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ON A MEASUREMENT OF STRENGTH OF FINAL FOCUSING QUADRUPOLE MAGNET FOR A LINEAR COLLIDER.

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Abstract As a high energy accelerator for physics of next generation, research and development of a linear collider are in progress. Design studies of a final focusing system of linear collider indicate that the bore radius of the final focusing quadrupole is of the order of 0.1 mm. This bore radius is too small to measure field strength by a conventional harmonic coil method. In this work, we propose new measurement method of quadrupole component of magnetic field.

New measurement method stands on the fact that the vibration frequency of a wire in quadrupole magnet depends both on the strength of quadrupole component of the field there and on the electric current flowing in the wire. To measure this frequency, we use resonance of the wire by a small perturbation. This method requires only one wire, whereas the harmonic wire uses two. It is possible to apply the new method to a quadrupole magnet which has small aperture as a final focusing quad in linear collider. Preliminary results of the measurement are presented.

The principle of the measurement

In this work we propose the new method to measure the strength of quadrupole component of magnetic field. The new method stands on the fact that resonance frequency of wire in magnetic field depends on the quadrupole component of the magnetic field k and DC current I_0 following in the wire. Suppose the wire suspended in the magnetic field with tension T. Both ends of the wire are fixed at the distance L. Oscillation of the wire follows the equation,

$$\lambda \frac{\partial^2 \vec{z}}{\partial t^2} - T \frac{\partial^2 \vec{z}}{\partial s^2} = \vec{I} \times \vec{B} \qquad \text{(Eq. 1).}$$

Here λ is a line mass density of the wire. When the direction of the wire is parallel to the axis of the quadrupole magnet, the right hand side of the equation (Eq. 1) becomes,

$$\vec{I} \times \vec{B} = I_0 \cdot B_0 \vec{e}_x + I_0 \cdot k \cdot (x\vec{e}_x - y\vec{e}_y) \qquad \text{(Eq. 2).}$$

Dipole component of the magnetic field appears since the distance between the wire and the magnetic axis of the quadrupole magnet does not vanish. We decompose the oscillation of the wire into the eigen modes, $a_n(t)$ and $b_n(t)$ (n=1,...,∞), which are defined by the equation,

$$z = \sum_{n=1}^{\infty}\{a_n(t)\vec{e}_x + b_n(t)\vec{e}_y\}\sin\left(\frac{n\pi}{L}s\right) \quad \text{(Eq. 3).}$$

The resonance frequency of the n-th eigen mode ω_n is

$$\omega_n^{\pm 2} = \frac{1}{\lambda}(T\,(\frac{n\pi}{L})^2 \pm kI_0) \quad \text{(Eq. 4).}$$

It should be noted that $\omega_n^{\pm 2}$ depends linearly on the current I_0 in the wire. And that its coefficient is determined by the strength of quadrupole component of magnetic field k and the line density of the wire λ. The method we proposing stands on this fact.

If we measure the resonance frequency of the wire as a function of the current I_0 in the wire, we will know the quadrupole components of magnetic field k using the equation

$$k = \lambda\frac{d\omega_n^{\pm 2}}{dI_0} \quad \text{or} \quad k = 4\pi^2\lambda\frac{df_n^{\pm 2}}{dI_0}. \quad \text{(Eq. 2)}$$

<u>Setup of the Equipments</u>

The principle was tested with the quadrupole magnet which is the same type used in the injection line of the TRISTAN Accumulation Ring(AR). The equipments used in this measurement and set-up of these equipment are shown in the Figure 1. We use a 0.2mm ϕ Cu wire as a pick up. The line density λ is 269.5 mgr/m and the length of sense wire L is 50.0 cm. The weight which ranges from 70 gr to 200 gr is used to give tension to the wire. The wire is located parallel to the axis of the quadrupole magnet with small offset. This offset gives a dipole magnetic field in the equation (Eq. 2). DC current source supplies DC current I_0 to the wire. Oscillation of the wire is excited by the interaction between the dipole magnetic field B_0 and AC current i_0 supplied by the spectrum analyzer. The signal produced by this oscillation is detected and analyzed by the spectrum analyzer. The resonance frequency is found as a position of the peak of the signal. A typical output of the spectrum analyzer is shown in the Figure 2. A horizontal axis is a frequency of excitation signal ν_p and a vertical axis indicate a response of the system. Dependence of the resonance frequency on the DC current I_0 is shown in the Figures 3a and 3b.

