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QUANTUM BEAMSTRAHLUNG AND ELECTROPRODUCTION OF THE PAIRS IN LINEAR COLLIDERS

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Abstract The processes of beamstrahlung and e^+e^- pair production at beam-beam collisions are discussed on the basis of the quasiclassical operator approach, taking the presence of the field, it's ingomogeneity, end effect and finite beam sizes into account.

INTRODUCTION

The particle interaction at beam-beam collision in linear colliders occurs in an electromagnetic field provided by the beams. As a result, firstly, the phenomena induced by this field turn out to be essential and, secondly, the cross sections of the main quantum electrodynamics processes are drastically modified compared to the case of free particle.

The general theory of electromagnetic interactions of relativistic particles in external fields was given elsewhere.[1] It is based on the quasiclassical operator approach which[2] consistently takes into account the recoil effects at interaction and neglects quantum character of the motion itself. Using this approach, the general problem of radiation at quasi-periodic motion was solved[3]. The motion of a particle in the field of oncoming beam belongs to this very type. In this case the accuracy of quasiclassical description is $\sqrt{1+\mathcal{D}}/N\alpha$, $\mathcal{D}$ is the disruption parameter, N is the total number of particles in the bunch, $\alpha = e^2 = 1/137$ (the system of units $\hbar=c=1$ is used). Recently this theory served as the foundation for the specific crystal electrodynamics[4,5,6], being confirmed in the series of experiments. In the present paper the quasiclassical theory is applied to description of e^+e^- pair creation and radiation at beam-beam collision. The radiation energy losses are widely discussed at present[7,8,9,10,11].

RADIATION

The polarization of the particles involved is naturally described within the quasiclassical approach[1,3]. Here, for the sake of simplicity we give the spectral density of the radiation probability for unpolarized initial particle, summed up over the polarizations of the final particles and integrated over the photon emission angles:

$$\frac{dw_\gamma}{d\omega} = \frac{i\alpha}{2\pi}\int_{-\infty}^{\infty}\frac{dt\,d\tau}{\tau - i0}\cdot\left[\frac{1}{\gamma^2} + \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right)\left(\frac{\bar{v}(t_1) - \bar{v}(t_2)}{2}\right)^2\right]\cdot$$

$$\cdot\exp\left\{-\frac{i\omega\varepsilon\tau}{2\varepsilon'}\left[\frac{1}{\gamma^2} + \frac{1}{\tau}\int_{t_1}^{t_2} dt'\bar{\Delta}^2(t')\right]\right\}, \qquad (2.1)$$

$$\bar{\Delta}(t') = \bar{v}(t') - \frac{1}{\tau}\int_{t_1}^{t_2} dx\,\bar{v}(x),$$

where $t_{2,1} = t \pm \tau/2$, $\gamma = \varepsilon/m$, $m(\varepsilon)$ is the electron mass (energy), $\varepsilon' = \varepsilon - \omega$ .Note,that the structure of Eq.(2.1) is typical for the quasiclassical approach,when the results obtained are expressed in terms of the velocity $\bar{v}(t)$ on the classical trajectory.The Eq.(2.1) represents the contribution of a certain trajectory.To describe the bunch radiation as a whole one should sum up the contributions from different trajectories.The same problem appeared at description of the radiation in crystals[6]. This procedure turns out to be especially simple at $\mathcal{D} \ll 1$ .Then Eq.(2.1) acquires a meaning of the contribution of the particle with definite impact parameter $\bar{\rho}$ and the summation is reduced to the averaging over particle distribution in the transverse plane.

If the field varies slightly along the trajectory, the integral $\int_{t_1}^{t_2} dt'\bar{\Delta}^2(t')$ and the quantity $(\bar{v}(t_1) - \bar{v}(t_2))^2$ in Eq.(2.1) can be expanded in powers of τ .The main term of this expansion gives the well known constant-field (CF) limit,called also synchrotron radiation limit:

$$\frac{dw_\gamma^{CF}}{d\omega} = \frac{\alpha}{\pi\gamma^2\sqrt{3}}\int_{-\infty}^{\infty} dt\,\Phi_\gamma(t),\ \Phi_\gamma(t) = \left(\frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon}\right)K_{\frac{2}{3}}(z) - \int_z^\infty dy\,K_{\frac{1}{3}}(y), \tag{2.2}$$

