CANCELLATION OF THE QUADRUPOLE EFFECT ON SPIN IN HIGH ENERGY ACCELERATORS

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A special arrangement of quadrupole and horizontally-bending-dipole magnets is being considered in order to cancel the depolarization effect of the quadrupoles to the first order. Also discussed is an alternative arrangement using combined-function magnets that contain both quadrupole and dipole fields in the same space. These quadrupole–dipole arrangements do not necessarily increase the needed integrated bending strength (in tesla-meter) of an accelerator. If this technique were to be used, two Siberian Snakes would be sufficient to maintain the beam polarization during acceleration at the SSC.

1. INTRODUCTION

It is well known that the main cause of beam depolarization during acceleration in large synchrotrons comes from spin perturbations at the quadrupoles. At energies above 1 TeV, one or two Siberian Snakes might not be sufficient to maintain polarization. Near intrinsic resonances, [1,2] the quadrupoles can reverse the polarization of a particle in a time less than the particle-revolution period. Although complete spin reversal of all particles in a beam does not cause depolarization, the situation is complicated by the presence of machine imperfection resonances that are close enough to overlap with the intrinsic ones. In order to answer the question of polarization stability one must investigate, theoretically and experimentally, the spin dynamics in the regions of overlapping resonances. Analytical and computer calculations have been made, [3] and an experimental study of simulated overlapping resonances at the IUCF Cooler Ring has been proposed. [4]

These difficulties foreseen in keeping polarization at very high energies call for the invention of special methods to decrease the effects of intrinsic resonances. This is theoretically possible because of the known origin, structure, and strength of these intrinsic resonances.

A few alternatives are suggested to achieve this objective. One possibility is to introduce a large number ($2N$) of Siberian Snakes in order to destroy the correlation between spin precession around the vertical field in the dipole magnets and particle oscillation in the quadrupole's field. As a result, the strength of the intrinsic resonances decreases by a factor of N,[5, 6] and it could in principle be reduced to zero by a special distribution of the Snakes in the ring.[7] A second method is to destroy this correlation with a relatively small modulation of the dipole field. One would decrease the strength of intrinsic resonances by a factor $\sqrt{M}$ for M modulation periods per revolution. Another method could be provided by a special choice of the lattice design[6, 7, 8] without using a large number of Snakes.

In this paper we consider methods of cancelling the quadrupole's effect on polarization, which do not depend on the structure of the lattice of the accelerator, and are applicable for both the intrinsic and the imperfection resonances. Two methods will be described. The first method involves splitting the quadrupoles into two equal pieces and inserting a dipole magnet between them. The second method involves using combined-function magnets.

2. SPLITTING THE QUADRUPOLES

Ideally, the equilibrium beam polarization in a storage ring with pairs of Siberian Snakes is in the vertical direction in regions outside the Snakes. Quadrupoles in these regions perturb the spins of particles which traverse the quadrupoles off-center vertically owing to either a closed-orbit error or their betatron oscillations. This perturbation due to quadrupoles is typically the main reason for beam depolarization in such accelerators. It is suggested below that, by a reconfiguration of these quadrupoles, their depolarization effects can be suppressed.

2.1. Short quadrupole approximation

Consider a particle moving through a quadrupole at an energy for which $G\gamma \gg 1$, where $G = g/2 - 1$ is the anomalous gyromagnetic ratio ($G = 1.79$ for protons) and γ is the relativistic factor. The perturbation on the vertical polarization S_y is due to spin precession around the horizontal axis. The precession angle is

$$\Delta\theta = -\frac{Ge}{Mc^2} \int_0^{\ell_q} B_x \, dz \tag{1}$$

where e and M are the particle's charge and mass; c is the speed of light; B_x is the horizontal field of the quadrupole; ℓ_q is the length of the quadrupole. We neglect the longitudinal field B_z everywhere and we assume that the precession angles in a single quadrupole are small. In linear approximation:

$$B_x = \frac{\partial B_x}{\partial y} \cdot y$$

$$B_y = \frac{\partial B_x}{\partial y} \cdot x$$

where y and x are the vertical and horizontal components of the particle's displacement from the quadrupole axis (both the betatron and the closed orbit contributions) and B_y

