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POLARIZED BEAMS OF HIGH ENERGY ELECTRONS AND POSITRONS

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Investigation of the spin dependence of the fundamental interactions necessary for the complete theory of elementary particles requires experiments using polarized particles in initial state. Such experiments demand considerable efforts to be spent on creation and conservation of the necessary direction and degree of polarization.

In contrast to the beams of heavy particles, use of polarized sources with a subsequent acceleration of polarized electrons or positrons in synchrotrons by the depolarization when spin resonances are crossed. The depolarization is caused by quantum fluctuations of synchrotron radiation. On the other hand, however, the very process of photon radiation in the magnetic field results in self-polarization of electrons and positrons. Such a radiative polarization can be achieved directly at high energy providing the only feasible source of high energy polarized positrons.

Up to now the process of radiative polarization has been many times observed in electron-positron storage rings at different energy /1-8/.

|  | VEPP | ACO | VEPP-2M | SPEAR | VEPP-4 | DORIS | CESR | PETRA |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{E}(\mathrm{GeV})$ | 0.65 | 0.53 | 0.65 | 3.7 | 5 | 5 | 5 | 16.5 |
| $\tau_{\rho}(\min )$ | 70 | 160 | 60 | 15 | 40 | 4 | 300 | 18 |
| $S_{\text {max }}$ | 0.8 | 0.9 | 0.9 | 0.7 | 0.8 | 0.8 | 0.8 | 0.8 |

The time of radiative polarization ranges from several minutes up to 2-3 hours in different storage rings. The achieved polarization degree $S_{\text {max }} i s$ high enough.

Effect of radiative self-polarization was theoretically predicted in 19639 for electrons and positrons moving
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in a homogeneous magnetic field. Real magnetic fields existing in the storage rings are of course far from homogeneous. However, the existing field inhomogeneity is small along the formation length of the photon $\Delta l \sim \frac{R}{\gamma}$ ( $R$-is the curvature radius) and does not influence the radiation process. After photon radiation the motion of both a particle and its spin essentially depends on the inhomogeneities.Let us remind in this connection the main features of spin dynamics in arbitrary magnetic $\vec{H}$ and electric $\vec{E}$ fields of the storage ring.

According to the BMT-equation $/ 10 / \frac{d \vec{S}}{d t}=\vec{\Omega} \times \vec{S}$, spin motion in the laboratory frame is precession at a frequency

$$
\begin{equation*}
\vec{\Omega}=-\left(\frac{q_{0}}{\gamma}+q^{\prime}\right) \vec{H}+\frac{\gamma}{\gamma+1} q^{\prime} \vec{V}(\vec{H} \cdot \vec{V})-\left(\frac{q_{0}}{\gamma+1}+q^{\prime}\right) \vec{E} \times \vec{V} \tag{1}
\end{equation*}
$$

where $\vec{v}$ is a particle velocity, $q^{\prime}$ and $q_{0}$ are the anomalous and the normal parts of the gyromagnetic ratio

$$
q=q_{0}+q^{\prime} .
$$

At $q^{\prime}=0$ the precession frequency during the motion in the magnetic field ( $\vec{E}=0$ ) coincides with the angular frequency of velocity rotation $\vec{\omega}=\frac{90 \vec{H}}{\ngtr}$, i.e. spin projection at the velocity direction $\vec{v}$ is conserved. The anomalous part of the magnetic moment makes spin motion more complicated, so that any stable polarization directions can be now achieved. It is the variation of the angle between the velocity and spin which is of physical interest. Therefore, spin dynamics can be naturally considered in a system of "swinging" reference frame related to velocity /11,12/:

$$
\begin{equation*}
\vec{e}_{1}=\frac{\vec{v} \times \vec{e}_{x}}{\left|\vec{v} \times \vec{e}_{2}\right|} ; \quad \vec{e}_{2}=\frac{\vec{v}}{|\vec{v}|} ; \quad \vec{e}_{3}=\vec{e}_{1} \times \vec{e}_{2} \tag{2}
\end{equation*}
$$

differing slightly from the accelerator orts $\vec{e}_{x}, \vec{e}_{y}, \vec{e}_{z}$. Besides that, for accelerators and storage rings it is convenient to study spin variation along the equilibrium orbit rather than its time dependence. To this end the generalized azimuthal angle $\Theta$ is used conventional of the orbital motion.