where K_ν are the MacDonald functions, $z = 2u/3\chi(t)$, $u = \omega/\varepsilon'$. The shape of the spectrum and the magnitude of the quantum effects in the radiation is determined by the parameter χ:

$$\chi = \sqrt{|(eF^{\mu\nu}p_\nu/m^3)^2|} = \gamma|\bar{F}|/H_0 \tag{2.3}$$

here $p^\nu(\varepsilon, \bar{p})$ is a particle four-momentum, $F^{\mu\nu}$ is an electromagnetic field tensor, $\bar{F} = \bar{E} - \bar{v}(\bar{v}\bar{E}) + \bar{v}\times\bar{H}$; $\bar{E}$ and $\bar{H}$ are electric and magnetic fields in the lab.system and $H_0 = m^2/e = 4.41\cdot 10^{13}$ Oe. The radiation is characterized by the length of photon formation ℓ_f corresponding to the values of τ in Eq.(2.1) yielding the phase of the exponent of order unity. One can use as estimate an expression $\ell_f \simeq \ell_0\chi^{-1}(1+\chi/u)^{1/3}$, $\ell_0 = \gamma\lambda_c$, $\lambda_c = 1/m$. We assume that the particles in each beam are in Gaussian distribution with standard deviations σ_x, σ_y and σ_z. The case $\sigma_x = \sigma_y = \sigma_\perp$ will be referred to as a round beam(rd) and the case $\sigma_x \gg \sigma_y$ as a flat beam(fl). Then we have in the region of time and transverse coordinates giving the main contribution, at fixed $\bar{\rho}$:

$$\chi(t) = \chi_0(\bar{\rho})\exp(-2t^2/\sigma_z^2),\ \chi_0^{rd} = \chi_m^{rd}\cdot f(\rho/\sigma_\perp)/f(\rho_0/\sigma_\perp),$$

$$\chi_0^{fl} = \chi_m^{fl}\exp(-x^2/2\sigma_x^2)\,\Phi\left(\sqrt{y^2/2\sigma_y^2}\right),\ f(s) = (1-e^{-s^2/2})/s, \tag{2.4}$$

$$\Phi(s) = \frac{2}{\sqrt{\pi}}\int_0^s dt\,e^{-t^2},\ \chi_m^{rd} = 0.72\,N\gamma\alpha\frac{\lambda_c^2}{\sigma_z\sigma_\perp},\ \chi_m^{fl} = 2N\gamma\alpha\frac{\lambda_c^2}{\sigma_z\sigma_x}.$$

The function $f(s)$ has a maximum at $s_0 = \rho_0/\sigma_\perp = 1.585$ and $f(s_0) = 0.451$.

Retaining next terms of expansion in powers of τ in Eq.(2.1) yields the probability in the form $dw_\gamma = dw_\gamma^{CF}$+corrections. Arising correction terms account for the field ingomogeneity along the length of formation. These gradient corrections were calculated first for the case of harmonic transverse motion[3]. For the general case their explicit form was given elsewhere[6,11]. The relative value of the corrections provided by longitudinal ingomogeneity is of order $(\ell_f/\sigma_z)^2$. The contribution of the transverse ingomogeneity is as a rule still more less[11]. We emphasize the validity of the formula(2.2) at large values of the parameter $\mathcal{D}$ as well. In this case one should take into account in Eq.(2.4) the time dependance of transverse coordinate on the particle trajectory.

For the frequencies ω contributing to the energy loos $\Delta\varepsilon$ and to the total number of photons n_γ emitted by one electron(positron) during collision, the relation $\ell_f/\sigma_z \lesssim 10^{-2}$ holds, providing a high accuracy of CF-limit. The numerical calculation of the relative energy loss $\Delta\varepsilon/\varepsilon$ and n_γ was carried out. After averaging over transverse distribution of radiating beam these quantities were fitted within 3% for arbitrary values of the parameter χ_m:

$$n_\gamma^{fl} = 0.64 \cdot \tau \cdot \chi_m \left(1 + 2.5\chi_m + 0.06\chi_m^2\right)^{-1/6}, \quad \tau = \alpha\sigma_z/\gamma\lambda_c$$

