	6.8		

* The last line corresponds to the single s.c. system version of Table I.
** Minimum bucket width, w'_1 obtained by choosing a different working point ($\Phi'_{S1} < \Phi_{S1}$) at low beam loading ($\delta f_2 \simeq 10$ kHz assumed).

different working points, for the BFI. It is clear that, if we want to preserve the matching condition at $I_{b\,\max}$,the value of β_1 is directly related to the chosen working point (Φ_{S1}, V_1). The example which was presented in Table I (column 5) corresponds to $\Phi_{S1} = 37.5°$.

At the beginning of the accumulation, when $V_2 \ll V_1$ ($V_2 \propto I_b$), the n.c. system alone must provide an RF bucket adequate for trapping of the injected bunches ($\sigma_s \simeq 8$ cm). As soon as $V_2 \gg V_1$, the bucket will be mainly determined by V_2. The first two columns of Table III give the correspondence between Φ_{S1} and the bucket width at 500 MHz (for $V_2 = 0$). The available power of 8 MW should permit a correct trapping, even for $V_2 = 0$.

The situation may be still improved by using the power reserve existing at low beam loading and choosing a working point different of that fixed at full beam current ($\Phi'_{S1} < \Phi_{S1}$, $V'_1 > V_1$), until $V_2 \gg V_1$. This process is strongly limited by the reinforced mismatch and its efficiency depends on the increasing rate of V_2 versus I_b (see next section). Examples of possible improvements are given in the two last columns of Table III†.

5.2 *Idle s.c System*

The voltage induced in the idle s.c. system is given by (12). For any particular cavity geometry ($(R/Q)_2$ and f_{r2} fixed), only one parameter is free, the detuning, δf_2 which will determine the increasing rate of V_2 versus I_b. During the accumulation, since the n.c. system alone is able to trap the injected bunches, δf_2 can be freely adjusted and fixed to its final value (imposed by the required bunch length), at the end of the filling operation. The minimization of the transient effects will certainly be preponderant in the choice of δf_2. The larger δf_2, the weaker the transient amplitude but the faster the frequency of the "free oscillations".

† The contribution of wigglers is included here[1]. Since they will probably be switched off during the accumulation, the situation will be more favourable.

TABLE IV

Typical Operating Conditions and Characteristics of the Transient "Free Oscillations" for the Idle s.c. System.

δf_2 [kHz]	ΔV_2 [kV]	$\Delta \hat{P}_2^{**}$ [kW]	$\Delta \hat{P}_2/P_1$ [%]	$\Delta\Phi_{S1}^{**}$ [°]
*8.2	320.0	350.0	5.0	1.7
25.0	100.0	115.0	1.7	0.6
50.0	50.0	60.0	0.9	0.3
150.0	17.0	20.0	0.3	0.1

$Q_2 = 10^9$; $(R/Q)_2 = 40\ \Omega$/cav; $n_{\text{cav}\,2} = 40$; $V_2 = 110$ MV; $P_{d2} = -P_{b2} = 4$ kW; $T_{f2} = 0.6$ s; f_s ($V_2 = 110$ MV) $= 43$ kHz; ΔI_b (amplitude of the injection pulses) $= 3.25$ mA.

* $\delta f_2 = 8.2$ kHz corresponds to the required value of the detuning in storage regime (for $V_2 = 110$ MV at $I_b = I_{b\max} = 1.12$ A).

** $\Delta \hat{P}_2$ is the maximum variation of the instantaneous power exchanged between the beam and the cavities (at $I_b = I_{b\max}$) during the transient build up of the RF voltage, in response to an injection pulse ($\Delta I_b = 3.25$ mA); $\Delta\Phi_{S1}$ is the equivalent phase change in the n.c. system.

The maximum admissible detuning will depend on how close the cavity resonance can be approached to the next beam spectrum line (310 kHz between two lines), without producing an instability. Since the growth rate of instabilities mainly is determined by the real part of the impedance (δf^{-2} dependance), this is probably not a too severe limitation.