At any field configurations their subsequent action
on spin during one particle turn is reduced to a simple protation around some direction $\vec{n}$ by an angle $\varphi$. The rotatron axis $\vec{n}=\vec{n}(\theta)$ and the angle are of course determined by the field structure along the entire orbit since they are functions of particle coordinates and momentum $\vec{n}(\theta)=$ $\vec{n}(\vec{p}, \overrightarrow{2})$. For the closed equilibrium orbit $\left(\vec{p}=\vec{p}_{s}, \vec{r}=\right.$
$\vec{z}_{s}$ ) the direction $\vec{n}(\theta)=\vec{n}_{s}$ will be repeated from one turn to another and in each point of the trajectory spin precesssos around $\vec{n}_{s}$ with a reduced frequency $\nu=\frac{\varphi}{2 \pi}$ conserving the projection $S_{n}=\vec{S} \cdot \vec{n}_{s}=$ const. Since particle motion is the oscillation around a closed orbit, for a group of the particles one obtains some distribution of $\vec{n}$ around $\vec{n}_{s}$ and after mixing over precession phases the average spin is directed along $\vec{n}_{s}$

$$
\langle\vec{S}\rangle \simeq\left\langle S_{n}\right\rangle \cdot \vec{n}_{s}
$$

The expression for the precession axis $\vec{n}$ can be genrally obtained if one introduces deviations $x$ and $\boldsymbol{Z}$ from some ideal equilibrium orbit $\overrightarrow{\mathbf{z}}=\overrightarrow{\mathbf{r}}_{s}(\theta)$ with curvature components $K_{z}=\frac{H_{z}}{\left\langle H_{z}\right\rangle}, K_{x}=\frac{H_{x}}{\left\langle H_{z}\right\rangle}$. Subtracting from (1) the angular precession of the basis (2) ( $\left.\vec{\Omega}_{e}=\frac{1}{2} \sum_{i}\left[\overrightarrow{\mathbf{e}}_{i} \times \overrightarrow{\mathbf{e}}_{i}^{\prime}\right]\right)$ one obtains the spin precession frequency around the "swinging" orts $\vec{W}=\frac{\vec{\Omega}}{\dot{\theta}}-\vec{\Omega}_{e}$ :

$$
\begin{aligned}
& W_{1}=\nu K_{x}+\nu\left(K_{x} \frac{\Delta \gamma}{\gamma}+z^{\prime \prime}\right) \\
& W_{2}=\frac{q}{90} K_{y}+\left[\frac{9 K_{0}}{9} y \frac{\Delta \gamma}{\gamma}+\frac{q}{90}\left(K_{x}^{\prime} x+k_{z}^{\prime} z\right)+\frac{q^{\prime}}{90}\left(K_{x} x^{\prime}+K_{z} z^{\prime}\right]\right. \\
& W_{3}=\nu K_{z}+\nu\left(K_{z} \frac{\Delta \gamma}{\gamma}-x^{\prime \prime}\right)
\end{aligned}
$$

