## SECTION XI

## POLARIZED BEAMS

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polarized proton beams*

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#### Abstract

The acceleration of polarized protons to high energy is made difficult by the resonances which arise whenever the spin precession frequency coincides with a spectral component of the particle oscillations. In moderate energy machines they can be combated by bruteforce methods, e.g. rapid jumping through resonances and meticulous corrections of orbit imperfections, one resonance at a time. This method becomes infeasible at higher energies because the resonances become more numerous as well as stronger. "Siberian Snakes" (180 degree spin rotators) may eliminate these resonances by making the spin precession frequency independent of energy and insensitive to small orbit perturbations. Snakes, which are combined horizontal and vertical orbit deflectors, have the disadvantage that they distort the orbit, especially at low energy, and seem feasible only for very high energies. They promise to make polarized proton acceleration possible in RHIC and the SSC, but are hard to model. A recent experiment at Indiana models the "snake" method using solenoids and verifies that a spin resonance is eliminated. Furthermore it has been proposed to combat spin resonances at the Brookhaven AGS by using partial snakes, e.g. short solenoids that rotate the spin by much less than 180 degrees.


## INTRODUCTION

The proton has spin. Just this fact would lead one to surmise that whatever protons do, the spin state is relevant, and therefore experiments with polarized proton beams may be expected to yield information beyond that obtainable with unpolarized proton beams.

Polarized proton beams have been accelerated in several

[^0]proton synchrotrons, including the ZGS at Argonne, the KEK proton synchrotron, SATURNE, and the Brookhaven AGS. It appears desirable to extend the energy of polarized beams up to the much higher ranges now available and in prospect, such as the proton accelerator mode of RHIC (250-300 GeV) and the SSC ( 20 TeV ). Recall that the W and Z particles are associated with the weak interaction which is inherently parity-nonconserving; therefore if they are produced in p-p collisions different helicity states are likely to make a difference.

To obtain high energy polarized beams one must either generate polarized protons in an ion source and accelerate them without depolarization, or else polarize the protons after they have reached full energy. The latter alternative is used for electrons or positrons, which polarize themselves spontaneously by the mechanism of spin-flip radiation. But the relaxation time for this process with protons is over a million years for the SSC and even longer at lower energies. Therefore polarization has to be produced at the source and maintained during acceleration.

## DEPOLARIZING_RESONANCES

The spin of a particle moving in a magnetic field precesses according to the law of spin motion ${ }^{1}$

$$
\begin{equation*}
\frac{d \vec{S}}{d t}=\frac{e}{\gamma m C} \vec{S} \times\left[(1+\gamma G) \vec{B}_{\perp}+(1+G) \vec{B}_{\|}\right] \tag{1}
\end{equation*}
$$

where $\vec{S}$ is the (normalized) spin vector, and $\vec{B}_{\|}$and $\vec{B}_{\perp}$ are the portions of the magnetic field parallel and perpendicular to the particle's instantaneous velocity; $\gamma$ is the relativistic Lorentz factor and $G \equiv(g-2) / 2$ is the anomalous magnetic moment coefficient.

As a consequence of eq. (1) the frequency of precession of a particle moving in a transverse magnetic field, such as in a circular accelerator or storage ring, is (1+rG) times the frequency of revolution - in effect the anomalous moment transforms with energy proportional to $\gamma$. On the other hand,
perturbing magnetic fields contain all harmonics of the revolution frequency, as well as the vertical betatron oscillation frequency. Therefore the perturbations can resonate with the spin precession frequency whenever the energy is such that

$$
\begin{equation*}
v_{s}=k \tag{2a}
\end{equation*}
$$

for resonances driven by imperfection fields, or

$$
\begin{equation*}
v_{s}=k P \pm v_{z} \tag{2b}
\end{equation*}
$$

for "intrinsic" resonances driven by vertical betatron oscillations. Here $v_{s}$ is the "spin tune" in the coordinate system rotating with the orbit; $k$ is any integer; $P$ is the number of identical periods in the magnet lattice, and $v_{z}$ is the vertical betatron oscillation "tune", i.e. frequency in units of the revolution frequency.

If the orbit is essentially plane with the bending accomplished by a vertical magnetic field, then eq. (1) ensures that

$$
\begin{equation*}
v_{\boldsymbol{s}}=\boldsymbol{\gamma} \boldsymbol{G} \tag{3}
\end{equation*}
$$

so that the imperfection resonances (2a) occur at all energies for which $\gamma G$ is an integer, while intrinsic resonances happen whenever $\gamma G$ satisfies (2b). For protons $G$ equals 1.793 and the mass is 0.938 GeV ; thus imperfection resonances are spaced 523 MeV apart while the location of intrinsic resonances depends on the characteristics of the lattice. If the lattice has periodicity $P$ there are two families of intrinsic resonances each with spacing of $523 \times P$ MeV .