Table IV gives typical operating conditions and the characteristics of the transient "free oscillations" for different values of δf_2, in the s.c. system.

Converted in terms of corresponding amplitude perturbations for the n.c. system, $\Delta \hat{P}_2/P_1$ or $\Delta\Phi_{S1}$, the effects seem quite tolerable. As already mentioned, this simple approach, where both systems are separately treated—whereas they are coupled together through the beam—is not sufficient for a correct estimate of the transient characteristics. This requires studying the global system response and evaluating the amplitudes and damping rates for its different eigenfrequencies.

Results of a preliminary analysis are presented in Table V. The poles of the equivalent transfer function (or roots of the characteristic equation) are calculated for different conditions. In the storage regime ($\delta f_2 \simeq 8.2$ kHz), the longest damping

TABLE V

Poles of the Global System Transfer Function versus δf_2 (at $I_b = I_{b\max} = 1.12$ A and $\Phi_{S1} = 30°$).

δf_2 [kHz]	poles 1, 2 [10^3 s^{-1}]	Poles 3, 4 [10^3 s^{-1}]	Poles 5, 6 [10^3 s^{-1}]
8.2	$-0.2 \mp 4.7j$	$-5.0 \mp 270.0j$	$-400.0 \mp 590.0j$
25.0	$-2.6 \mp 17.5j$	$-2.6 \mp 220.0j$	$-400.0 \mp 590.0j$
90.0	$-5.0 \mp 24.0j$	$-0.1 \mp 570.0j$	$-400.0 \mp 590.0j$
180.0	$-5.0 \mp 24.0j$	$-400.0 \mp 590.0j$	$-0.04 \mp 1130.0j$

time is about 5 ms (two orders of magnitude lower than the s.c. cavity filling time!). During the accumulation, the dynamic properties of the global system can be strongly modified by varying δf_2. The shortest damping times ($\simeq 400$ μs) are obtained for $\delta f_2 \simeq 25$ kHz. These levels of damping, together with the maximum amplitudes ("free oscillations") given in Table IV, tend to confirm that the transient effects are quite moderate and should not pose a serious problem. The condition which minimizes the damping times may not necessarily correspond to the optimum case; in the search for the best compromise, one must simultaneously take into account the relative values of the amplitudes, damping rates and eigenfrequencies. In addition, the optimization depends on I_b; as for the n.c. system, it may be useful to modify the working point (δf_2) during the accumulation. Further investigations are necessary in order to estimate the direct effects on the beam and then define a real "figure of merit" for this multiparameter optimization.

If it were confirmed that the transient effects do not require any special cure, the n.c. system might be simply equipped with "classical slow" amplitude and tuning control loops; for the s.c. system, a servo mechanism controlling the amplitude, V_2, via the action of a "slow" tuner, would be sufficient.

If it were necessary, the transient effects could be compensated with different means:

- by adding "fast" amplitude and phase loops on the n.c. system[7];
- by externally coupling the 2 systems so that a variation of amplitude, V_2, is automatically corrected by an adequate change of V_1 or Φ_{S1}.

Remark The previous analysis has pointed out that, for a given I_b, it was always possible to adjust δf_2 for getting the longest damping time much smaller than T_{f2}, the natural s.c. cavity filling time. However, for a wrong choice of δf_2, in the worst case, the damping time tends to T_{f2}. It could be reduced by coupling an external load to the s.c. cavities. Typically, a factor 10 may be achieved ($T_{f2} \simeq 60$ ms) while maintaining P_{b2} negligible.

6 HIGHER HARMONIC SYSTEM

In the examples previously treated, the frequency of both system was assumed to be 500 MHz. A higher harmonic could be envisaged for the s.c. system: 1.5 GHz, for example, which is a frequency already used in other laboratories (CEBAF, Cornell, Saclay). At higher frequency, for the same focusing strength, a lower voltage is required and one would profit from higher achievable E-field in the s.c. cavities[8].