$$n_\gamma^{rd} = 1.47 \tau \cdot \chi_m \left(1 + 4.35\chi_m + 0.2\chi_m^2\right)^{-1/6}, \tag{2.5}$$

$$\frac{(\Delta\varepsilon)^{fl}}{\varepsilon} = 0.114 \cdot \tau \cdot \chi_m^2 \left[1 + 1.7 \cdot (1+\chi_m)\ln(1+2.6\chi_m) + 0.29\chi_m^2\right]^{-2/3},$$

$$\frac{(\Delta\varepsilon)^{rd}}{\varepsilon} = 0.418 \cdot \tau \cdot \chi_m^2 \left[1 + 3(1+\chi_m)\ln(1+2\chi_m) + 0.8\chi_m^2\right]^{-2/3}.$$

The formulae (2.5) allow one to estimate the radiation characteristics in colliders avoiding tedious calculations. Note, that for the known projects we get $n_\gamma \sim 1$ and $\Delta\varepsilon/\varepsilon = (4\div10)\cdot10^{-2}$. The length of formation ℓ_f increases with decreasing frequency ω and CF-description becomes invalid at $\omega \sim \omega_0 = (2\ell_0/\sigma_z)^3 \cdot 4\varepsilon/\chi_0^2$. For the frequencies satisfying the condition $\omega_0 \ll \omega \ll \varepsilon' \ell_0/\sigma_z$ side by side with "synchrotron" photons the end (collinear) photons are radiated. The contribution of the end photons to the energy losses is always much less, than one provided by CF-mechanism. Their spectral distribution was derived elsewhere[11] :

$$\frac{dw_\gamma}{d\omega}=\frac{\alpha}{\pi\omega}\left\{\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\left[\ln\left(\frac{2l_0\varepsilon'}{\omega\sigma_z}\sqrt{\ln\frac{\omega}{\omega_0}}\right)-1.714\right]-2\frac{\varepsilon'}{\varepsilon}\right\}. \tag{2.6}$$

The momentum transfer along the length of formation causes an influence of an external field on incoherent processes with χ/u being the parameter.The problem was solved first to logarithmic[12] and then to power accuracy[13] for $\chi/u\ll 1$.Under this condition the radiation of a virtual photon only is modified resulting in an increase of the minimal momentum transfer q_{min} by the factor $(4\gamma^2\chi/u)^{1/3}\gg 1$ and logarithmic decrease of the cross section.When the parameter χ/u is not small the radiation vertex of a real photon changes and the cross section decreases as $(u/\chi)^{2/3}$. As a result we have[14] at $\chi/u\gg 1$:

$$\frac{d\sigma}{d\omega}=\frac{2\alpha^3\Gamma(1/3)}{5m^2\omega}\left(\frac{u}{3\chi}\right)^{2/3}\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\ln\frac{q_{max}}{q_{min}}, \tag{2.7}$$

here $q_{max}=m(\chi/u)^{1/3}$, $q_{min}=m\chi\,\gamma^{-4/3}(u/\chi)^{2/9}$.

The suppression of incoherent radiation owing to the finiteness of transverse beam sizes was investigated in INP both experimentally[15] and theoretically[16].The phenomenon can be understood directly from the uncertainty relation: $q_\perp\sigma_y\gtrsim 1$ and one should substitute $q_{min}\to\sigma_y^{-1}$ in Eq.(2.7) at $q_{min}<\sigma_y^{-1}$.The spectrum of equivalent photons changes correspondingly:

$$n(\omega)=\frac{2\alpha}{\pi\omega}\,\ln\left(\Delta/q(\omega)\right) \tag{2.8}$$

$$q(\omega)=\omega\gamma^{-1}(1+\varepsilon\chi/\omega)^{1/3}+\sigma_y^{-1}\equiv q_F(\omega)+q_\sigma,\quad q_\sigma=\sigma_y^{-1},$$

here Δ is the upper bound of a transverse momentum transfer,depending on the process, the virtual photon is involved.