Similarly to the orbital motion let us separate in the precession frequency the equilibrium periodical part $\vec{W}_{s}$ and the perturbation $\vec{w}$ related to the particle deviations from the closed orbit $\times, z, \frac{\Delta \gamma}{\gamma}$. Solving the equation $\vec{s}^{\prime}=\vec{W}_{s} \times \vec{S}$ one finds the periodical precession axis $\vec{n}_{\mathbf{s}}$ and two orthogoral vectors $\vec{\eta}$ and $\vec{\eta}^{*}$ rotating around $\vec{n}_{s}$ with a irequincy $\nu$;
$\left.(\vec{\eta}(\theta+2 \pi))^{\eta}=e^{-2 \pi i \nu} \eta(\theta)\right)$
In the general case because of the smallness of martiche deviation from the equilibrium orbit the axis $\vec{n}$ can be found from the perturbation theory. In the linear appro-
ximation

$$
\begin{equation*}
\vec{n} \simeq \vec{n}_{s}+\operatorname{Im} \vec{\eta} \int_{-\infty}^{\theta}\left(\vec{w} \cdot \vec{\eta}^{\star}\right) d \theta \tag{4}
\end{equation*}
$$

It is clear that the frequency spectrum of $\vec{n}$ contains frequencies of orbital motion. The $\vec{n}$ axis possesses a maximum sensitivity to the trajectory parameters near spin resonances when the average precession frequency is close to the combination of frequencies of vertical $\nu_{z}$ and radial
$\nu_{x}$ betatron oscillations as well as synchrotron $\nu_{\gamma}$ oscillations

$$
\begin{equation*}
\nu=\nu_{k}=k+e \nu_{z}+m \nu_{x}+n \nu_{\gamma} \tag{5}
\end{equation*}
$$

## RADIATIVE POLARIZATION

If one takes into account spin interaction with radiation fields the projection $S_{n}$ is not conserved and in the first approximation of the perturbation theory one can write

$$
\dot{S}_{n}=(\vec{s} \cdot \vec{n})=\dot{\vec{s}} \cdot \vec{n}+\vec{s} \cdot \dot{\vec{n}}=\left[\vec{w}_{2} \times \vec{s}\right] \cdot \vec{n}+\vec{s} \cdot \vec{f}_{2} \frac{\partial \vec{n}}{\partial \vec{p}}
$$

The first term in this expression describes direct action of radiation fields on the spin. The precession frequency $\vec{w}_{2}$ contains generally all radiation fields, but because of their smallness the linearly accumulating effect of spin rotation arises only due to the fields rotating simultaneously with the spin. The field of the rotation of the magnetic dipole is an example $\vec{H}_{\mu}=\frac{2}{3} \frac{\vec{\mu}}{c^{3}}$ (Fig. 1). Variation of the projection $S_{n}$ due to this field $\left(\dot{S}_{n}\right)_{\mu} 1-S_{n}^{2} \rightarrow r e-$ suits in a slow alignment of $\vec{S}$ along the direction $\vec{n}_{s}$ as in the case of the homogeneous magnetic field.

Another mechanism of radiation action on spin is relate to the radiation reaction force. In the "quasiclassical" case the force of radiative reaction is written as follows /13/:

$$
\begin{equation*}
\vec{f}_{2}=-\frac{2}{3} e^{2}|\dot{\vec{v}}|^{3} \gamma^{4} \vec{v}\left[1-3 \frac{|\dot{\vec{v}}|}{m^{2}}(\vec{s} \cdot \vec{b})\right] \tag{6}
\end{equation*}
$$

where $\vec{b}=\frac{\dot{\vec{v}} \times \vec{v}}{|\vec{v} \times \vec{v}|}$ is a unit vector along the guiding magnetic field and interference of the charge radiation and magnetic moment has been taken into account. The spin part of this force results in the modulation of the precession axis $\vec{n}(\vec{p}, \overrightarrow{2})$ with the frequency of spin precession when $\vec{n}$ does not coincide with $\vec{b}$. Taking into account that gredients of $\vec{n}$ in longitudinal and transverse directions are generally of the same order, whereas the radiation reaction force is mainly longitudinal, one can write the average rate of $S_{n}$ variation as

$$
\left(\dot{S}_{n}\right)_{f}=\left\langle\vec{S} \cdot \vec{f}_{2} \frac{\partial \vec{n}}{\partial \vec{p}}\right\rangle \simeq q_{0}^{2} \gamma^{5}\langle(\vec{B} \cdot \vec{d})| \vec{v}^{3}| \rangle\left(1-S_{n}^{2}\right)
$$

where

$$
\vec{d}=\gamma \frac{\partial \vec{n}}{\partial \gamma} \text { is a vector of spin-orbit coupling. }
$$

