Particle Accelerators, 1990, Vol. 28, pp. 101–106
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NEW ASPECTS OF THE THEORY OF RESONANT BEAM EXTRACTION FROM SYNCHROTRONS (STRETCHERS)

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Abstract The work is devoted to a detailed study of particles radial motion in synchrotrons near the third integral resonance. The well-known generalized equations describing the radial motion of a particle from cycle to cycle (one cycle equals three revolutions) in the general case are shown to describe the process of particle ejection from synchrotrons. It has been proved that in order to describe the process of slow ejection on the third integral resonance correctly, it is necessary to follow the particle position at a given azimuth, at least after each its revolution. From the derived linearized equations of the particle r-motion with respect to revolutions two new criteria have been obtained for the ejection system optimization in the form of two additional conditions which the ejection system must satisfy so that the well-known phase pattern (the triangular separatrix, etc.) of particle motion would really take place.

The theory of extraction at the third integral resonance is known for a long time (1955, Barton[1]) and the estimates of the parameters of the extracted beam are based on the corresponding pattern of the particle r-motion on the normalized phase plane $(\eta,\dot{\eta})$; $(\eta=r\beta^{-1/2}$; $\dot{\eta}=\beta^{+1/2}(dr/ds - 1/2\ d\beta/ds \cdot \beta^{-1}r))$. However, our detailed investigations show that the generally accepted theory of resonant extraction (Barton[1], Mashke[2], Hemmie[3]) is based on incorrect assumptions. Indeed, while the existence of a triangular separatrix on the plane $(\eta,\dot{\eta})$ really follows from the linearized "cycle equations", the application of this theory to explain the process of the particles throwing into the septum-magnet contains a wrong assumption, namely, during a cycle (three revolutions), after each revolution, the phase pattern is rotated so that the separatrix branches cyclically interchange their places. But in reality (see below), for this to take place it is necessary that the ejection system (the number of sextupoles, their location, etc.) satisfies certain conditions. Only under such conditions the separatrix branches will be really "identical" at the moment of the beam throwing into the septum-magnet. That is why we call them conditions of the beam extraction system optimization.

The investigations are based on the equation

$$r''_s + K(s)\, r = -\, g(s) r^2 \qquad (1)$$

where g is a small magnitude.

Assume that there are j "thin" sextupoles with length Δs_i $(i=1,2,\ldots,j)$ on the azimuths s_i. Let us introduce the small parameter ε, $2\pi Q=(2\pi m+\varepsilon)/3$, which characterizes closeness of the oscillation frequency Q to its resonance value m/3 and take three revolutions (a cycle) as unit time of changing of particle coordinates. Then in the polar coordinate frame (R,α) with $\eta=\cos\alpha$, $\dot{\eta}=R\sin\alpha$, and in the linear approximation of g and ε one can write the well-known equation of motion in the following recurrent form:

$$R_{n+1} = R_n \sqrt{U_n^2 + V_n^2}\,; \quad \alpha_{n+1} = \alpha_n - \mathrm{arctg}\,\frac{V_n}{U_n}\,,$$

$$U_n = 1-3ER_n \sin(\varphi+3\alpha_n),\; V_n = \varepsilon + 3ER_n \cos(\varphi+3\alpha_n)$$

where n is the cycle number and

$$E = \sqrt{C^2+D^2}\,;\; \varphi = \mathrm{arctg}\; C/D;$$

$$C = 1/4 \sum_{i=1}^{j} g_i \Delta s_i \beta_i^{3/2} \cos 3\psi_i\,;\; D = -1/4 \sum_{i=1}^{j} g_i \Delta s_i \beta_i^{3/2} \sin 3\psi_i$$

β_i and ψ_i are amplitude and phase functions on the azimuth of the i-th sextupole, respectively. From (2) follows the well-known triangular separatrix with rectilinear branches which allow to visualize the particles throwing into the septum. Far from the resonance the triangle's area is considerably larger than the emittance of the circulating beam. The closer to the resonance $(\varepsilon\to 0)$, the smaller the triangle's area:

$$S_{\Delta} = \frac{3\sqrt{3}}{4}\,\eta_o^2\ ; \qquad \eta_o = \frac{|\varepsilon|}{3E}\ .$$

