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EFFECTS OF CHARGE DISTRIBUTION IN A DRIVING BUNCH ON NONLINEAR PLASMA WAKEFIELD ACCELERATION

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Abstract We investigate the nonlinear plasma wave phenomena induced by the injected electron bunch, laying emphasis on the effects of charge distribution of the driving bunch on the wakefields. The acceleration gradient can be increased with the beam-to-plasma density ratio or the bunch length of the driving beam. In the nonlinear regime it is possible to increase the transformer ratio without highly-controlled beam shaping, which is essential in the linear regime of the plasma wakefield acceleration.

Introduction

Plasma wakefield acceleration (PWFA) uses wakefields excited by a relativistic electron bunch in a plasma to accelerate trailing charged particles to ultra-high energies.[1] The accelerating gradient and the transformer ratio are major parameters which must have large values to realize an accelerator using this principle. The wakefield theorem limits the transformer ratio to less than 2 for a symmetric driving bunch. This limitation can be overcome by introducing asymmetric charge distributions for the driver, such as a triangular or a door-step distribution.[2, 3] In this case, however, requirements on beam shaping, in particular on the sharpness of the cut-off, may prove to be the toughest technological challenges in conventional accelerator technologies.

Plasma oscillation gets nonlinear as the charge density disturbance in the plasma becomes non-negligible compared with the plasma density.[4] It has been proposed to utilize this nonlinear regime for the PWFA, because higher electric fields can be obtained with lower plasma density than in the linear regime, and because the elongated driving beam enhances the transformer ratio without complex maneuvering of the charge distribution in the beam.[5, 6, 7]

Analyses in the nonlinear regime hitherto have assumed a flat distributed charge density in the driver. The actual beam, however, has nearly Gaussian or Lorenzian density distribution longitudinally. It appears yet to be difficult to produce long and flat distributed beams which have sharply cut-off edges. It is important to analyze the PWFA processes including the charge distribution to help plan experiments, to estimate correctly the results of the experiments, and to clarify the requirements for the pulse shape control of the nonlinear PWFA. We have made simulation calculations for the nonlinear wakefild generation and have studied the effects of the charge distribution in the driver on the PWFA.

Nonlinear PWFA equations

The nonlinear electrostatic motion of the electrons in a cold plasma with stationary ions is given by the following equations,[4, 5]

$$\nabla\cdot \mathrm{E} = \frac{e}{\varepsilon_0}(n_0 - n - n_b)\,, \tag{1}$$

$$\frac{\partial}{\partial t}(n_0 - n - n_b) + \nabla\cdot(-n\mathbf{v} - n\mathbf{v}_b) = 0\,, \tag{2}$$

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{p} = -e\,\mathbf{E}\,, \tag{3}$$

where n_0 is the plasma density, n is the plasma electron density, n_b is the beam electron density, $\mathbf{v}$ is the plasma electron velocity, $\mathbf{v}_b$ is the beam electron velocity and other quantities have their usual meaning. The plasma electron momentum $\mathbf{p}$ is defined as,

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-(v/c)^2}}\,, \tag{4}$$

which includes the relativistic variation of the mass of the plasma electrons. Assuming that the wave motion is one-dimensional and longitudinal, we obtain nondimensionalized formulation of the fluid equations described as a function only of the nondimensional variable $\tau = \omega_p(t - z/v_{ph})$, where $\omega_p = \sqrt{n_0 e^2/\varepsilon_0 m}$ is the plasma frequency and v_{ph} is the phase velocity of the plasma wave. These equations are as follows:

$$\frac{\partial \mathcal{E}}{\partial \tau} = N + N_b - 1\,, \tag{5}$$

$$\frac{\partial N}{\partial \tau} = \frac{N}{1-V}\frac{\partial V}{\partial \tau}\,, \tag{6}$$

$$\frac{\partial}{\partial\tau}\left(\frac{V}{\sqrt{1-V^2}}\right) = -\frac{\mathcal{E}}{1-V}, \tag{7}$$

where $\mathcal{E}$, N, N_b, V are the nondimensional forms of the electric field, the plasma electron density, the beam electron density and the plasma electron velocity, respectively. These variables have been nondimensionalized using the wave-breaking electric field $v_{ph}\omega_p m/e$, the plasma density n_0 and the wave phase velocity v_{ph}. The equations (5) - (7) are solved numerically to predict the nonlinear wave phenomena with various distributions of the driving bunch, $N_b = N_b(\tau)$.