Fig. 2 shows the action of spin-orbit coupling in the case when $\vec{n}$ and $\vec{b}$ do not coincide. For two particles with equal energies $\gamma=\gamma_{s}$ spins $\vec{S}_{1}$ and $\vec{S}_{2}$ have initial projections $S_{n_{1}}=S_{n_{2}}$. Radiation of the photons with the energy $\hbar \omega=\delta \gamma$ will result in a jump of the precession axis by $\delta \vec{n}$ to $\vec{n}$. $S_{n_{1}}$ becomes smaller and $S_{n_{2}}$ increases (we do not consider rare processes with spin flip). Then $\vec{n}$ due to the relaxation of the orbital motion to the equilibrium slowly damps to $\vec{n}_{s}$ conserving $S_{n_{1}}^{*}$ and $S_{n_{2}}^{*}$. According to (6) the radiation probability (intensity) depends on the projection $\vec{S}$ on the direction of the magnetic field $\vec{b}$ and finally spins are aligned along $\vec{n}_{3}$.

These considerations are based on the quantum character of the radiation. From the same picture it is clear that charge radiation which is many orders of magnitude more intense than that of the magnetic moment will result in diffuse decrease of the projection $S_{n}$. Spin diffusion with a rate proportional to $|\vec{d}|^{2}=\left|\gamma \frac{\partial \vec{n}}{\partial \gamma}\right|^{2}$ restricts the achievable polarization degree.

Consistent consideration of the kinetics of radiative polarization in Ref. / $14 /$ taking into account both direct action of radiation on spin and via spin-orbit coupling gives the following formula for the equilibrium polarization degree

$$
\begin{equation*}
S_{\text {max }}=\frac{8}{5 \sqrt{3}} \frac{\left.\left.\langle | H\right|^{3} \vec{b}(\vec{n}-\vec{d})\right\rangle}{\left.\langle | H\right|^{3}\left[1-\frac{2}{9}(\vec{n} \cdot \vec{v})^{2}+\frac{11}{18}|\vec{l}|^{2}\right\rangle} \tag{7}
\end{equation*}
$$

As seen from this formula spin-orbit coupling can both decrease and increase the polarization degree. Besides that, if usual radiative polarization is absent (for example, $\vec{n}_{s}(\theta)$ is in the orbit plane) the polarization degree can be $60-70 \%$ due to spin-orbit coupling. However, increase of the spin-orbit coupling ( $d \geqslant 1$ ) always results in a lower level of polarization. Further it will be shown that the contribution of the spin-orbit coupling to the diffusion increases with the electron energy and special analysis is needed to understand how spin-orbit coupling arises and can be suppressed in storage rings with the energy of $10-100 \mathrm{Gev}$.

## SPIN-ORBIT COUPLING

Let us start from the usual storage ring with a planar orbit, i.e. $K_{x}=0, K_{y}=0$. Since bending magnets can be a little bit inclined and focusing lenses are vertivally shiftted, the radial field $h_{x}$ appears leading to the distortion of the equilibrium orbit $\boldsymbol{Z}_{\mathbf{s}}$. As a rule, $\boldsymbol{z}_{\boldsymbol{s}}$ is considerably higher than a beam size ( $\boldsymbol{z}_{\mathbf{s}}>\boldsymbol{\sigma}_{\mathbf{z}}$ ).