However, the bucket width provided by the higher harmonic system is not sufficient for trapping of the injected bunches. A means for making $V_2 \ll V_1$ during the accumulation must therefore be found. The problem is nearly the same with a powered or an idle system: due to the high beam loading, it is obviously not helpful just to switch off the RF source and unpractical amounts of external loading or detuning are required to satisfy this condition. Moreover, the difficulty for extracting the HOM power is strongly increased at high frequency.

TABLE VI

Examples of RF Systems for Other Versions of 10^{34} cm^{-2} s^{-1} BFI;
a) *Machine* 2: Asymmetric Round Beams ($n_{\text{bunch}} = 320$, $\sigma_s = 12$ mm, $E = 3.5 * 8$ GeV, $I_b = 3.3 * 1.45$ A);

	n.c. 3.5 GeV	n.c. 8 GeV	s.c. 8 GeV	n.c./s.c.* 8 GeV	n.c./s.c.** 8 GeV
n_{cav}	8	44	36	32/4	36/4
V_{acc}[MV]	3.5	20.5	20.5	9.0/12.0	9.5/11.0
E_{acc}[MV/m]	1.5	1.5	1.9	0.9/9.6	0.9/8.8
$P_{\text{dis}}^{1\text{cav}}$[kW]	27.0	30.0	—	11.0/–	11.0/—
$P_{\text{HOM}}^{1\text{cav}}$[kW]	23.0	4.5	4.5	4.5/4.5	4.5/4.5
P_{beam}[MW]	1.3	8.4	8.3	7.4/0.95	8.3/0.0
$P_{\text{input}}^{1\text{cav}}$[kW]	195.0	220.0	230.0	240.0/235.0	240.0
$P_{\text{tot}}^{\text{RF}}$[MW]	1.6	9.7	8.3	7.7/0.95	8.7/0.0
n_{klyst}	2	11	9	8/1	9/0

b) *Machine* 3: Symmetric Crab Crossing ($n_{\text{bunch}} = 800$, $\sigma_s = 8$ mm, $E = 5.3 * 5.3$ GeV, $I_b = 2.8 * 2.8$ A).

	n.c.	s.c.	n.c./s.c.*	n.c./s.c.**
n_{cav}	24	16	12/4	16/4
V_{acc}[MV]	17.1	17.1	4.9/12.2	5.0/12.1
E_{acc}[MV/m]	2.4	3.5	1.35/10.0	1.05/10.0
$P_{\text{dis}}^{1\text{cav}}$[kW]	72.5	—	24.0/—	15.0/—
$P_{\text{HOM}}^{1\text{cav}}$[kW]	10.0	10.0	10.0/10.0	10.0/10.0
P_{beam}[MW]	3.5	3.4	2.5/0.9	3.4/0
$P_{\text{input}}^{1\text{cav}}$[kW]	217.0	215.0	235.0/225.0	230.0/0.0
$P_{\text{tot}}^{\text{RF}}$[MW]	5.2	3.4	2.8/0.9	3.65/0.0
n_{klyst}	6	4	3/1	4/0

Remark Versions of the $L \simeq 10^{34}$ cm^{-2} s^{-1} BFI, quite different from that presented in Table I and used as reference for our study, have been proposed (Tables VI-a and b)[1,2,3]. Since these machines are fully realizable with n.c. cavities, the possibility of combining cavities of different frequencies has been studied. This solution does not really present any advantage with n.c. cavities. The other possible options are presented in Tables VI-a and b.

7 CONCLUSIONS

This study shows that a hybrid n.c./s.c. RF system is an attractive solution for high luminosity e^+e^- circular colliders, especially with the s.c. cavities operated in an idle mode (no external RF source). A first approach to the problems concerning the stability and control did not point out any particular critical issue as compared to a conventional system. On the contrary, the partial or complete separation of functions (powering and focusing), characteristic of this system, makes it particularly flexible. Further investigations in the domain of evaluating the transient effects are necessary. Assuming that the present ideas are confirmed by more detailed studies, one could envisage an experiment in a real beam. For such a test, the CERN SPS with its unloaded s.c. cavity seems well suited[9].