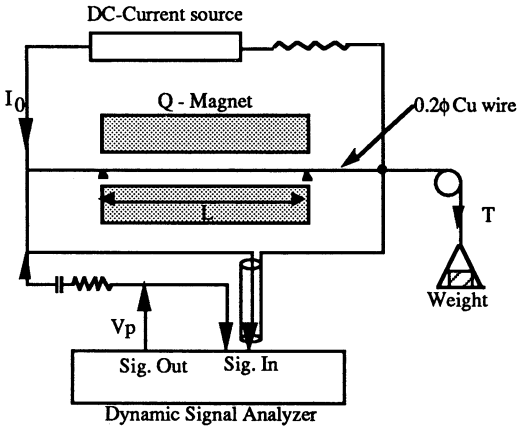

FIGURE 1.
Schematic diagram of setup of the equipments used in the measurement.

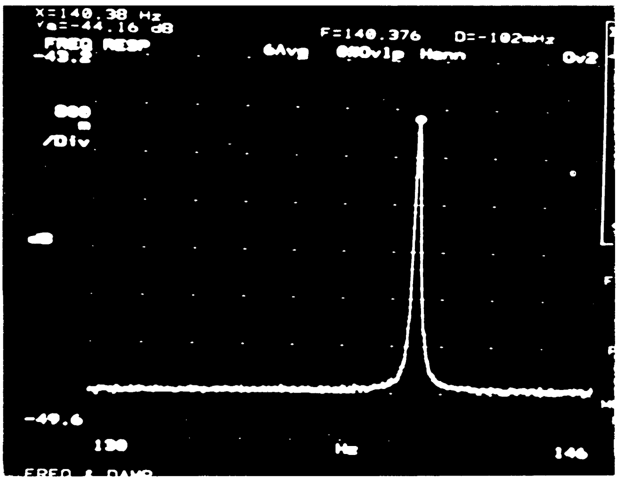

FIGURE 2.
Example of a output signal on the dynamic signal analyzer.

<u>Data Analysis</u>

The dependence of resonance frequency on the DC current I_0 is approximated by the polynomial, The coefficients a_i are determined by using the least square method. The coefficient a_1 is related to the strength of quadrupole components k as

$$k = 4\pi\lambda a_1 \qquad \text{(Eq. 4).}$$

The highest order of polynomials n is fixed so that a quantity called AIC[1]

$$v = N_{data} \log\left(\sum_{data}\left(f^2 - \sum_{i=0}^{n} a_i I_0^{\,i}\right)^2\right) + 2n \qquad \text{(Eq. 5)}$$

becomes minimum. This method is known as the AIC method. For the most of data we got, optimum degree of polynomials is 3 or 4 (see Figure 4). For data shown in Figure 3a and 3b, we found the coefficients a_1 's as shown in TABLE 1. The origin of a_2 and a_3 is not well understood yet. Curvature of equilibrium state of the wire is one possiblity. Error dk/k is estimated and alos shown in Figure 4. The minimum value of dk/k is less than 0.4%. This value of accuracy is worst case in our measurement. Estimated error can be less than 0.1%.

Dependence of quadrupole component on a position of the wire and on a quadrupole excitation current in Figures 5 and 6.

TABLE 1.
Coefficients a_1 's to fit data shown in the Figures 3a and 3b.

Fig.	a_0	a_1	a_2	a_3
3a	4522.600	231.6528	1.976652	-.3079836
3b	4523.605	234.4437	20.69971	-3.382414

Discussion

In this work we proposed the new method of measurement of quadrupole magnet and also show a example data proving that the principle of the method works. Relative accuracy of the measurement is in the range 0.1 ~ 0.3 % . We plan to test this method with a quadrupole magnet with small bore. To work at higher frequency region, we will use 30μm ϕ tungsten wire as a pick up in future measurement.

G.E. Fisher et al. proposed a similar single wire method to find a magnetic center of a quadrupole magnet[2]. In their work, oscillation is excited mechanically. If the frequency of eigen modes of oscillation of a wire is measured in their method, it is possible to apply the principle of our method to evaluate strength of quadrupole magnet.

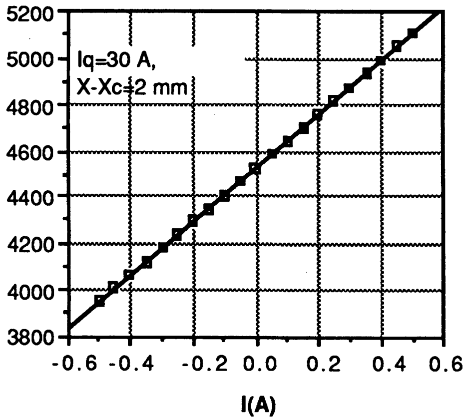

FIGURE 3a

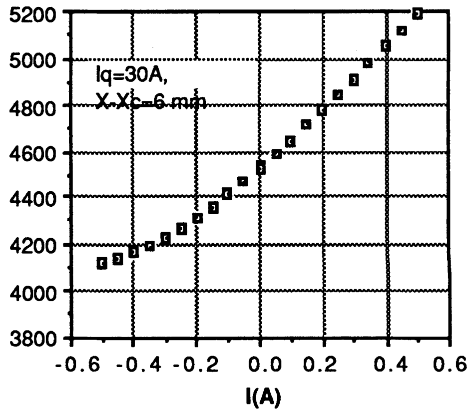

FIGURE 3b.