Spin-orbit coupling is in this case related to the energy dependence of spin precession frequency and grows rapiddly near integer resonances $\nu=k$. Outside the resonances the precession axis differs slightly from the basis vector $\vec{e}_{3}$ and from (3) and (4) one obtains
where

$$
\begin{aligned}
\vec{n} & \simeq \vec{e}_{3}+\operatorname{Im}\left\{\left(\vec{e}_{1}-i \vec{e}_{2}\right) e^{i \nu \tilde{k}} \int_{-\infty}^{\theta} z_{s}^{\prime \prime} e^{-i \nu \tilde{k}} d \theta\right\} \\
\tilde{k} & =\int_{0}^{\theta} k_{2} d \theta ; \quad \nu=\gamma \frac{q^{\prime}}{9_{0}}
\end{aligned}
$$

Expanding the integrand in the Fourier series over integer resonances one has

$$
\begin{equation*}
|\vec{d}|^{2}=\left|\gamma \frac{\partial \vec{h}}{\partial \gamma}\right|^{2}=\nu^{2} \sum \frac{\left|w_{k}\right|^{2}}{(\nu-k)^{4}} \tag{9}
\end{equation*}
$$

The resonance amplitude (Fourier harmonic) $\boldsymbol{w}_{k}$ can be represented as

$$
\begin{gather*}
v_{k}=\frac{\nu}{2 \pi R} \int_{0}^{2 \pi} z_{s}^{\prime \prime} e^{-i \nu(\tilde{k}-\theta)-i k \theta} d \theta=  \tag{10}\\
=\frac{\nu^{2}}{2 \pi R} \int_{0}^{2 \pi} k_{z} z_{s}^{\prime} e^{-i k \tilde{k}} d \theta=\frac{\nu}{2 \pi} \int_{0}^{2 \pi} h_{x} F^{\nu=k} e^{-i k \theta} d \theta
\end{gather*}
$$

where $F^{\nu}$ is the characteristic response function of the storage ring determined by the Floquet solution $f_{z}$ of the equation of vertical betatron oscillations /15/:

$$
\begin{align*}
& F^{\nu}=\frac{\nu}{2}\left[f_{z} \int_{-\infty}^{\theta} k_{z} f_{z}^{\alpha^{\prime}} e^{-i \nu \tilde{k}} d \theta-f_{z}^{*} \int_{-\infty}^{\theta} k_{x} f_{z}^{\prime} e^{-i \nu \tilde{k}} d \theta\right] e^{-i \nu \theta}= \\
& =\frac{\nu}{2}\left\{\frac{f_{z} \int_{\theta-2 \pi / \rho}^{\theta} k_{z} f_{z}^{\kappa^{\prime}} e^{-i \nu \tilde{k}} d \theta}{1-e^{i \frac{2 \pi}{p}\left(\nu+\nu_{z}\right)}}-\frac{f_{z}^{*} \int_{-2}^{\theta} K_{z} f_{z}^{\prime} e^{-i \nu \tilde{k}} d \theta}{1-e^{i \frac{2 \pi}{p}\left(\nu-\nu_{z}\right)}}\right\} \tag{11}
\end{align*}
$$

The response function $F^{\nu}$ takes into account spin rotations due to radial fields arising during vertical oscillations. It increases sharply near $\nu=m \cdot p \pm \nu_{z}$ resonances ( $p$ is the number of superperiods). Fig. 3 gives a plot of $|F \nu|$ calculated for the HERA storage ring at three different values of energy.

One can see from (9) that for substantial decrease of the spin-orbit coupling it is sufficient to suppress the influence of the closest resonances. One can reach that by the compensation of the corresponding $K$-harmonic of the
$h_{x}$ perturbation and by the minimization of the response function $F^{\nu}$.

The idea of suppressing the closest harmonics of the vertical orbit distortions ( $K=37$ and 38 ) was successfully applied to the PETRA storage ring at the energy of 16.5 GeV ( $\boldsymbol{\nu}=37.5$ ) where the polarization degree of $80 \%$ was achieved $/ 16 /$. The variation of the orbit position during this procedure is not important ( $\leqslant 0.1 \mathrm{~mm}$ ) since $\nu_{z}=23.3$. Corrector tuning at the maximum polarization was performed using the polarimeter which measured the asymmetry of the Compton scattering of circularly polarized laser photons.