The Brookhaven AGS, for example, has about 55 imperfection and 10 intrinsic resonances in its operating range up to 30 GeV . These have varying strengths; dynamical calculations - and experience - show that about a dozen of these are strong enough to depolarize an initial polarized beam very substantially if not completely. Fig. 1 shows these resonances computed with random alignment errors of $\pm 1 \mathrm{~mm}$;
the horizontal lines show the limits above which the spin is reversed by $99 \%$ and the limit below which it is preserved to the extent of $99 \%$. Note that imperfection resonances tend to be strongest near the intrinsic resonances.

In order to preserve polarization as the beam is accelerated to high energy, it is necessary to compensate or eliminate all these resonances. At the AGS - and other proton accelerators such as the Argonne ZGS; SATURNE at Saclay and the proton synchrotron at KEK - two methods are used:

Jump rapidly through intrinsic resonances with the aid of very rapidly pulsed (and expensive) auxiliary quadrupoles. The strength of the quadrupoles needed for this purpose increases approximately with the $3 / 2$ power of the energY.

Alleviate imperfection resonances by energizing appropriately distributed correction magnets - 96 of them in the AGS - at every resonant energy. This has to be done essentially every 0.523 GeV (although some resonances can possibly be skipped); the process of setting the correctors at each step and then programming the magnet cycle accordingly is very laborious.

With the use of these two techniques the AGS has succeeded in maintaining a reasonable degree ( $\simeq 50 \%$ ) of polarization up to 22 GeV , and somewhat higher energies are in prospect. But for prospective or actual machines of much higher energies these methods will rapidly run out of steam - both because the resonances become absolutely stronger with higher energy (and the compensation magnets needed to correct even a given strength also become stronger), and because there are just too many: the SSC would have over $10^{5}$ resonances within its range, and some of them - at high energy - have strengths up to ~ 50 units!

## SIBERIAN_SNAKES

The "Siberian Snake" technique promises to resolve this dilemma. Derbenev and Kondratenko ${ }^{2}$, working in Novosibirsk,

USSR, showed that spin-orbit resonances can be eliminated rather than just alleviated by introducing judiciously placed spin rotators into the lattice of a circular accelerator or storage ring. Conceptually the simplest variant of this method is a solenoid placed at one azimuth, with a strength such that it rotates the spin about the longitudinal axis by an angle of $\pi=180^{\circ}$, i.e.

$$
\begin{equation*}
B \ell=\frac{\pi}{1+G}(B \rho)=3.5206 \sqrt{\gamma^{2}-1} \text { Tesla-m } \tag{4}
\end{equation*}
$$

where $B \rho=\left(m c^{2} / e c\right) \beta \gamma=\left(m c^{2} / e c\right) \sqrt{\gamma^{2}-1}$ is the magnetic rigidity of the particle.

The effect of this spin rotator on the spin is as follows: For any magnetic field pattern there exists a "periodic" spin direction, which we denote by $\vec{n}(s)$, at each point of the azimuth, defined by the requirement that if the spin has this direction initially it returns to the same orientation after one turn when the particle moves along its closed orbit. With the $180^{\circ}$ solenoidal rotator, this direction is just longitudinal at the point of the orbit opposite $\left(180^{\circ}\right.$ away from) the solenoid. To see this, note that if the spin is longitudinal at the point opposite the solenoid, it will then precess in the horizontal plane, through an angle of $\pi \gamma G$, by the time it reaches the solenoid, so that it is then horizontal and at an angle of $\pi \gamma G$ to the beam axis. The solenoid then rotates it through $\pi$ about the solenoid axis, and the spin ends up again horizontal, but making an angle of $-\pi \gamma G$ with the beam axis. It then goes through the second half of the ring and precesses through an additional angle of $\pi \gamma G$, so that by the time it gets back to the original point the angle with the beam axis is again zero.

Now consider a particle whose spin at the initial point deviates from $\vec{n}(s)$ and has a transverse component as well. The longitudinal component will still return to the same value as initially. If the extra component is vertical (up), it will stay that way through the first half arc; the solenoid will flip it down, and it will arrive at the starting
point with spin down. The next turn brings it up again: A spin deviation from the periodic spin repeats in two turns, and thus has a precession frequency $v_{s}=1 / 2$. (It is easily seen that the same applies to an initial deviation component in the horizontal plane).

The crucial point is that this behavior, $\vec{n}(s)$ along the beam axis at 180 degrees and $\nu_{s}=1 / 2$, is independent of energy as long as the spin rotation by the solenoid is always $\pi$. Thus the resonance conditions (2) are simply never satisfied, and there are no depolarizing resonances, i.e. small depolarization fields (due to either imperfections or betatron oscillations) will come at different precession phases at successive revolutions of a particle, so that their effects do not accumulate.