The triangle "squeezes" the emittance in three directions. In a certain instant of time the accelerated beam "occupies" the triangle. Then, when $\varepsilon \to 0$, part of particles is "forced out" of the triangle through all its vertices simultaneously. Therefore, the particles turn out to be distributed near the separatrix branches, and moving along them depart from the beam core. Choosing the number of sextupoles, j, and their location sites, s_i, relative to s_{sep} one can achieve that on the septum azimuth s_{sep} one of the separatrix branches is parallel to the η axis. This requires satisfaction of the condition C=0. Then, depending on what side of the orbit the septum-magnet is placed and on the sign of ε, one provides such a sign of D that the particle motion along the separatrix branch parallel to the η axis turns out to be directed to the septum. Then, with a definite probability, the particles extracted from the region of stability will reach the septum and part of them will appear beyond it.

From such a "behaviour" of the beam phase pattern from cycle to cycle, however, itfollows that only a third of the beam will be thrown into the septum (without account of losses on the "blade" of the septum), since it is not clear how the particles moving along the two other branches will "behave" relative to the septum. It is just here that the generally accepted theory makes baseless assumptions that the separatrix branches interchange their places within a cycle. First, it supposes existence of a triangle separatrix within a cycle, at least after each revolution. And second, it supposes that after each revolution the separatrix is rotated by an angle equal to exactly $2\pi m/3$. But it is not proved. If it is really so, then it must be followed from the investigations of the motion from turn to turn, just what we have done.

In contrast to the generally accepted theory we followed the position of particles on the given azimuth after each revolution. In the polar coordinate frame, and in the same approximation the motion of particles from turn to turn is described by the recurrent formulae

$$R_{k+1} = R_k\sqrt{U_k^2 + V_k^2}\,;\quad \alpha_{k+1} = \alpha_k - \frac{2\pi m}{3} - \operatorname{arctg}\frac{V_k}{U_k};$$
$$U_k = 1-ER_k\left[\sin(\varphi+3\alpha_k) - A\cos\alpha_k + B\sin\alpha_k\right]; \qquad (3)$$
$$V_k = \varepsilon/3 + ER_k\left[\cos(\varphi+3\alpha_k) + 3A\sin\alpha_k + 3B\cos\alpha_k\right]$$

where k is the number of revolutions and

$$A = 1/4E\sum_{i=1}^{i} g_i\Delta s_i\beta_i^{3/2}\sin\psi_i;\quad B = 1/4E\sum_{i=1}^{i} g_i\Delta s_i\beta_i^{3/2}\cos\psi_i$$

A simple comparison of (2) with (3) shows that the motion from turn to turn depends on the parameters A, B, m, whereas the "cycle equations" lack these parameters. Therefore, on the first stage the investigations were carried out according to (3), assuming A=B=0. It became clear (see fig.1) that only at A=B=0there really exists a triangular separatrix with rectilinear branches which after each revolution are rotated by $2\pi m/3$ and the beam phase pattern exactly corresponds to the generally accepted notion. In all other cases (see, e.g. fig.2). i.e. when $A\neq0$, $B\neq0$, there is no such separatrix. In other words, in the general case the "cycle equations" describe the particles motion near the resonance region incorrectly and hence, it is at least incorrect to compare the experiment with the predictions of the generally accepted theory. The investigations via (3) have shown that when $A\neq0$, $B\neq0$ the quadratic nonlinearity leads to deformation of the beam phase volume, which is different at different values of A and B. At $\varepsilon\simeq1$ we get more accurate formulae (without linearization over ε)

$$R_{k+1} = R_k\sqrt{U_k^2 + V_k^2}\,;\quad \alpha_{k+1} = \alpha_k - 2\pi Q - \operatorname{arctg}\frac{V_k}{U_k},$$
$$U_k = 1-ER_k\left[\sin(\varphi+3\alpha_k) - A\cos\alpha_k + B\sin\alpha_k\right], \qquad (4)$$
$$V_k = ER_k\left[\cos(\varphi+3\alpha_k) + 3A\sin\alpha_k + 3B\cos\alpha_k\right].$$

The conditions of existence of quasistationary points on the plane (η,η), i.e. points cyclically interchanging their places after each revolution, are:

$$R^{st}_{k+1} = R^{st}_{k} \ ; \qquad \alpha^{st}_{k+1} = \alpha^{st}_{k} - \frac{2\pi m}{3}. \tag{5}$$