Results and discussions

Three distributions, flat, triangular and Gaussian, have been considered for the longitudinal charge distributions in the driving beam. These shapes are schematically shown in Figure 1. The flat distribution is used for comparison with the theoretical analyses, while the triangular is one of the asymmetric distributions and is nearly optimum to attain the highest transformer ratio in the linear regime, and the Gaussian is adopted to represent and simulate the actual beams.

Figure 2 is the calculated transformer ratios of the nonlinear PWFA as a function of the driving bunch length normalized with the plasma wavelength λ_p. The approximate square-root increase for the flat beam is shown to be consistent with the analytical predictions in reference 6. For the other distributions, the transformer ratios seem to saturate at the longer bunch lengths.

Distributions of the decelerating electric fields excited in the driving bunchs are shown in Figure 3. The flat beam has a nearly uniform decelerating field, which in turn implies that this is an adequate shape to attain a high transformer ratio. In the triangular and the Gaussian beams the retarding fields oscillate at the head of the beam where the beam-to-plasma charge density ratio is low, while they increase rapidly with the density ratio in the latter or the

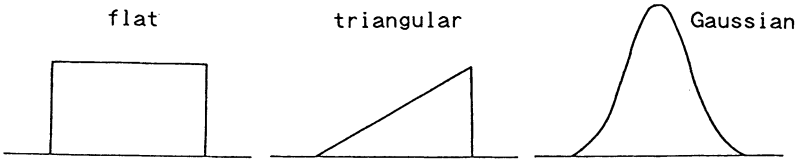

Figure 1. Schematic charge distributions in driving beams.

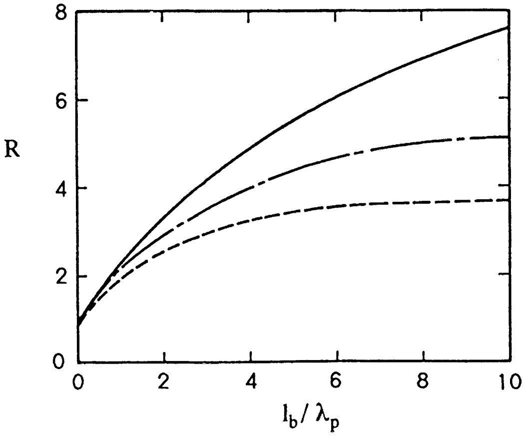

Figure 2. Transformer ratio as a function of nondimensional bunch length. Beam-to-plasma density ratio is 0.5. Solid line: flat beam, dash-and-dot line: triangular beam, dotted line: Gaussian beam.

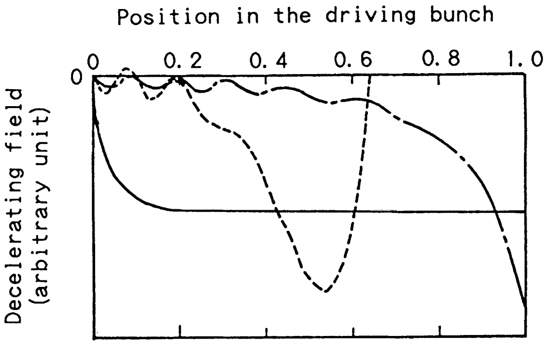

Figure 3. Distribution of decelerating fields in the driving bunch. Solid line: flat, dash-and-dot line: triangular, dotted line: Gaussian.

middle part of the beam where the degree of the nonlinearity becomes notable.