is the vertical component of quadrupole's field. The spin precession angle due to the quadrupole is equal to

$$\Delta\theta = -\frac{Ge}{Mc^2}\frac{\partial B_x}{\partial y}\int_o^{\ell_q} y(z)dz \qquad (2)$$

where $y(z)$ satisfies the equation

$$y'' + ny = 0$$

with

$$n = -\frac{e}{\gamma Mc^2}\frac{\partial B_x}{\partial y}$$

In high energy accelerators the length of a single quadrupole is small compared with the distance between the quadrupoles and we can neglect the change of the vertical displacement y inside the quadrupole:

$$\Delta\theta = \gamma Gn\ell_q y \qquad (3)$$

To first order in $\Delta\theta$, the change in vertical polarization is given by

$$\Delta S_y = S_y\cos\ \Delta\theta + S_z\sin\ \Delta\theta - S_y \approx S_z\Delta\theta$$

Particles move from one quadrupole to another through the dipoles which rotate the polarization (S_x, S_z) in the orbital plane. Near the depolarization resonances the value

$$\sum_q \Delta S_y$$

increases at each revolution, leading to the depolarization of the beam.

We can cancel the effect (3) in a simple and efficient way. We split the quadrupole in the middle and insert between the two pieces a dipole magnet with a vertical field that rotates the spin by 180°. Consequently, the spin component S_z will be equal and opposite in these two pieces of the quadrupole. This results in

$$\Delta S_y = \gamma Gny\left(\frac{\ell_q}{2}S_z - \frac{\ell_q}{2}S_z\right) = 0$$

More precisely, if we take into account that $y' \neq 0$, we obtain

$$\Delta S_y \approx \frac{1}{4}\gamma Gn\ell_q S_z y' \cdot (\ell_q + 2\ell_r) \qquad (4)$$

where ℓ_r is the dipole length, and we have assumed here that y' does not change much in the region and that S_z does not change much across the quadrupoles. The comparison with Eq. (3) shows that such a method allows us to suppress the depolarization resonance strength due to quadrupoles by a factor

$$k \simeq \frac{4\beta_q}{\ell_q + 2\ell_r} \qquad (5)$$

where β_q is the β-function in the quadrupole. This factor is about 60 for the SSC on the average, although it varies because of the variation of β_q in different quadrupoles.

2.2. Orbit matching in the horizontal plane

The inserted dipole will also change the particle velocity by an angle $\pi/\gamma G$, which depends on the particle's energy. We must then add two compensating dipoles at both ends of the quadrupole in order to exclude any closed-orbit distortions outside the region. Each of the compensating dipoles is half the length of the inserted dipole. The maximum orbit distortion in the middle of this arrangement is

$$(\Delta x)_{\max} \approx \frac{\pi}{4\gamma_{\mathrm{inj}} G}(\ell_r + \ell_q) \qquad (6)$$

where γ_{inj} is the γ-factor at injection. The value ℓ_r can be found from the condition

$$\frac{Ge}{Mc^2} B_r \cdot \ell_r = \pi$$

If we assume $B_r = 5.4$ T, then $\ell_r = 1$ m. In the SSC case we have

$$(\Delta x)_{\max} \simeq \begin{cases} 0.1 \text{ cm} & \text{in the Main Ring } (\ell_q = 5 \text{ m}, \ \gamma_{\mathrm{inj}} = 2 \times 10^3) \\ 0.35 \text{ cm} & \text{in the High-Energy Booster } (\ell_q = 1.5 \text{ m}, \ \gamma_{\mathrm{inj}} = 200) \end{cases}$$

Note that it may be desirable, in practice, for the string of dipoles to be used for normal bending so that the maximum needed integrated bending strength remains the same as the nominal design. This can be accomplished by changing the compensating field to (approximately):

$$B_c = B_r \left(2\frac{\gamma}{\gamma_{\max}} - 1\right)$$

so, at the highest energy ($\gamma = \gamma_{\max}$) all the dipoles work, as usual, with the normal bending magnets.