A relative failure of this method at the energy of 19 GeV was probably due to the necessity to take into account the response function $F^{\nu}$ (see (10)) that had not been done.

One can use the formulae (9-11) to estimate the accuracy of manufacturing and mounting the magnetic elements of the storage ring lattice designed to obtain polarized electrons. The resulting action of $N$ sections with a length
$\Delta l_{i}$ each is determined by the sum $\left|w_{k}\right|^{2}=\sum_{i}^{N}\left|w_{k}^{i}\right|^{2}=\nu^{2} \sum\left(\frac{l_{i}}{2 \pi R}\right)^{2}$. - $h_{x}^{i} \cdot\left|F_{i}^{\nu=k}\right|^{2} \quad$ since they are not correlated. Then from (7) one obtains

$$
\sum_{i}\left(\frac{e_{i}}{2 \pi R}\right)^{2} h_{x}^{i}\left|F_{i}^{\nu=k}\right|^{2} \leqslant \frac{54}{11 \pi^{4}} \frac{\sin ^{4} \pi \nu}{\nu^{4}\left(1+2 \cos ^{2} \pi \nu\right)}
$$

For example, at $\nu=[\nu]+\frac{1}{2}$ to obtain the polarization degree higher than $50 \%$ at the energy of 50 GeV one should provide the positioning accuracy $\boldsymbol{\Delta} \mathbf{Z}$ of lenses with a gradient $g$

$$
\sqrt{(\Delta Z)^{2}} \leq \frac{2 \sqrt{5 y / 11} R^{2} \sqrt{N}}{\pi \nu^{2} g \sqrt{\sum e_{i}^{2}\left|F_{i}\right|^{2}}} \leq 10^{-2} \mathrm{~cm}
$$

The numerical computations of the influence of vertical distortions have been performed in /17/for the storage rings VEPP-4 ( $E=5 \mathrm{GeV}$ ) and $\operatorname{LEP}(E=50 \mathrm{GeV})$.

Similar analysis can be performed for the allowed rotation angles of the quadrupole magnets $\underset{\sim}{\alpha}$ at which the coupling of $X$ and $Z$ oscillations arises /15,17/. For the coupling having harmonics with any number $K$ there are oscillations with the frequencies $K, K \pm \nu_{z}$ and $K \pm \nu_{x}$ in $\boldsymbol{Z}$-motion of the particle. From (3,4) it is clear that spin diffusion is strongly enhanced if the precession frequency is close to the frequencies of vertical oscillations. Outside the resonances under the same conditions ( $E=50 \mathrm{GeV}$ and $\nu=[\nu]+\frac{1}{2}$ ) one can obtain the allowed angle of lens rotation if error independence is assumed

$$
\sqrt{\alpha^{2}} \leq \frac{\nu_{x}^{2} \sqrt{N} \cdot R}{\nu g \sqrt{\sum e_{i}^{2}\left|F_{i}\right|^{2}}} \leqslant 1 \cdot 10^{-3}
$$

A method to calculate the value of the spin-orbit coupling using a formalism of $8 \times 8$ matrices had been developed in / 18 / . The computer code SLIM takes into account possible perturbations and their influence on the precession axis $\vec{n}_{s}$ and $\vec{d}$ which are later averaged with formula (7). A disadvantage of the matrix approach is that it is impossible to take into account nonlinear effects such as beam-beam effects, influence of frequency spread and so on.

Difficulties with the computation of the nonlinear spin resonances had been mostly overcome in the code SMILE /19 / . The author had developed an algorithm of calculating a strength of the high order resonances at which the perturbation $\boldsymbol{w}$ is linear in perticle deviations. Such approach is probably justified at high energy ( $\nu \gg 1$ ) when, e.g., the quadrupole contribution to the amplitude of the second order resonance $\nu=2 \nu_{x}+k$ is proportional to $\nu^{2} x^{2}$ and for sextupoles it is $\nu x^{2}$. Analysis of the experimental data from the SPEAR storage ring performed with this code provides reasonable explanation for most of the spin resonances observed / $20 /$.