PAIRPRODUCTION

As it was shown in the previous section,the density of the photons arising during the collision time is comparable to the density of charged particles in the bunches. We will consider below the ways of converting these photons into e^+e^- pairs, which can serve as unavoidable source of the background. The angular distribution of the outgoing pairs depends on their energy spectrum owing to an rapid increase of the particle deflection angle in the field of the oncoming beam with decrease of the particle's energy.

Pair Creation in the Field of the Oncoming Beam

The probabilities of a photon radiation by a charged particle and pair creation by a photon are interrelated[1]. The corresponding formulae are mutually derived from each other with substitutions:

$$\varepsilon \to -\varepsilon, \quad \omega \to -\omega, \quad \omega^2 d\omega \to -\varepsilon^2 d\varepsilon . \tag{3.1}$$

For example, using Eq.(3.1) we get from Eq.(2.2) the spectral probability of e^+e^- pair creation by the photon for an unpolarized case in CF-limit:

$$\frac{dw_e^{CF}}{dE} = \frac{\alpha m^2}{\pi \omega^2 \sqrt{3}} \int\limits_{-\infty}^{\infty} dt\, \varphi_e(t), \quad \varphi_e(t) = \left(\frac{E}{E'} + \frac{E'}{E}\right) K_{\frac{2}{3}}(y) + \int\limits_{y}^{\infty} ds\, K_{\frac{1}{3}}(s). \tag{3.2}$$

Where E is the energy of one of the created particles, $E' = \omega - E$, $y = 2\omega^2/(3\varkappa(t) E E')$. The parameter $\varkappa$ is determined by Eq.(2.3), if one changes the particle momentum p^ν to photon momentum k^ν i.e. $\gamma \to \omega/m$, $\bar{v} \to \bar{n} = \bar{k}/\omega$. The corresponding length of formation is $l_p \simeq \omega m^{-2} \varkappa^{-1} (1 + \varkappa E' E/\omega^2)^{1/3}$. Explicit formulae, describing corrections to CF-limit at photoproduction were given elsewhere[5,17]. The results for the magnitude of the gradient corrections and end effects discussed above are valid in the case of photoproduction[17].

We get for the probability of cascade process of the photon radiation and subsequent pair creation in the field of the oncoming beam (coherent cascade):

$$\frac{dw_r}{dE} = \frac{r^2}{6\pi^2} \int\limits_{-\infty}^{\infty} dt_1 dt_2 \int\limits_{E}^{\varepsilon} \frac{d\omega}{\omega^2} \left[\varphi_\gamma(t_1) \varphi_e(t_2) - K_{2/3}(z(t_1)) K_{2/3}(y(t_2)) \right]. \tag{3.3}$$

The functions Φ_γ, Φ_e are defined in Eqs.(2.2),(3.2).Remind that $z(t)=2\omega/3\varepsilon'\chi(t)$, $y(t)=2\omega^2/(3\varepsilon\varepsilon'\varkappa(t))$, $\tau=\alpha\sigma_z/\gamma\lambda_c$. We stress the point that the probability (3.3) is not reduced to a product of the probabilities (2.2) and (3.2).We mean not a trivial factor 1/2 appearing in transformations of integrals over time, but an additional term $K_{2/3}\cdot K_{2/3}$ in Eq. (3.3).It appears owing to the fact,that radiated photons have a definite polarization and in turn the probability of pair creation depends on this polarization.Fig.1 shows the

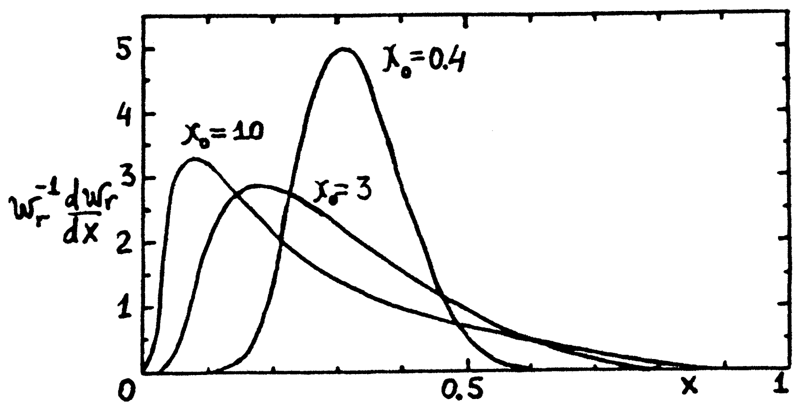

FIGURE 1 The energy spectrum in coherent cascade.