2.3. Betatron Tune Shift

We also have to estimate the vertical betatron tune shift $\Delta\nu_y$ due to the horizontal orbit distortions in the field $B_y(z)$ of the inserted dipole, particularly at energies $\gamma \ll \gamma_{\max}$. Neglecting the change of the β-function in the quadrupole section, we have [10]

$$\Delta\nu_y = \frac{\beta_y}{4\pi}\left(\frac{e}{\gamma Mc^2}\right)^2 \int B_y^2 \, dz$$

In the case where the quadrupoles are split into two pieces, we have

$$(\Delta\nu_y)_{\mathrm{tot}} = \frac{\pi N_q}{2(\gamma G)^2} \cdot \frac{\bar{\beta}_y}{\ell_r}$$

where N_q is the total number of quadrupoles (in the original nominal design) and $\bar{\beta}_y$ is the average value of β_y in the quadrupoles. At injection energies the values of $(\Delta\nu_y)_{\mathrm{tot}}$ will be approximately 0.01 for the SSC Main Ring and 0.3 for the SSC High Energy Booster, respectively. Such tune shifts will require compensation. However, values are small relative to the ν-values 123 and 33, and therefore it should be possible to provide constant values of the betatron tune during acceleration.

2.4. Precise cancellation of the quadrupole effect on the spin (Linear Theory)

If it is desirable to cancel the residual quadrupole effect (4), we will need an additional degree of freedom to operate in integral (2). In order to do this we can split the quadrupole into three pieces (of lengths ℓ_1, ℓ_2, and ℓ_1) and insert between them two horizontal π-rotators and add two dipoles at each end of the arrangement.

We assume the lattice to be geometrically symmetric with respect to the middle of the central quadrupole piece ($z = 0$). The compensation condition of spin perturbation is as follows:

$$\int_{-z_1}^{-z_1+\ell_1} y(z) \, dz - \int_{-\ell_2/2}^{\ell_2/2} y(z) \, dz + \int_{z_1-\ell_1}^{z_1} y(z) \, dz = 0 \qquad (7)$$

where $y(z)$ is the general solution of the equation

$$y''(z) + n(z)y(z) = 0$$

between the beginning of the left quadrupole piece and end of the right piece ($n = 0$ between the quadrupole pieces where the spin is rotating around uniform vertical field). With the symmetry referred to above, we can choose the general solution $y(z)$ as an arbitrary linear combination of the symmetric solution $y_s(z)$ and the antisymmetric one $y_a(z)$:

$$y_s(z) = y_s(-z); \qquad y_a(z) = -y_a(-z)$$

Then the condition (7) is satisfied automatically for $y_a(z)$ and we only need

$$\int_0^{\ell_2/2} y_s(z) \, dz = \int_{z_1}^{z_1+\ell_1} y_s(z) \, dz \qquad (8)$$

It is obvious that this requirement could always be satisfied by a suitable ratio between ℓ_1 and ℓ_2; the distances between the quadrupole pieces being fixed by the spin π-rotators.

Considering the question concerning the compensation of the orbit distortion, we have to note that rotators can have either the same or opposite signs. The last case is preferable if one has strong limitations on orbit distortions inside the quadrupole, although this technique does not allow one to use these inserted dipole magnets for normal bending.

3. USING COMBINED-FUNCTION MAGNETS

Now let us consider the spin motion through a combined-function magnet with a vertical dipole field B_y and a quadrupole gradient $\partial B_x/\partial y$. The change of the vertical polarization S_y in a quadrupole is

$$\Delta S_y = -\frac{Ge}{Mc^2} \frac{\partial B_x}{\partial y} \int_{-\ell_q/2}^{\ell_q/2} y(z) \, S_z(z) \, dz \qquad (9)$$

and the evolution of $S_z(z)$ to lowest order is described by the formula:

$$S_z(z) = S_z(0) \cos\psi + S_x(0) \sin\psi$$
$$\psi = \frac{Ge}{Mc^2} B_y z = \Omega z$$

where S_x is the horizontal spin component and B_y is the uniform dipole field of the combined-function magnet.