Square of the measured resonance frequency f^2 is shown as a function of the DC current in the wire. In the neighbor of I=0A, f^2 depends linearly on I.

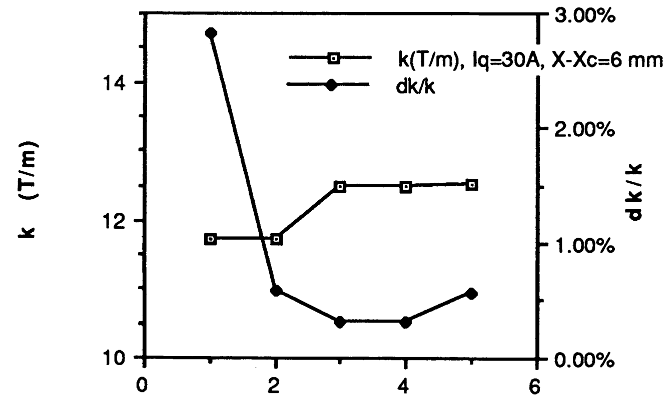

FIGURE 4.

Strength of Quadrupole(T/m) vs. the order of fitting polynomials n. Error in k estimated by the least square method dk/k is also shown.

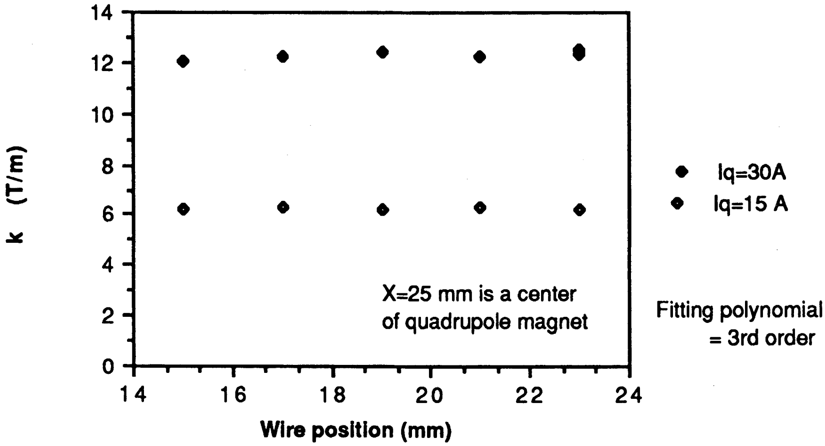

FIGURE 5.

Strength of quadrupole magnetic field in horizontal plane.
A mechanical center of the quadrupole magnet is at x=25 mm.

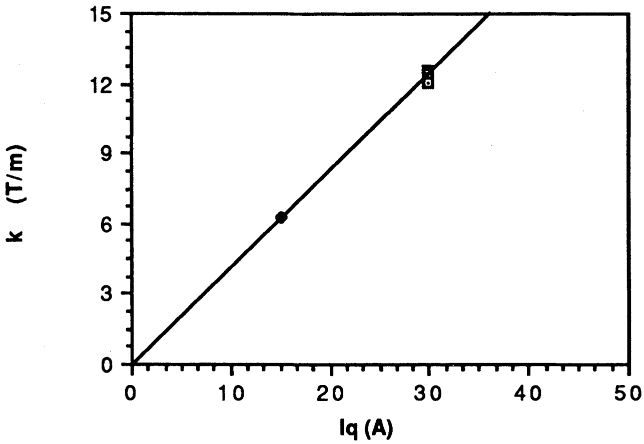

FIGURE 6.

Measurement was made for the quadrupole excitation current I_q=15A and I_q=30A.

References

1. H. Akaike, Mathematical Sciences, 213, 7,1981; K. Takeuchi, Mathematical Sciences, 219, 5,1981

2. G.E. Fisher,J.K. Cobb, D.R. Jensen, "Finding the Magnetic Center of a Quadrupole to High Resolution - a Draft Proposal - ", SLAC-TN-89-01