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EVOLUTION OF LONGITUDINAL DISTRIBUTION FUNCTION OF AN INTENSE BUNCH IN A PROTON SYNCHROTRON

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Abstract Evolution of stationary longitudinal distribution of particles in a bunch is investigated by means of computer calculations. The bunch is supposed to be circulating in a proton synchrotron with a space charge parameter slowly increasing. The coupling impedance is represented by a low-Q and high-F resonant cavity-like element. The longitudinal distribution is shown to tend to Gauss one and does not depend on the initial distribution, provided the impedance value is sufficiently large. The bunch parameters satisfy the well-known Boussard criterion commonly interpreted as a microwave instability threshold.

There are two view-points regarding the so called microwave instability. The first one identifies it with the "fast (or turbulent)" instability, whose typical time is much less than the synchrotron oscillation period, and a typical distance is much shorter than the bunch length[1,2]. It is considered that in this view one can ignore the synchrotron oscillations as well as the beam bunching, and use the coasting beam model. This model leads to the threshold relation[3] which is in this case should be interpreted as the condition for the absence of the fast instability:

$$\left|\frac{Z_n}{n}\right| < \frac{E\beta^2|\eta|\,B}{eJ}\left(\frac{\Delta p}{p}\right)^2 . \qquad (1)$$

Here Z_n is the vacuum chamber impedance at the n-th harmonic of the revolution frequency, $E=mc^2\gamma$ is the energy, p is the momentum, β is the normalized velocity of a particle, J is the average current, B is the bunching factor, Δp is the specific momentum spread of the beam, $\eta=\alpha-\gamma^{2}$, α is the momentum compaction factor.

The shortcomings of such an approach were considered in[4,5]. The principal one is that in the above assumptions there are no conditions for a positive feedback in the bunched beam, which is necessary for the usual (regenerative) instability to appear. At the same time it has been pointed out that sometimes the violation of (1) leads to the powerful strengthening of the bunch response to the external perturbations, always occurring, say, due to continuous deviation from the stationary state during the standard acceleration. Therefore the supposition has been made that inequality (1) determines the minimum momentum spread of the bunch, which is in thermodynamic balance with its own radiation (short-wave and fast-waked). The results of computational analysis are demonstrated below, which can, to our mind, clarify the problem.

We cannot restrict ourselves to solving the stationary problem because it has an infinite number of solutions, and it can hardly be expected that all of them will satisfy the inequality (1). It seems more realistic to investigate a non-stationary process when condition (1) is first definitely fulfilled but violated later due to a change of the external conditions. A typical example is the parameter $|\eta|$ diminishing when the beam energy approaches transition. This case has been considered in Ref.[6]. Here another process, when the impedance gradually increases in time, is investigated. A similar situation takes place at the IHEP accelerator (U-70), the coupling impedance of which is of the high-F and low-Q resonant type, and the shunt impedance increases by an order of magnitude in the 4÷6 GeV range[7,8]. Our computations have been performed with the parameters of the U-70 at the 5 GeV flat top. We have used the macroparticles method (up to 40.000 per bunch), when each particle was ascribed a certain weight factor ("the charge"). The initial longitudinal phase space distribution function was formed by an appropriate arrangement of the macroparticles with a certain "charge" in the phase plane. The coupling impedance is represented by a short low-Q and high-F resonant cavity-like element whose characteristic is

$$Z(f) = \frac{2iRf\,\Delta f}{f^2+2if\Delta f-f_o^2}, \tag{2}$$

where f is the frequency, f_o is the resonant frequency, $2\Delta f$ is the bandwidth, R is the shunt impedance. The parasitic cavity (to be termed just "cavity" further on) is assumed to be placed at the same azimuth as the accelerating RF-cavity. In order to find the instantaneous current the bunch in the phase space plane is divided by a large number of vertical strips (up to 130 ones). Then the charge of each strip is calculated. The voltage induced by a strip after it has passed through the cavity can be found using expression (2). The resulting voltage was smoothed by means of spline interpolation in order to diminish the noise due to the discrete nature of macroparticles. The necessary parameters used in the program are listed in Table I.

TABLE I. U-70, Parasitic Cavity and Initial Bunch Parameters.

Average radius	R_o=236 m
Harmonic number	q=30
Momentum compaction factor	α = 0.01112
Peak RF-voltage	V=0.35 MV
Resonant frequency	f_o=300 MHz
Bandwidth	$2\Delta f$=32 MHz
Longitudinal emittance (total)	ε=74 MeV/c·m
Bunch population	N=$1.7\cdot 10^{12}$ protons/bunch
Synchrotron period	T_s=1.74 ms

We take into account a single bunch, but there is no loss of generality as the bunches in the machine do not influence each other provided Δf is sufficiently large.

The typical result is that the high-frequency modulation of the bunch current appears, when the shunt impedance exceeds some threshold value. The modulation does not grow much with R increasing further, but the growth of the effective bunch phase-space area becomes essential. If the rate of the impedance growth is not too high we find the bunch in a quasi-stationary state, i.e. its longitudinal phase space distribution function depends, to a first approximation, only on the phase oscillation energy.