Figure 4 and 5 are the transformer ratio and the nondimensional accelerating fields at the wake of the flat and the Gaussian beams, as a function of the beam-to-plasma density ratio which is a measure of the nonlinearity. The transformer ratio has a peak at $n_b/n_0=0.5$ and decreases with increasing n_b/n_0. The peak is broader and lower for the Gaussian beam. The controll of the density ratio to about 0.5 is critical for the flat beam to attain the high transformer ratio, because it is extremely sensitive to n_b/n_0. The accelerating fields increase with n_b/n_0 surpassing the wave-breaking limit by far, which is one of the major advantages of the nonlinear PWFA

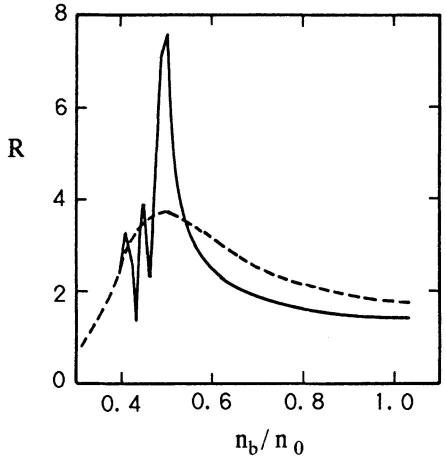

Figure 4. Transformer ratio as a function of beam-to-plasma density ratio. $l_b/\lambda_p = 10$
Solid line: flat, dotted line: Gaussian beam.

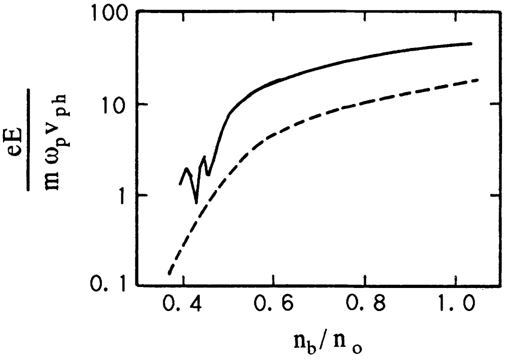

Figure 5. Nondimensional accelerating field in the wake of the driving beam. $l_b/\lambda_p = 10$
Solid line: flat, dotted line: Gaussian beam.

We have studied the effects of sharpness at the front and the back edges of the charge distribution of the flat beam. Figure 6 shows these effects on the transformer ratio for the cases when there is a linear rise at the front, a linear fall at the back and linear changes at both edges in the charge density distribution. The transformer ratio is not so affected sensitively by the sharpness of the rise and the fall of the charge distribution as compared with the linear regime where the cut-off must be shorter than $\lambda_p/2\pi$ which corresponds about 0.16 of the horizontal axis of Figure 6..

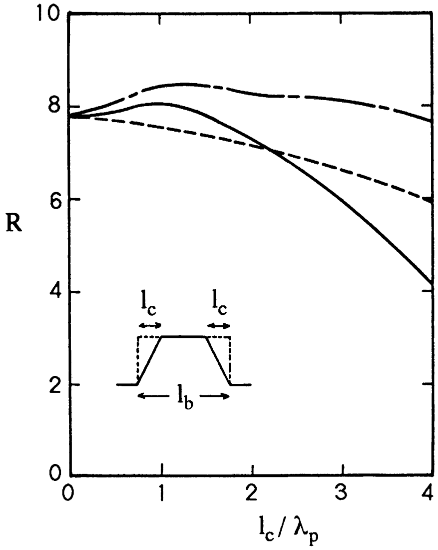

Figure 6. Change of the transformer ratio with the length in which charge density rises or falls. Dash-and-dot line: rise at front, dotted line: fall at back, solid line: rise at front and fall at back. $l_b/\lambda_p = 10$

Summary

In the nonlinear PWFA the flat charge distribution in the driving bunch is most effective to realize a high transformer ratio. With the Gaussian or asymmetric beams we can get a transformer ratio greater than 2, but cannot increase it by elongating the beam length. In the nonlinear regime, the sharpness of the rise and the fall of the charge density at the edges of the driving beam has smaller effects on PWFA performance than in the linear regime. The beam shaping needed for the nonlinear PWFA is not far from what can be expected from extensions of present accelerator technologies.

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