But the price is high. The strength of the solenoid needed for a $180^{\circ}$ spin rotation increases proportional to the particle momentum (see (4)) and becomes prohibitive at high particle energies - e.g. $37 \mathrm{~T}-\mathrm{m}$ at $10 \mathrm{GeV}, 74 \mathrm{~T}-\mathrm{m}$ at 20 GeV etc. But the Siberians noted that a spin rotation around the longitudinal axis can also be accomplished by a sequence of transverse deflecting magnets, with some deflecting vertically and some radially. Because of the factor $\gamma G$ in the $B_{\perp}$ term of (1) the absolute strength of the magnets needed for a given spin rotation is essentially independent of energy. A $180^{\circ}$ spin rotation about the beam axis can, for example, be accomplished by a sequence of magnets each of which rotates the spin around a vertical or horizontal axis: $H V H^{-2}$ $V^{-2} H H^{2} H^{-2} V^{-1} H$, where $V$ and $H$ stands for a magnet which turns the spin through $45^{\circ}$ about a vertical and horizontal axis, respectively; $H^{-1}$ and $V^{-1}$ rotate in the opposite direction, and $H^{2}$ and $V^{2}$ stand for $90^{\circ}$ rotators. From (1) it follows that a $45^{\circ}$ rotator requires a strength of

$$
\begin{equation*}
B \ell=\frac{\pi}{4 \gamma G}(B \rho)=1.372 \beta \mathrm{~T}-\mathrm{m} \tag{5}
\end{equation*}
$$

so that for all the magnets one needs a total of $19.19 \beta$ $T-m$, independent of energy except for the factor $\beta$. These
magnets will, in addition to rotating the spin, also deflect the beam up and down and sideways in a twisting curve; this fact has led to the term "snake" for the spin rotator. The lateral deflection of the beam as it passes through the snake decreases with increasing energy, since the ratio of spin precession angle to orbit deflection angle in a given magnet is $\gamma G$. In the snake configuration described above the maximum deflection (for $\gamma \geqslant 1$ ) is

$$
\begin{equation*}
x_{\max }=z_{\max }=0.75 \frac{(B \ell)^{2}}{B(B \rho)}=\frac{1.802 \beta}{\gamma B} \mathrm{~m} \tag{6}
\end{equation*}
$$

where $B \ell$ is given by (5). For 30 GeV protons and $B=2$ Tesla this comes to 3 cm ; it is clear that the orbit excursion in the snake becomes prohibitive at energies below about 20 GeV. Furthermore it is desirable to have available "snake" configurations which rotate about the transverse horizontal axis, or axes in between transverse or longitudinal; it turns out that these tend to entail larger orbit excursions than (6). The advantage of employing such configurations is that if a ring has two snakes placed $180^{\circ}$ apart in the ring with their rotation axes, both in the horizontal plane, having an angle of $90^{\circ}$ between them, then the periodic spin $\vec{n}(s)$ will be vertical, up in half the ring and down in the other half, while the spin precession tune $\nu_{s}$ will still be 1/2. Thus resonances are still eliminated (in fact, this configuration is more stable than with only one snake). Variants with more than one pair of snakes enhance stability further, especially in large rings.

It seems impossible to incorporate full snakes in existing machines such as the Brookhaven AGS. But a partial snake, rotating the spin by an angle less than $180^{\circ}$, may still do some good $^{3}$. A partial snake will produce a spin tune which, while it is not independent of energy, will never reach an integral value; therefore it still eliminates imperfection resonances and, if the machine tune is close to an integer, it also avoids intrinsic resonances. Schemes for a short partial snake, fitting into a 3 m straight section at the Brookhaven AGS, are now being discussed.

## HIGH_ENERGY_PROSPECTS

Because of the factor $(1+\gamma G)$ in (1) the excitation strength of resonances increases with energy. As a result the resonances can become too strong for the siberian snakes to overcome them. This will be the case when the strength - which is also the width - becomes comparable to one unit, for then the resonances overlap. One thing that helps is to subdivide the ring into several sectors with a pair of snakes in each sector, for then the direction of spin precession reverses every time a snake is traversed; thus a multiplicity of snakes reduces the number of precessions between reversals. At a recent workshop ${ }^{4}$ a consensus was reached that of the order of 10 pairs of snakes should be sufficient for the intrinsic resonances even in the SSC.