spectrum (3.3) normalized to unity: $W_r^{-1}dW_r/dx$, $x=E/\varepsilon$ for different values of the parameter χ_0 .At $\chi_0 \ll 1$ there is a peak at x=1/3 i.e. the energy of the initial particle is divided approximately into equal parts between three final particles.With the parameter χ_0 growing, the position of the spectral maximum x_m moves to the left: $x_m \simeq (3+\chi_0)^{-1}$. At large $\chi_0 \gg 1$ the spectrum in a wide region $x > x_m$ is proportional to x^{-1}.The fitting procedure (within 3.3% for flat and 5%for round beam) gives for numerically calculated probabilities after averaging over transverse coordinates

$$\overline{W}_r^{fl} = \frac{0.369\cdot\tau^2\chi_m^2}{(1+\chi_m^{-1})^{3/2}}\cdot\frac{(25+\chi_m)^{2/3}}{(25+4\chi_m)^{4/3}}\ln\left(1+\frac{\chi_m}{50}\right)\exp\left(-\frac{16}{3\chi_m}\right) \qquad (3.4)$$

$$\overline{W}_r^{rd} = \frac{1.446\,\tau^2\chi_m^{5/2}}{(7+3\chi_m)^2}\cdot(10+\chi_m)^{5/6}\cdot \ln\left(1+\frac{\chi_m}{30}\right)\cdot\exp\left(-\frac{16}{3\chi_m}\right).$$

The quntities χ_m and τ are defined by Eqs.(2.4) and (2.5). The probability $\overline{W}_r$ is negligibly small at $\chi_m \ll 1$, but with the parameter χ_m growing it increases very rapidly, so that at $\chi_m > 1$ the contribution of the coherent cascade becomes dominant.

The direct electroproduction process ($e \to 3e$) owing to virtual intermidiate photons occurs as well. This process was investigated[19,20] in CF-case. The total probability of $e \to 3e$ process becomes comparable to $\overline{W}_r$ (see Eq.(8) in Ref.18) only at $\chi_m \gg 1$, when the spectrum falls as $x^{-4/3}$ for $x > x_m \simeq \chi^{-1}$. At $\chi_m \lesssim 1$ the probability $\overline{W}_r$ is approximately $\sigma_z/\ell_f \sim 10^2$ times larger.

The density of the real photons radiated incoherently is very small, that is why the corresponding number of pairs created by these photons in the field of the oncoming beam is under real conditions a few orders of magnitude less comparing to the main processes. To get idea about the scale of the effect we give asymptotic expressions for the probability of this process $W_p^{(i)}$ for round beams

$$\overline{W}_p^{(i)}\Big|_{\chi_m \ll 1} = 2.75\cdot 10^{-4}\,\alpha\,\tau^2\chi_m^2\cdot\frac{\lambda_c}{\sigma_\perp}\cdot\ln\left(\frac{\sigma_\perp}{\lambda_c}\right)\exp\left(-\frac{8}{3\chi_m}\right), \tag{3.5}$$

$$\overline{W}_p^{(i)}\Big|_{\chi_m \gg 1} = 0.21\,\alpha\,\tau^2\chi_m^2\cdot\frac{\lambda_c}{\sigma_\perp}\cdot\ln\left(\frac{\sigma_\perp}{\lambda_c}\chi_m^{1/3}\right).$$

The main contribution to the probability $\overline{W}_p^{(i)}$ is given by $\omega \sim \varepsilon$ at arbitrary values of the parameter χ_m, so that the spectrum in this process has no peculiarities.