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APPENDIX 1

Steady state description of the fundamental beam loading ($T_b/T_f \ll 1$, short bunches)

The fundamental interaction between the beam and the RF system may be represented by the equivalent lumped circuit of Figure 1-a.

—The generator is described by its fundamental short circuit current, $2\tilde{i}_g$ and its internal impedance R_g†. The power delivered to a matched load is equal to the generator power:

$$P_l = P_g = \tfrac{1}{2} R_g i_g^2$$

If the load is not matched,

$$P_l = P_g - P_r, \qquad \text{where } P_r \text{ is the reflected power}$$

—The beam is represented by its fundamental component $\tilde{i}_b$; $|\tilde{i}_b|$ is twice the average beam current, I_b, for short bunches.

† These are the values "seen" by the beam (i.e., transformed by the coupling circuit). R_g may represent a circulator.

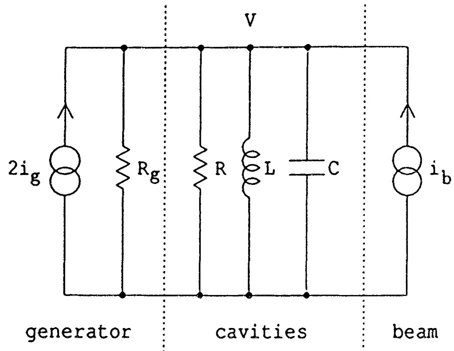

FIGURE 1-a Beam-cavity-generator equivalent lumped circuit.

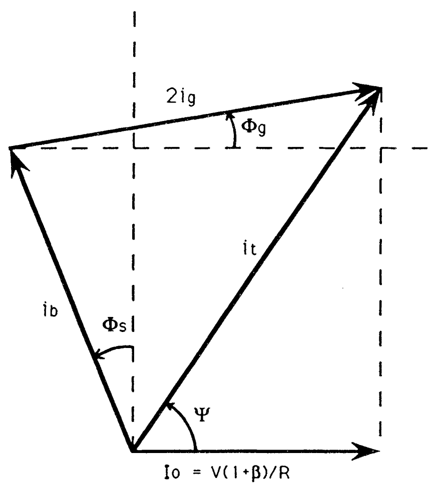

FIGURE 1-b Steady state phasor representation.

—The RF cavities are modelled by an RLC circuit with the following definitions:

$(LC)^{-1/2}/2\pi = f_r$, resonant frequency;
$(L/C)^{1/2} = R/Q$, specific shunt impedance;
R, shunt impedance;
Q, unloaded quality factor;
$\beta = R/R_g$, coupling factor;
$T_f = Q/\pi\ f_r(1 + \beta)$, filling time of the loaded cavity;
V, cavity voltage.

Any steady state condition may be represented by the corresponding phasor diagram of Figure 1-b. The cavity voltage $\tilde{V}$ is chosen as phase reference (real axis). Φ_s is the synchronous phase, Φ_g is the generator phase, h is the harmonic number, f_0 is the revolution frequency, and Ψ represents the cavity detuning:

$$\tan \Psi = \frac{2Q\,\delta f}{(1+\beta)f_r} \qquad \text{where } \delta f = hf_0 - f_r$$

The power delivered to the beam is

$$P_b = VI_b \sin \Phi_s$$

and the cavity dissipation is

$$P_d = \frac{1}{2}\frac{V^2}{R}$$

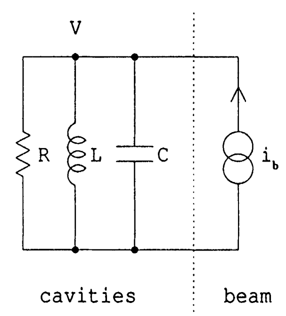

FIGURE 2-a Equivalent circuit for idle cavities.