The evolution of the longitudinal distribution function averaged over the phases of synchrotron oscillations is shown in Fig.1. The shunt impedance increases linearly from 0 to R/n=85 Ohm in 28 ms (16 phase oscillation periods) and further is kept constant. Here $n=f_o/f_s$, f_s is the revolution frequency. The normalized square of the phase oscillation amplitude is plotted horizontally. The initial distribution function $f(x)\sim\sqrt{1-x^2}$ changes in time and tends to be exponential, at least in the bunch core. The final distribution appears to be actually independent on the initial one. This is illustrated in Fig.2, where the previous case is considered for different initial distributions. The distributions (1,2,3) in 40 T_s's come to the same final state (a). The result is slightly different (b) for the initial distribution f(x)=const (4). The reason is that 40 T_s's are insufficient for such a distribution to come to the stationary one, so more prolonged runs are required which would consume too much of the computer time.

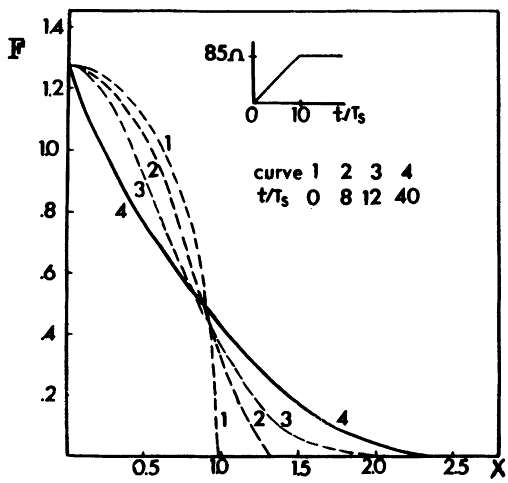

FIGURE 1. Evolution of the longitudinal distribution function; x is the energy of phase oscillations.

Analyzing these results we have come to the conclusion that the interaction of a single bunch with the high-frequency and low-Q impedance leads to a new stationary distribution. For the longitudinal coordinates and momenta this distribution is Gaussian:

$$F\sim\exp\left[-\frac{s^2}{2\sigma_s^2}-\frac{(p-p_s)^2}{2\sigma_p^2}\right]=\exp\left(-\frac{\varepsilon}{2\varepsilon_o}\right) , \qquad (3)$$

where ε is the phase space area (divided by π) enclosed within a trajectory in the coordinate-momentum phase plane, $\varepsilon_o=\sigma_s\sigma_p$ is the rms longitudinal bunch emittance. We show in Fig.3 ε_o as a function of shunt impedance. The bunch parameters are listed in Table I and the initial distribution was combined of two conjugated parabolas (see curve 1 in Fig.2). Two ways of the impedance variation were investigated:

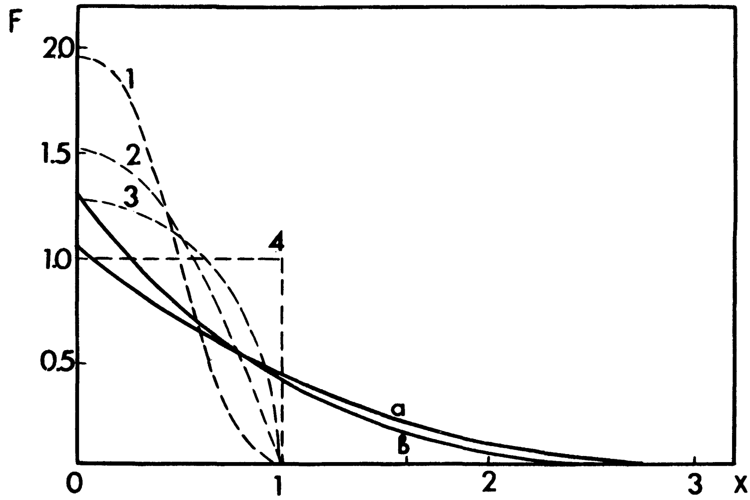

FIGURE 2. Final stationary distribution function (solid lines) for different initial distributions (dashed lines).

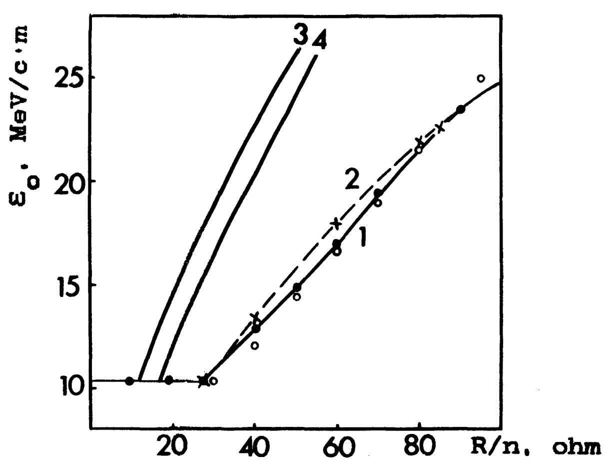

FIGURE 3. RMS longitudinal emittance versus impedance.