Incoherent Processes

The CF-photon incoherent conversion into the pair (mixed cascade) is of importance at $\chi_m < 1$ only. In this case the parameter $\varkappa = \chi\cdot\omega/\varepsilon$ of the radiated photon is small enough and incoherent photoproduction cross section σ_p is nearly independent of the photon frequency, owing to the finite beam size influence

$$\sigma_p = \frac{28}{9}\alpha^3 \lambda_c^2 \cdot Q \cdot \ln\left(\frac{\sigma_y}{\lambda_c}\right) \quad , \quad Q = 1 + \frac{396}{1225}\varkappa^2 . \tag{3.6}$$

Using Eq.(2.2) one can show, that $\overline{\omega^2}/\varepsilon^2 < 1/6$. Then the probability of the process under discussion $w_p^{(2)}$ is factorized, if neglecting corrections $\sim \varkappa^2$ (letting $Q=1$ in Eq. (3.6)):

$$w_p^{(2)}(\bar{\rho}) = \frac{1}{2} N\, n_\perp(\bar{\rho}) n_\gamma(\bar{\rho}) \cdot \sigma_p \tag{3.7}$$

where $n_\perp(\bar{\rho})$ is the transverse distribution in the oncoming beam. At averaging of Eq.(3.7) with a transverse distribution in the second beam it is convenient to use an expression fitting the number of photons at fixed $\bar{\rho}$ within 1% :

$$n_\gamma(\bar{\rho}) = 1.81\, \tau \chi_0 \left[1 + 1.5(1+\chi_0)\ln(1+3\chi_0) + 0.3\chi_0^2\right]^{-1/6} \tag{3.8}$$

After averaging mentioned we have at $\chi_m \ll 1$

$$\left.\overline{w}_p^{(2)}\right|_{\chi_m \ll 1} = C_p \cdot \alpha \tau^2 \chi_m^2 \cdot \frac{\lambda_c}{\sigma_y} \cdot \ln\left(\frac{\sigma_y}{\lambda_c}\right) \tag{3.9}$$

where $C_p^{rd} = 0.224$ and $C_p^{fl} = 3.58 \cdot 10^{-2}$. The probability $\overline{w}_p^{(2)}$ is negligibly small at $\chi_m \gg 1$, comparing to the coherent cascade case, e.g. one has for the round beams

$$\left.\overline{w}_p^{(2)}/\overline{w}_r\right|_{\chi_m \gg 1} \simeq 5.5\alpha \cdot \frac{\lambda_c}{\sigma_y} \cdot \frac{\ln(\sigma_y/\lambda_c)}{\ln \chi_m} . \tag{3.10}$$

To describe the pair creation by two virtual photons we will use the equivalent photon approximation. As the photoprocess is suppressed at $\varkappa \gg 1$ by the factor $\sim \varkappa^{-2/3}$, the upper bound on contributing frequencies is determined by the condition $\varkappa(\omega) = \chi\omega/\varepsilon \lesssim 1$, so that $\omega_{max} = \varepsilon/(1+\chi)$. The lower bound is determined as in a free particle case by the relation $\omega_1 \omega_2 = m^2$, giving $\omega_{min} = m(1+\chi)/\gamma$. At $\omega < \omega_{max}$ one can neglect the field dependence of the two photon process cross section and the quantity Δ in Eq.(2.8) Let the frequency ω_σ be defined by the equality $q_F(\omega_\sigma) = q_\sigma$ (q_F, q_σ are given in Eq.(2.8)) then the lower bound on momentum transfer $q(\omega)$ is determined by the beam sizes at $\omega \lesssim \omega_\sigma$ and by the field at $\omega \gtrsim \omega_\sigma$. One finds from Eq.(2.8) $\omega_\sigma = \omega_{min}\cdot(q_\sigma/q_F(\omega_{min}))^{3/2}$. Letting $\Delta = m$ we have in the main logarithmic approximation for the cross section of the $2e \to 4e$ process

$$\sigma(2e\to 4e) = \frac{28}{3\pi}\alpha^4 \lambda_c^2 \left[L_1 L_2^2 - L_2 L_3^2 + \varphi\left(\left(\frac{m}{\omega_\sigma}\right)^{4/3}\right)\right],$$

$$L_1 = \frac{2}{3}\ln\left(\frac{\omega_{max}}{\omega_{min}}\right),\quad L_2 = \ln\left(\frac{\sigma_y}{\lambda_c}\right),\quad L_3 = \frac{2}{3}\ln\left(\frac{\omega_{max}}{\omega_\sigma}\right), \tag{3.11}$$

$$\varphi(z) = \int_0^\infty \frac{dx}{x}\ln(1+x)\cdot\ln\left(1+\frac{z}{x}\right).$$