But imperfection resonances are largely excited by random errors, which are of course uncorrelated between one snake sector and another; thus the frequent spin reversal does not help with imperfections. The only hope is to apply orbit corrections that are good enough to reduce the resonance strengths without snakes to tolerable values. Yokoya estimates that the closed orbit corrections have to be good enough to reduce orbit excursions to about 0.1 mm rms in order that Siberian snakes can eliminate resonances - a very difficult requirement. On the other hand, Derbenev and Kondratenko ${ }^{5}$ conclude that less stringent tolerances, of the order of a few mm, are sufficient. Steffen ${ }^{6}$ has devised a correction scheme which appears capable of correcting spin resonances beyond the level reached by orbit correction schemes alone.

It seems clear that in the case of a ring the size of RHIC or the SSC injector (a few hundred to 2000 GeV ) one to three Siberian snake pairs, together with closed orbit correction of the quality that is desirable in any case, should be adequate to enable one to maintain proton polarization.

## THE_IUCF COOLER EXPERIMENT

All this is a nice theory, but how can it be tested experimentally? since real wiggly snakes, as we have seen, cause large orbit perturbations at low energies, it is not feasible to add them to existing low or moderate energy machines (furthermore they require around $20 \mathrm{~T}-\mathrm{m}$ of magnetic field, so that long straight sections would be needed). On the other hand, not everybody has such great faith in theory that they are willing to add these rather massive and expensive devices to a new high-energy project without some test.

For low energies, in the hundreds of MeV , the magnet requirements for the solenoid version of the snake are not so excessive, and so it occurred to Alan Krisch and others a few years ago that the Indiana Cyclotron (IUCF) cooler ring is almost unique in having rather long straight sections and low energy, so that a test of the principle with a solenoid snake appeared feasible. One complication: A solenoid produces pretty extensive changes to the focusing properties of the lattice, including a rotation of the oscillation (hori-zontal-vertical coupling). S. Mane has worked out a scheme for compensating these effects with skew quadrupoles, and that was incorporated in the machine at Indiana.

The first experimental results from this experiment ${ }^{7}$ appear to confirm that the snake kills a resonance. A superconducting solenoid with compensation quadrupoles is used. The imperfection resonance $\gamma G=2$ is at 108 MeV ; it is excited by energizing additional solenoids (which are part of the regular IUCF Cooler ring). At the nearby energy of 104 MeV , and with the snake off, the polarization is very sensitive to these solenoids; with the snake on the polarization is about constant (and large) independent of the strength of the imperfection.

## CONCLUSIONS

Snakes are good, but they are not a panacea. At very high energies the resonances may be so strong that snakes cannot
eliminate their effects completely. In addition, Tepikian and Lee ${ }^{8}$ have shown that, if the betatron oscillation resonances in the absence of snakes are strong enough, "snake resonances" appear (at certain specific values of the tune) which cause depolarization even with snakes.

With siberian snakes one can contemplate polarized proton beams in very high energy accelerators. Specifically, they are being seriously considered for the Canadian KAON project and the related European Hadron Facility proposals; for the p-p option of the RHIC project at Brookhaven, and probably for the SSC. It is, however, not yet certain whether they can indeed cope with the depolarization problems at SSC where, with the very high value of $\gamma G$, even very small orbit imperfections can make resonances strong enough so that they may overlap. Further studies are needed.
${ }^{1}$ V. Bargmann, L. Michel, V. L. Telegdi, Phys Rev Letters 2,435 (1959)
M. Froissart and R. Stora, Nucl. Inst. and Methods 1, 297 (1960)
${ }^{2}$ Ya. S. Derbenev and A. M. Kondratenko, Proceedings of X-th International Conference on High Energy Accelerators, Protvino, USSR, 1977
${ }^{3}$ T. Roser, High-Energy Spin Physics Symposium, Minneapolis 1988 (AIP Conference Proceedings \# 187, 1989), pp.1442-1446
${ }^{4} \mathrm{~K}$. Yokoya, Report of Workshop on Siberian Snakes for the SSC, report SSC-SR-1036 (1988)
${ }^{5}$ Ya. S. Derbenev and A. M. Kondratenko, High Energy Spin Physics Symposium, Minneapolis 1988 (AIP Conference Proceedings \#187, 1989), 1474-1485
${ }^{6}$ K. Steffen, report DESY 89-024 (1989)
${ }^{7}$ A. D. Krisch et al., submitted to Phys. Rev. Letters (1989)
${ }^{8}$ S. Tepikian, PhD Thesis (SUNY, Stony Brook, 1986)
S. Tepikian, High-Energy Spin Physics Symposium, Minneapolis 1988 (AIP Conference Proceedings \# 187, 1989), 14501460
S. Y. Lee and S. Tepikian, Phys. Rev. Letters 56, 1635-1638 (1986)


Depolarization resonances for SSC



[^0]:    Work performed under auspices of the US Department of Energy