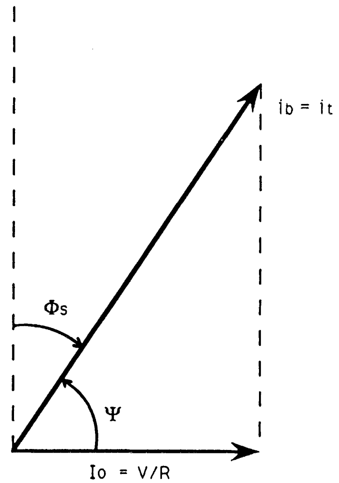

FIGURE 2-b Phasor diagram for idle cavities.

For a given steady state, the minimum generator power requirement, $P_g = P_b + P_d$ (no reflected power), corresponds to the matched condition, when:

i) the reactive power is compensated by a proper detuning, δf ($\Phi_g = 0$),

ii) the coupling factor is adjusted such $\beta = 1 + P_b/P_d$.

An equilibrium or steady state condition is obtained for:

$$P_b = VI_b \sin \Phi_S = I_b \Delta U_0/e$$

where ΔU_0 is the total beam energy loss per turn (including HOM losses). The relation between the RMS bunch length σ_s and the RF voltage V is then:

$$\sigma_s \propto (hV \cos \Phi_S)^{-1/2}$$

Idle RF cavities

In the particular case of idle RF cavities (no external RF source) the equivalent circuit and phasor diagram are reduced to those of Figure 2-a and b. The characteristic of the idle system is that $\Phi_s = \Psi - \pi/2$; the beam induced voltage is then:

$$V = 2RI_b \cos \Psi$$

For unloaded s.c. cavities and large I_b, $\Psi \simeq \pi/2$ ($\Phi_S \simeq 0$, $\cos \Psi \simeq 1/\tan \Psi$) and one gets:

$$V \simeq \frac{2RI_b}{\tan \Psi} = \frac{(R/Q)I_b f_r}{\delta f}$$

$$P_b = VI_b \sin \Phi_S = -P_d = -\frac{1}{2}\frac{V^2}{R}$$

APPENDIX 2

Stability criterion for a double RF system

In order to determine whether a particular steady state condition is stable or not, the response of the global system (beam + RF system 1 + RF system 2) to small phase oscillations (linear approximation) has to be analyzed.

Beam equations

In the presence of a double RF system providing a voltage

$$V = V_1 \sin(2\pi h_1 f_0 t + \Phi_{S1}) + V_2 \sin(2\pi h_2 f_0 t + \Phi_{S2})$$

the equations of motion become:

$$\frac{d}{dt}\left(\frac{\Delta E}{ef_0}\right) = \Delta\Phi_{S1} V_1 \cos \Phi_{S1} + \Delta\Phi_{S2} V_2 \cos \Phi_{S2} - \Delta\Phi_{V1} V_1 \cos \Phi_{S1}$$
$$- \Delta\Phi_{V2} V_2 \cos \Phi_{S2} + \Delta V_1 \sin \Phi_{S1} + \Delta V_2 \sin \Phi_{S2}$$

$$\frac{d}{dt}(\Delta\Phi_{s1}) = -2\pi f_0 \alpha_p h_1 \frac{\Delta E}{E}$$

$$\Delta\Phi_{S2} = \frac{h_2}{h_1} \Delta\Phi_{S1} \quad \text{and} \quad \frac{d}{dt}(\Delta\Phi_{S2}) = \frac{h_2}{h_1}\frac{d}{dt}(\Delta\Phi_{S1})$$

ΔX and $\Delta\Phi_X$ represent small variations of the phase and amplitude of the phasor $\tilde{X}$, respectively; E is the beam energy, α_p the momentum compaction factor and the other parameters are defined as in Appendix 1.