The approach developed for SMILE allows also to calculate the resonances related to synchrotron oscillations of the particle energy $\gamma=\gamma_{s}+\Delta \gamma \cos \nu_{\gamma} \theta$. However, since
$\nu_{\gamma} \ll 1$ synchrotron oscillations can be considered as modulation of time offset $\delta=\nu-K=\Delta \cos \nu_{\gamma} \theta$ from the integer resonance $\bar{\nu}=K$ where $\Delta=\nu \frac{\Delta \gamma}{\gamma}$. Using the expansion $e^{-i \frac{\Delta}{\gamma_{\gamma}} \sin \nu_{\gamma} \theta}=\sum_{m} J_{m}\left(\frac{\Delta}{\gamma_{\gamma}}\right) e^{-i m \nu_{\gamma} \theta \gamma} \quad$ where $J_{m}$ is the Bessel function, (9) can be transformed to

$$
\begin{equation*}
|\vec{d}|^{2}=\nu^{2} \sum_{n, m} \frac{\left|w_{k}\right|^{2}\left\langle\eta_{n}^{2}\left(\frac{\Delta}{v_{r}}\right)\right\rangle}{\left[\left(k-\bar{\nu}-m \nu_{\gamma}\right)^{2}-\nu_{k}^{2}\right]^{2}}=\sum_{k} C_{k} \frac{\left|\vartheta_{k}\right|^{2}}{(\bar{\nu}-k)^{y}} \tag{12}
\end{equation*}
$$

Depolarization increase near the side-band resonances $K-\bar{\nu}-m \nu_{\gamma}=0$ strongly depends on the modulation index $P=\frac{\Delta}{\nu_{\gamma}}$. For the LEP energies this index $p \sim 1$ and modulation resonances are very important as shown in /21,22/.

In previous considerations it was assumed that the spread of precession frequencies $\Delta \nu \ll 1$, i.e. much less than a distance between the spin resonances. As seen from the calculations for LEP / 21/ such an approach is valid up to the energy $E \simeq 100 \mathrm{GeV}$. Construction of storage rings for higher energies is connected with known difficulties not taking into account polarization and is hardly possible because of the high cost of the projects.

Another scheme of obtaining polarized beams exists for linear electron-positron colliders at the energy of 100$1000 \mathrm{GeV} / 23 /$. In this scheme recuperation of the beams and their polarizations is performed via radiation by the beam ( $E \geqslant 100 \mathrm{eV}$ ) of circularly polarized quanta in a helical undulator with a period of 1 cm. "Photons" ( $\hbar \omega \simeq 10+20$ MeV ) from the entire length of the undulator ( $100-200 \mathrm{~m}$ ) were converted into electron-positron pairs, so that a particle with an energy $0.9 \hbar \omega$ also had longitudinal polarization due to the helicity conservation. Further treatment of polarization, for example rotation to a transverse one and back at the input and output of the cooling storage ring will of course require additional measures, but no essential difficulties are envisaged.

## SPIN ROTATORS AND POLARIZATION CONTROL

Unfortunately, for cyclic machines there are problems with using spin rotators making longitudinal polarization of the beams difficult. Several schemes have been suggested to this end. At the energy of several GeV if a booster storage ring exists, one can apply a scheme referred to as a Siberian snake / $24 /$. The precession axis $\vec{h}_{s}$ in this scheme is in the orbit plane ( $\left.\vec{n}_{5} \cdot \vec{b}=0\right)$. Spin-orbit coupling is given by the expression $|\vec{d}|^{2}=\nu^{2}(\pi-\widetilde{\kappa})+\frac{\pi^{2}}{4} \sin \pi \nu$ and one can see that the life time of longitudinal polarization in this scheme rapidly decreases with the increase of
electron energy.
Other rotator schemes are characteristic of the common approach to obtaining longitudinal polarization. The closed solution $\vec{n}_{s}$ is directed along the field in the main part of the ring with the exception of the sections in which subsequent action of different magnetic fields transforms the polarization into the longitudinal one in the interaction region and later is made again vertical. In the simplest variant rotations can be performed by radial fields only / 25,26/. In other cases one can use a combination of longitudinal and vertical fields / $24,27,28 /$ and radial and vertical fields / $29 /$.