We first assume that the magnet is short enough to neglect the effect of quadrupole field on the particle trajectory across the magnet, i.e.

$$\sqrt{\frac{e}{\gamma Mc^2}\left|\frac{\partial B_x}{\partial y}\right|} \cdot \ell_q \ll 1$$

Such a situation is typical for high-energy accelerators. Particle motion in the region is then approximated as if in free space. Equation (9) then gives

$$\Delta S_y = -\frac{2}{B_y}\frac{\partial B_x}{\partial y}\left[y(0) \cdot \sin\frac{\psi_q}{2} \cdot S_z(0) - y'(0) \cdot S_x(0)\left(\frac{\ell_q}{2}\cos\frac{\psi_q}{2} - \frac{\sin\frac{\psi_q}{2}}{\Omega}\right)\right]$$

where $\psi_q = \Omega\ell_q$.

The orders of magnitude of $y'(0)$ and $y(0)$ are related as follows:

$$y'(0) \sim y(0)/\beta_q$$

where β_q is the β-function inside a quadrupole. Assuming $\beta_q \gg \ell_q$ the value $|\Delta S_y|$ is close to a minimum when $\psi_q = 2\pi$, i.e.

$$B_y \cdot \ell_q = 2\pi\frac{Mc^2}{eG} \tag{10}$$

then

$$\Delta S_y \approx -\frac{1}{B_y}\frac{\partial B_x}{\partial y} \cdot y'(0) \cdot \ell_q \cdot S_x(0)$$

Compared with Eq. (3), i.e. the case when no dipole field was imposed to make the quadrupole magnet a combined-function one, the depolarization resonance strength has been suppressed by a factor

$$k = \frac{2\pi\beta_q}{\ell_q}$$

If it is desirable to completely cancel the integral (9), one could switch the sign of the dipole field at the middle of the magnet. In the approximation $y' = $ const, the integral (9) vanishes if

$$|B_y|\ell_q \approx 4\pi\frac{Mc^2}{eG} \tag{11}$$

For long quadrupoles with $y' \neq$ constant, cancellation of the integral (9) could be achieved by some small variation of $|B_y(z)|$, symmetrically relative to the middle of the magnet. In this scenario, dipoles will have to be added at the ends of the combined-function magnet in order to remove the closed-orbit effect outside the system.

Another possibility that matches the closed orbit automatically is to change the combined-function dipole field in such a way that the first 1/4 and the last 1/4 of the magnet have their field opposite in sign to the middle 1/2 of the magnet, and the field strength is given by

$$|B_y|\ell_q \approx 8\pi\frac{Mc^2}{eG} \tag{12}$$

Still another possibility would be to split the combined-function magnet and insert a dipole π-rotator between the pieces.

For the scenario described by Eq. (10) or (11), as mentioned, it would be necessary to add compensating dipoles to the ends of the combined-function region in order to avoid orbit distortions outside the region. An alternative is to have the arrangement described above in Eq. (12), in which case the orbit distortion is automatically compensated. In the case described by Eq. (10), still another, more complicated but profitable, method of matching is to use the couple of dipoles as part of the bending magnets. The compensating dipole field $B_c(\gamma)$ is determined as a function of γ from the condition that the design beam trajectory is bent by a fixed angle θ_{tot}:

$$\theta_{\text{tot}} \equiv \frac{2\pi}{\gamma_{\max}G} + 2\frac{e\ell_c}{Mc^2}\frac{B_{\max}}{\gamma_{\max}} = \frac{2\pi}{\gamma G} + 2\frac{e\ell_c}{Mc^2}\frac{B_c(\gamma)}{\gamma}$$

where ℓ_c is the length of the compensating dipoles, and $\gamma_{\max}$ and $B_{\max}$ are the maximum values of γ and B_c (at the peak energy). Note that $B_c(\gamma)$ can change its sign at $\gamma \ll \gamma_{\max}$.