Combining these three equations and introducing the Laplace operator p, we get:

$$\Delta\Phi_{S1}\left(\frac{p^2 + \omega_{S0}^2}{\omega_{S0}^2}\right) = \frac{P_1'}{D'} \Delta\Phi_{V1} + \frac{P_2'}{D'} \Delta\Phi_{V2} - \frac{A_1'}{D'} \Delta V_1 - \frac{A_2'}{D'} \Delta V_2$$

$$\Delta\Phi_{S2} = \frac{h_2}{h_1} \Delta\Phi_{S1}$$

where

$$P_1' = h_1 V_1 \cos \Phi_{S1}, \quad P_2' = h_1 V_2 \cos \Phi_{S2}$$
$$A_1' = h_1 \sin \Phi_{S1}, \qquad A_2' = h_1 \sin \Phi_{s2}$$
$$D' = h_1 V_1 \cos \Phi_{S1} + h_2 V_2 \cos \Phi_{S2}$$
$$\omega_{S0} = \left(\frac{2\pi f_0^2 \alpha_p}{E/e}(h_1 V_1 \cos \Phi_{S1} + h_2 V_2 \cos \Phi_{S2}\right)^{1/2}$$

$f_{S0} = \omega_{S0}/2\pi$, is the frequency of free synchrotron oscillations.

Cavity transfer function

In Laplace transform, the cavity voltage and current are related by:

$$\tilde{V}(p) = \frac{R\alpha}{p + \alpha + j\alpha \tan \Psi} \tilde{I}(p) \quad (\alpha = 1/T_f)$$

The relation between $\Delta\tilde{V}$ and $\Delta\tilde{I}$ can then be expressed:

$$\Delta V(p + \alpha) - \Delta\Phi_V V\alpha \tan \Psi = R\alpha\Delta I$$
$$\Delta V\alpha \tan \Psi + \Delta\Phi_V V(p + \alpha) = R\alpha I \Delta\Phi_I$$

or:

$$\Delta V = \frac{R\alpha(p+\alpha)}{D}\Delta I + \frac{R\alpha^2 I \tan\Psi}{D}\Delta\Phi_I$$

$$V\Delta\Phi_V = -\frac{R\alpha^2\tan\Psi}{D}\Delta I + \frac{R\alpha I(p+\alpha)}{D}\Delta\Phi_I$$

with

$$D = p^2 + 2\alpha p + \alpha^2(1+\tan^2\Psi)$$

For $\Phi_{ib} = \pi/2 + \Phi_s$, $|\tilde{i}_b|$ and $\tilde{i}_g$ constant, the relation between $\Delta\tilde{i}_b$ and $\Delta\tilde{I}$ is given by:

$$\Delta I = -i_b \cos\Phi_S \Delta\Phi_S$$

$$I\Delta\Phi_I = -i_b \sin\Phi_S \Delta\Phi_S$$

Combining these results, we get:

$$\frac{\Delta V}{V} = -\frac{\alpha Y}{D}[\alpha(\tan\Psi\sin\Phi_S + \cos\Phi_S) + p\cos\Phi_S]\Delta\Phi_S$$

$$\Delta\Phi_V = \frac{\alpha Y}{D}[\alpha(\tan\Psi\cos\Phi_S - \sin\Phi_S) - p\sin\Phi_S]\Delta\Phi_S$$

with $Y = i_b/I_0 = 2I_b/I_0$

Cavity-beam transfer function

If we now put together the results obtained for the beam and the RF systems, we finally get the global transfer function represented by the block diagram shown in Figure 3 where

$$B = \omega_{S0}^2/(p^2+\omega_{S0}^2),\quad D' = h_1 V_1 \cos\Phi_{S1} + h_2 V_2 \cos\Phi_{S2}$$

$$A_1 = -h_1 \sin\Phi_{S1}/D',\quad A_2 = -h_1 \sin\Phi_{S2}/D'$$

$$P_1 = h_1 V_1 \cos\Phi_{S1}/D',\quad P_2 = h_1 V_2 \cos\Phi_{S2}/D'$$

$$CA_i = -\frac{Y_i V_i \alpha_i^2(\tan\Psi_i \sin\Phi_{Si} + \cos\Phi_{Si})}{D_i} - \frac{Y_i V_i \alpha_i \cos\Phi_{Si}}{D_i}p$$