It is easy to show that at any "k" the expressions (5) have solutions only when A=B=0, which have the form:

$$R^{st} = \frac{2}{E} \ \sin\frac{|\varepsilon|}{6} \ ; \qquad \alpha^{st}_{i} = \delta_{\pm} + (i-1)\,\frac{2\pi}{3} \ ; \quad (i=0,1,2)$$

$$\delta_{+} = -\frac{\varphi}{3} - \frac{|\varepsilon|}{18} + \frac{\pi}{3} \ ; \qquad \delta_{-} = -\frac{\varphi}{3} + \frac{|\varepsilon|}{18} \tag{6}$$

where $(\pm)$ is the sign of ε.

In conclusion let us elucidate the physical meaning of the conditions A=B=0. To do this, come back to the eq.(1), the solution of which we represent as:

$$r(s) = r_{o}(s) + r_{1}(s) \tag{7}$$

where $r_{o}(s)$ is the solution of (1) at g=0. Substituting (7) into (1), in an approximation linear over g we obtain

$$r_{1}'' + \left[K(s) + 2gr_{o}\right]\, r_{1} = -\, gr_{o}^{2}. \tag{8}$$

When deriving the relations (2)÷(4), we did not take account of the second term in square brackets which, as is known, gives correction to the frequency Q. In our case it has the form:

$$\Delta Q_{k} = \frac{2E}{\pi}\left\{A\cdot r_{k}(0)\beta^{-1/2}(0) + B\cdot\beta^{1/2}(0)\left[r_{k}(0)+\alpha(0)\beta^{-1}(0)r_{k}(0)\right]\right\} \tag{9}$$

whence it follows that the conditions A=B=0 assure independence of Q on the amplitude of oscillations.

The results of calculations according to (4) have been compared with the numerical computations using the parameters of our synchrotron. The smaller the value of $|g|$, the better the agreement between the results.

Thus, the new approach developed allows to make the ejection system optimization easy and simple.

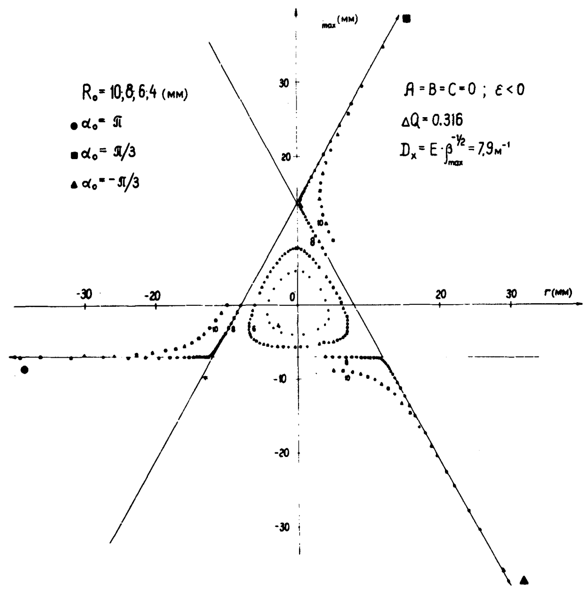

Fig.1

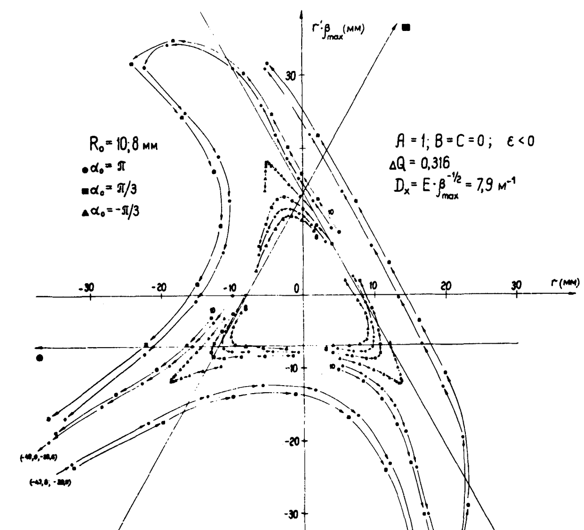

Fig.2

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