1. The shunt impedance increased gradually with a rate $\dot{R}/n$=765 Ohm/s (curve 1). Two different cavities were considered in this variant:

a) f_0=300 MHz, Δf=16 MHz (•)

b) f_0=900 MHz, Δf=16 MHz (o)

The results have actually coincided, so further calculations were performed for case a) only.

2. The shunt impedance increased gradually in the same manner as above until it has reached a certain value and kept constant afterwards. The emittance was calculated when it became actually stationary. These

stationary values are shown as crosses in Fig.3. The dashed curve 2 is plotted only for illustration purposes. As seen, the bunch sometimes has no time to reach the quasi stationary state in the case when the impedance increases gradually.

Curve 2 corresponds well to the law $\varepsilon_0 \sim \left(\frac{R}{n}\right)^{2/3}$. The law follows from expression (2) and is shown by curve 3 in Fig.3. Some discrepancy ($\sim$ 1.7 times) is explained by the fact that the cavity filling time was in our calculations comparable with the bunch duration (10 ns and 20 ns respectively), which is equivalent to some increase of the bunching factor. To make sure of that we have repeated the calculation for the case 1a, using a smaller filling time, τ=2 ns (Δf=80 MHz) and $\dot{R}/n$= =380 Ohm/s. As a result we have curve 4 which is much closer to the "theoretical" curve 3.

The dependence of ε_0 on Δf is plotted in Fig.4. The case 2a has been considered with the final value of the impedance R/n=60 Ohm. The emittance satisfies condition (1) at $\tau \leqslant 0.1\,\tau_b$ (the dashed line shows the value that follows from (1)). We have no visible effect on the initial bunch for $\tau \geqslant 2\tau_b$ though the impedance is about twice as much as the formal "threshold" (1).

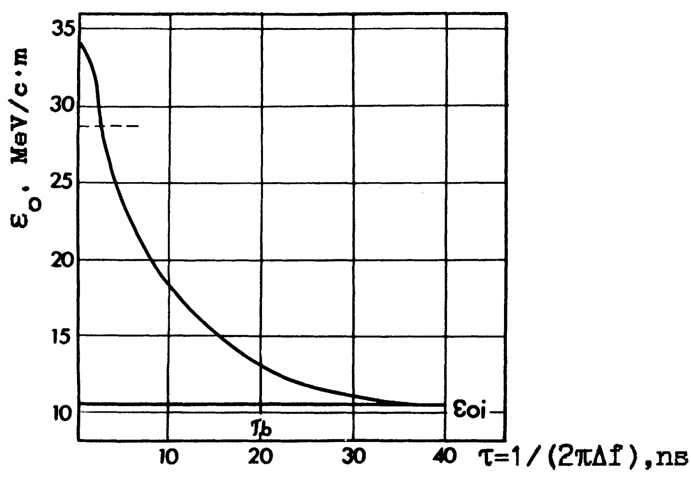

FIGURE 4. RMS longitudinal emittance versus cavity bandwidth.

If the impedance grows too quickly there is no time for a quasi - stationary state to establish. A powerful high-frequency modulation of the bunch current develops which leads to an additional bunch blow-up (see Fig.5). The value of R/n increased from 0 to 85 Ohm in a manner shown in Fig.1, the rising time varying in rather a wide range. The two curves correspond to the initial distributions noted as in Fig.1. It is seen that the "adiabaticity" condition is more difficult to be fulfilled for the distributions with sharp boundaries.

Thus, we can state that under the conditions considered in this work the longitudinal distribution function tends to that expressed by (3). The value of $\Delta p \simeq 2.87\,\sigma_p$ is determined by Boussard criterion (1) The process of the distribution function reconstruction is quasi adiabatic, its rate is determined by the rate of variation of the external conditions.

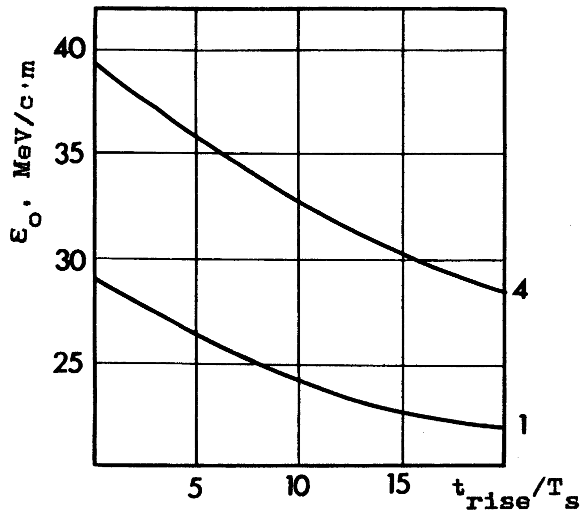

FIGURE 5. RMS final emittance versus the time rise of R/n from 0 to 85 Ohm.

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