$$CP_i = \frac{Y_i \alpha_i^2(\tan\Psi_i \cos\Phi_{Si} - \sin\Phi_{Si})}{D_i} - \frac{Y_i \alpha_i \sin\Phi_{Si}}{D_i}p$$

$$D_i = p^2 + 2\alpha_i p + \alpha_i^2(1+\tan^2\Psi_i)$$

From the block diagram shown in Figure 3, we can deduce the sixth order characteristic equation:

$$1 - B * \left[P_1 * CP_1 + A_1 * CA_1 + \frac{h_2}{h_1}(P_2 * CP_2 + A_2 * CA_2)\right] = 0$$

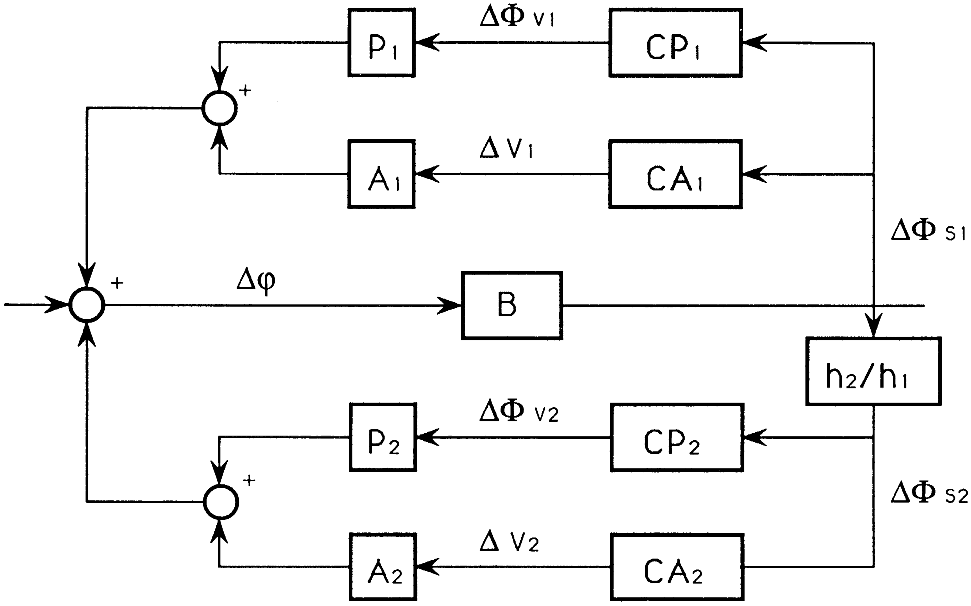

FIGURE 3 Block diagram of cavity beam transfer function.

The stability criterion originally obtained by Robinson[5] for a single RF system:

$$0 < \sin 2\Psi < 2 \cos \Phi_S / Y$$

together with our preceding results, suggest expressing the high current stability limit for a double RF system ($0 < \Psi_{1,2} < \pi/2$)† as:

$$h_1 Y_1 V_1 \sin 2\Psi_1 + h_2 Y_2 V_2 \sin 2\Psi_2 < 2(h_1 V_1 \cos \Phi_{S1} + h_2 V_2 \cos \Phi_{S2})$$

or

$$\sin 2\Psi_1 < \frac{2 \cos \Phi_{S1}}{Y_1} + \frac{h_2 V_2}{h_1 V_1 Y_1} (2 \cos \Phi_{S2} - Y_2 \sin 2\Psi_2)$$

The numerical solution of the characteristic equation—roots with negative real part correspond to stability—and our "intuitive" analytic expression show perfect agreement.