One should keep the term $\varphi(z)$ in Eq.(3.11) at $z \gg 1$ only, when $\varphi(z) \simeq (\ln z)^3/6$. The ratio of this cross section at $\chi_m \lesssim 1$ to the standard Landau-Lifshitz[21] one σ_{LL} is roughly about

$$\sigma(2e\to 4e)/\sigma_{LL} \approx \ln(\sigma_y/\lambda_c)/\ln\gamma^2 \tag{3.12}$$

The total number and the spectrum of created pairs at $\chi_m < 1$ are determined just by the $2e \to 4e$ process and by the mixed cascade with approximately equal contributons.

For the pair production by two real photons ($\gamma\gamma$-process) we consider more general case, when created particles have the mass μ and spin s= 0 or s=1/2. One should average the corresponding two photon cross section with two radiation spectra (2.2). The interval of the contributed frequencies is determined by the properties of these spectra and by the mentioned suppression of the photoproducton at $\varkappa \gg 1$. The minimum frequency coincides with $\omega_0 = 4\varepsilon(2\gamma\lambda_c/\sigma_z)^3/\chi_0^2$ introduced above and $\omega_m = \mu^2/\omega_0 \lesssim \varepsilon\chi_0/(1+\chi_0)^2$. The effective cross section was derived[18] at $\omega_0 \ll \mu$ to logarithmic accuracy. One get the total number of pairs after averaging $\sigma_{e\zeta}^{(s)}$ with $N^2 n_\perp^2(\bar\rho)$,

$$\sigma_{ef}^{(s)} = 1.54\, C^{(s)} \alpha^4 \sigma_z^2 \left(\frac{\chi_0 m}{\gamma^2 \mu}\right)^{4/3} \ln\left(\frac{\omega_m}{\bar{\omega}_0}\right), \tag{3.13}$$

where $C^{(1/2)}=1$ and $C^{(0)}=1/8$. When an opposite inequality is fullfilled $\omega_0 \gg \mu$ the pair production by $\gamma\gamma$-process is suppressed as $(\mu/\omega_0)^{4/3}$. Note, that for e^+e^- production it's contribution is $1 \div 2$ orders of magnitude less comparing to other incoherent processes. For the creation of heavy particles with $\mu \gg m$ the parameter $\chi(\mu)=\chi(m)\cdot(m/\mu)^3$ is small and the incoherent processes only are of importance. For $\mu \gg m$ the probabilities of the mixed cascade and $2e \to 4e$ process acquire a factor $(m/\mu)^2$, whilst the $\gamma\gamma$-process according to Eq.(3.13) get a factor $(m/\mu)^{4/3}$. The rough estimate of the ratio the number of pairs created by $\gamma\gamma$-mechanism to one by mixed cascade gives $(\mu/\bar{\omega})^{2/3}\cdot n_\gamma/\alpha$, where $\bar{\omega} \simeq \varepsilon\chi_0/(3+4\chi_0)$. So, the $\gamma\gamma$-process determines the heavy particles creation at $\mu \gg \alpha^{3/2}\bar{\omega}$.

TABLE I Characteristics of radiation and pair creation

Project (energy TeV)	χ_m	Radiation n_γ	$\frac{\Delta\varepsilon}{\varepsilon}\cdot 10^2$	Number of pairs per beam: Coherent	$2e \to 4e$
KEK (0.5)	0.14	2.25	6.5	$2.7\cdot 10^{-9}$	750
INP (0.5)	0.19	1.64	4.0	$8.3\cdot 10^{-6}$	$1.8\cdot 10^4$
INP (1)	0.38	1.57	6.2	49	$3.6\cdot 10^4$
CERN(1)	0.42	1.51	8.9	34	390
SLAC(0.5)	2.39	0.85	8.7	$5.5\cdot 10^6$	$9.6\cdot 10^3$
SUPER(5)	4600	0.41	10.3	$4.4\cdot 10^6$	$1.1\cdot 10^3$

Some quantities characterizing the radiation and pair creation are given in Table I. Owing to the permanent modification of the designes, the figures rather represent the scale of the effects, but the formulae derived allow one to estimate simply the situation for arbitrary set of the parameters.

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