While calculating rotator schemes one should take into account direct depolarizing action of transverse fields and, besides matching orbital motion of the particles, take also care of the compensation of spin-orbit coupling necessarily arising inside the rotators. Localization of the problems is achieved by the condition $\vec{d}=0$ at the entrance and exit of the rotator section. In this case, a rate of spin diffusion is determined by quantum fluctuations at the energy only at the specially inserted section (besides "usual" non-idealities). For diffusion suppression one should perform spin rotation in rotators by as weak transverse fields as possible.

In experiments with the polarized beams it is desirable to be able to change the direction and degree of polarization. As we had seen, in electron-positron storage rings polarization is aligned along some determined direction. Use of rotators allows a necessary helicity sign to be obtained by changing their polarity. This procedure is, however, rather slow and not very convenient. For example, the HERA rotator needs mechanical displacements of the magnets and the vacuum pipe by a distance of 50 cm .

It is attractive to use for reversing polarization a radio frequency electromagnetic field $\widetilde{H}$ which is in resonance with a spin precession frequency and perpendicular to $\overrightarrow{\boldsymbol{r}}_{s}$. Other beam parameters (energy, the location of
closed orbit, betatron tunes, etc.) are constant because
$\tilde{H}$ is not in resonance with the orbital motion. It is quite possible technically to provide an adiabaticity condition during resonance crossing by changing the frequency of an RF-resonator. Experimentally the feasibility of the RF-flipper had been studied at the storage ring VEPP-2M where this device was used in $(g-2)$ experiments / $30 /$. The RF field applied at the orbit section with a length $l$ where $\vec{n}_{s}$ is for example vertical, produces a resonant harmonic $\boldsymbol{w}_{k}$ which for a longitudinal field $\boldsymbol{w}_{k y}=H$. $=\frac{\tilde{H}_{y}}{\left\langle H_{l}\right.} \frac{e}{2 l}$ whereas for a radial one it is calculated from

$$
w_{k_{x}}=\nu \frac{\tilde{H}_{x}}{\left\langle H_{z}\right\rangle} \frac{e}{2 L}\left(1+F^{\nu}(\theta=0)\right)
$$

where $F^{\gamma}(\theta)$ is a response function determined in (11). Diffusion due to quantum fluctuationsof the radiation restricts application of the RF-flipper at the energy higher than $20-30 \mathrm{GeV}$. However, the same diffusion at relatively low RF fields can provide a rapid depolarization in a narrow region of "artificial" resonance which can be useful for reference measurements. Application of the travelling wave where $\left|\tilde{H}_{x}\right|=\left|\widetilde{E}_{x}\right|$ in experiments with the colliding beams allows (when there is no reflection) to depolarize a beam moving counter to the wave practically unaffecting the polarization of the other beam. Besides that the selective depolarization of the bunches inside one beam is possible due to the application of the short pulses phased with a resolution frequency whose pulse height is modulated by a frequency in resonance with spin $/ 31 /$.

Described methods are aimed at eliminating systematical uncertainties in experiments with the polarized beams. However, the resonance beam depolarization found interesting application in electron-positron storage rings. The high accuracy in the knowledge of the ratio of anomalous and magnetic moments ( $\simeq 10^{-8}$ ) and a small spread of spin frequencies of electrons in a storage ring $\left(10^{-5}-10^{-6}\right) / 32 /$ provide a basis for the method of energy calibration using resonance depolarization which is widely used today in high
precision measurements of the masses of the particles produced in electron-positron interactions /33/.

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Fig. 1


Fig. 3