The maximum orbit distortion inside the combined-function magnet is

$$(\Delta x)_{\max} \approx \frac{\pi\ell}{4G\gamma_{\text{inj}}} \tag{13}$$

where ℓ is the total length of the system and γ_{inj} is the γ-factor for the injection energy. Equation (13) gives a similar result to the split quadrupole scheme, Eq. (6). In the case of reversing dipoles, the value of $(\Delta x)_{\max}$ would be less, but such a dipole string cannot serve for normal bending of the circumference.

Note that the quadrupole's contribution to the spin phase ψ_q, due to the orbit distortion, is small and does not change with energy during acceleration. In fact, it is reduced to a small renormalization of the dipole field B_y. A similar situation occurs for the case of Eq. (10).

We may compare the features of the combined-function magnet method and the split-quadrupole method as follows:

1. The total length of the magnet strings and the maximum orbit distortion are the same for both methods.

2. The combined-function method is schematically simpler and offers an advantage in space usage, at the cost of a more complex magnet design.

4.   CONCLUSION

Our results indicate that it is possible to cancel the quadrupole influence on vertical polarization by the intrinsic and imperfection depolarization resonances. With the use of such a technique, keeping the beam polarized during acceleration is reduced, mainly, to the limitation on random dipole angular misalignment α and vertical displacement of quadrupole pieces Δz_q using two Siberian Snakes: [9]

$$\alpha \quad \ll \quad 1/\sqrt{G\gamma_{\max}}$$

$$\Delta z_q \quad \ll \quad R\sqrt{R/\ell_q}/\nu^2 G\gamma_{\max}$$

Here R and ν are the machine radius and betatron tune. This technique could allow the possibility of polarized beams for energies as high as 10^5–10^6 GeV. Theoretically, additional requirements to the beam emittance can be formulated taking into consideration the higher order effects of the quadrupoles on the spin. Here we only note that the cancellation considered above substantially reduces these effects simultaneously.

To summarize, we briefly discuss the advantages and disadvantages of the quadrupole–dipole coupling methods:

1. From the physical point of view, these methods seem to be the most efficient way to suppress the effects of the beam emittance and machine imperfections on spin. Two Siberian Snakes would then be sufficient to overcome all dangerous spin resonances in a machine such as the SSC.

2. Matching the dipoles considered above does not necessarily lead to increased integrated magnetic strength around the circumference because these dipoles can be used to serve the normal bending function. The total length of these compensating dipoles would be small compared with the total length of normal dipoles in a lattice. One disadvantage of these methods is the need for some special care of orbit corrections at injection in view of the relatively high value of the dipole field.

3. A final consideration would be the technical design and the cost of the considered magnet systems. Accordingly, these methods should be justified critically as compared with the options where large numbers of Siberian Snakes are used in order to suppress the strength of the depolarization resonances.

REFERENCES

1. Polarized Beams at SSC, AIP Conf. Proc. **145** (New York) 1986.
2. A. W. Chao, E. D. Courant, S. Y. Lee, L. C. Teng and K. Yokoya, Report of the 1988 Workshop on Siberian Snakes at the SSC, ed. K. Yokoya, SSC report, SSC-SR-1036 (1988).
3. S. Y. Lee and E. D. Courant, Phys. Rev. **D41** (1990) 292.
4. A. D. Krisch, et al., Proposal for approved IUCF experiment CE-20, "Study of Overlapping Depolarizing Resonances using an RF solenoid".
5. Ya. S. Derbenev and A. M. Kondratenko, Sov. Phys. Dokl. **24** (1979) 366.
6. Ya. S. Derbenev and A. M. Kondratenko, in Proc. of the 12-th International Conference on High Energy Accelerators (Fermilab, 1983) 413.
7. K. Steffen, Part. Acc. **24** (1988) 45.
8. A. W. Chao, AIP Conf. Proc. **95**, High Energy Spin Physics (BNL, 1982), 458 (New York 1983).
9. Ya. S. Derbenev and A. M. Kondratenko, AIP Conf. Proc. **187**, v. 2, High Energy Spin Physics, VIII International Symposium (Minneapolis, MN 1988), 1474.
10. Ya. S. Derbenev, in Proceedings of the 9-th International Symposium on High Energy Spin Physics (Bonn 1990), UM HE 90-21.