For the particular case where the 2nd system is idle ($\Phi_{S2} = \Psi_2 - \pi/2$, $Y_2 = 1/\cos \Psi_2$), the 2nd term of the right hand side cancels and once again, we get the classical Robinson limit for the first system, as if it was alone:

$$\sin 2\Psi_1 < 2 \cos \Phi_{S1} / Y_1$$

† $\Psi_1 > 0$ for reactive beam loading compensation and $\Psi_2 > 0$ for focusing.

This result is consistent with the physical interpretation of the high current Robinson limit given in Reference 10: stability is lost when the generator driven voltage and the beam current become in phase. Such a condition is obviously independent on the presence of an additional beam induced voltage in the idle RF system.

APPENDIX 3

Transient build up of the RF voltage in response to a step change ΔI_b

Following P. B. Wilson's approach[4], the transient response of RF cavities to a step change, ΔI_b of I_b, may be described by:

$$\tilde{V}(n) = \tilde{V}_i + \Delta\tilde{V}(n)$$

$$\Delta\tilde{V}(n) = \tilde{V}_{b0}\left(\sum_{k=0}^{n} e^{-k(\tau - j\delta)} - \frac{1}{2}\right)$$

$$\tilde{V}_f = \tilde{V}_i + \Delta\tilde{V}$$

$$\Delta\tilde{V} = \tilde{V}_{b0}\left(\frac{1}{1 - e^{-\tau + j\delta}} - \frac{1}{2}\right)$$

In the notations already used appendix 1 and 2, $\tau = T_b/T_f$, $\delta = \tau \tan \Psi$, $V_{b0} = (R/Q)\, 2\pi f_r T_b \Delta I_b$ (single pass induced voltage), $\tilde{V}(n)$ represents the voltage at the nth bunch passage, $\tilde{V}_i$ and $\tilde{V}_f$, the initial and final steady state, respectively.

For our particular case with unloaded s.c. cavities ($\tau \ll 1$, $\delta \simeq 0$, $\Psi \simeq \pi/2$, $V_{b0} \ll \Delta V$), we get:

$$V_i = (R/Q) I_b f_r / \delta f$$

$$V_f = V_i + \Delta V$$

$$\Delta V = (R/Q) \Delta I_b f_r / \delta f$$

$$\Delta\tilde{V}(t) = \Delta V(1 - e^{j\,2\pi\,\delta f\, t} e^{-\alpha t})$$

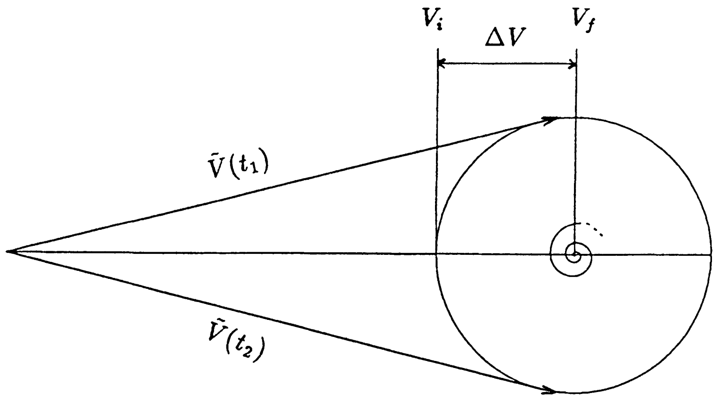

FIGURE 4 Transient build up of the cavity voltage in response to a step change, ΔI_b ("free oscillations").

The resulting oscillations of $\tilde{V}$ (Figure 4) correspond to a modulation of the power exchanged between the beam and the RF system. The instantaneous power is maximum when the beam current is in phase with $\Delta\tilde{V}(t)$; $\Delta\hat{P} \simeq I_b \Delta V$. The frequency of the modulation is δf and its damping rate, T_f. The transient amplitude thus is inversely proportional to δf and the strongest effect will be produced at the end of the accumulation ($I_b = I_{b\,\max}$).

Remark

These results are valid for a single RF system. In the double RF system configuration, both systems are coupled together through the beam and a complete treatment requires considering the global response.