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LANDAU DAMPING DUE TO TUNE SPREADS IN BETATRON AMPLITUDE AND MOMENTUM

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Abstract Due to the large space charge transverse impedance in a low energy synchrotron, the coherent tune shift causes the Landau damping to be ineffective in damping the transverse coherent motion. We analyze the effect of Landau damping that is caused by the tune spreads of the betatron amplitude (space charge and/or octupole) and momentum. We find that the Landau damping becomes more significant in our two dimensional analysis.

INTRODUCTION

Usually, the transverse space charge impedance for the high intensity, low energy Booster synchrotron is very large. For the AGS Booster, the impedance is given by,

$$Z_\perp = i\,\frac{RZo}{\beta^2\gamma^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \begin{cases} i\ 9.2\ \mathrm{M\Omega/m} - \text{proton at 200 MeV} \\ i\ 2\ \mathrm{G\Omega/m} - \text{Au at 45 MeV/u} \end{cases}$$

where $Zo = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377\Omega$, and R, a, b are the radius of the accelerator, the beam size and the radius of the vacuum chamber respectively. The equation of motion for a single particle under the influence of a wake field is given by,

$$x + Q^2\omega^2\, x = 2\, Q\omega\Delta\omega\, \langle x\rangle \tag{1}$$

where $\langle x\rangle$ is the average centroid of the beam, Q is the betatron tune, and ω is the revolution frequency. The frequency shift ΔQ is related to the broad band impedance by the following expression,

$$\Delta\Omega = +i\frac{e^2\beta^2 N_B c Z_\perp}{2Q\omega\gamma m_o\sigma_\ell} = U + (1 + i)\ V \tag{2}$$

where N_B/σ_ℓ is related to the bunch line density. The growth rate is given by,

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$$\frac{1}{\tau} = \mathrm{Im}\,\Delta\Omega = \Omega_i$$

At injection energy, the U, V parameter of Ref. 1 for the AGS Booster is given by

	Proton	S	Au
U [s^{-1}]	-40000.	-50000	-3500
V [s^{-1}]	400.	3	0.6

where the resistive wall of an Inconel 625 vacuum chamber was used. Note here that U>>V is a feature common to all low energy synchrotrons.

For the broad band impedance case, the transverse single bunch impedance is equivalent to the impedance of the coasting beam. The dispersion integral can then be derived from Eq. (1) as[1,2],

$$1 = [U + (V+iV)] \int \frac{f(p)dp\ g(J)d_J}{\Omega - (n-Q)\omega} \qquad (3)$$

with the fast wave component being neglected. The distribution functions of the bunch for the momentum and the betatron emittance space are normalized respectively,

$$\int_{\infty}^{\infty} f(p)dp = 1 \quad ; \quad \int_{0}^{\infty} g(J)dJ = 1 \qquad (4)$$

There are many theoretical analyses of Landau damping via the frequency spread in momentum space[2] or betatron space[3]. Möhl and Schönauer[3] have studied Landau damping due to betatron nonlinearity in tune spreads. Stability diagrams show interesting information for the octupole needed to increase the tune spread. In the limit of vanishing octupole strength, it is known that the incoherent space charge force does not influence dipole motion[4]. This is due to the fact that the incoherent space charge force is measured relative to the center of the beam. The damping of the transverse coherent motion arises from the tune spread relative to the center of the central orbit. On the other hand, the incoherent space charge force in the presence of synchrotron motion can induce Landau damping of the dipole mode. The effect of Landau damping due to momentum and betatron space together should become more effective.

In this paper, we look at the solution of Eq. (3) for some simple models. We shall examine the model of (1) no Landau damping (2) damping due to betatron space, (3) damping due to momentum frequency spread and (4) damping due to momentum and betatron space.

No Landau Damping Case

When $f(p) = \delta(p-p_o)$ and Q is independent of the emittance J, the imaginary part of the frequency shift is easily evaluated to be equal to V, i.e., $\Omega_i = V$. Thus, only the resistive wall contributes to the growth in this case.

Damping in the Betatron Space

Let $f(p) = \delta(p-p_o)$ and let Q depend on the emittance, such that $Q = Q_o + \Delta Q\,(J-\epsilon_o)/\epsilon_o$, where ΔQ is the tune spread of the beam. We assume further that

$$g(J) = \begin{cases} \frac{1}{\epsilon_o} & 0 \le J \le \epsilon_o \\ 0 & \text{elsewhere} \end{cases}$$

The dispersion integral of Eq (4) becomes,

$$\frac{U+V+iV}{\Delta Q\omega_o}\; ln\, \frac{\Omega-(n-Q_o)\omega_o}{\Omega - (n-Q_o+\Delta Q)\omega_o} = 1 \qquad (5)$$

By defining μ and ν as follows,

$$\mu = \Delta Q\omega_o \frac{U+V}{(U+V)^2+V^2} \qquad (6a)$$

$$\nu = \Delta Q\omega_o \frac{V}{(U+V)^2+V^2} \qquad (6b)$$

the imaginary part of the frequency can be written as,

$$\Omega_i = \Delta Q\omega_o \frac{e^{-\mu}\sin\nu}{1-2e^{-\mu}\cos\nu + e^{-2\mu}} \qquad (7)$$

By expanding Eq. (7) in a power series of the tune spread parameter, we express the imaginary part of the frequency shift as

$$\Omega_i = V\,[1 - \frac{(U+V)\;\Delta Q\omega_o}{(U+V)^2 + V^2} + \ldots] \qquad (8)$$

From this equation, we observe that the amplitude dependence of the tune may alter the growth rate through Landau damping. The tune spread that is given in this scenario may arise from octupole magnet elements or from incoherent space charge detuning, together with the synchrotron motion. It is easily seen from Eq. (8) that when the frequency spread $\Delta Q\omega_o$ is much smaller than the coherent tune shift U+V, the Landau damping becomes ineffective. The growth rate is dominated by V.

Damping in the Momentum Frequency Spread

Let $\delta = (p-p_o)/p_o$ and

$$f(p) = \begin{cases} \frac{1}{2\sigma_p} & -\sigma_p < \delta < \sigma_p \\ 0 & \text{elsewhere} \end{cases}$$

$$Q = Q_o + \xi Q_o \delta$$

$$\omega_o = \omega_o + \eta\omega_o\delta$$

where ξ and η are the chromaticity and the momentum slip factor respectively. The dispersion integral becomes

$$\frac{U + V + iV}{2\Delta S} \; ln \; \frac{\Omega-(n-Q_o)\omega_o+\Delta S}{\Omega-(n-Q_o)\omega_o-\Delta S} = 1 \tag{9}$$

with $\Delta S = \sigma_p\omega_o \; (Q_o\xi+\eta(n-Q_o)$. Likewise, we can define the parameters μ and ν as,

$$\mu = 2\Delta S \; \frac{U + V}{(U+V)^2+V^2} \tag{10a}$$

$$\nu = 2\Delta S \; \frac{V}{(U+V)^2+V^2} \tag{10b}$$

We obtain a similar result to that of Eq. (8) as,

$$\Omega_i = 2\Delta S \; \frac{e^{-\mu}\sin\nu}{1-2e^{-\mu}\cos\nu+e^{-2\mu}} \tag{11}$$

Similarly, when the coherent tune shift U >> ΔS of the tune spread, the imaginary part of the frequency shift becomes,

$$\Omega_i = V \left(1 - \frac{2\Delta S(U+V)}{(U+V)^2 + V^2} + \cdots\right) \tag{12}$$

This feature is identical to that of Eq. (8). This equation shows that the tune spread in momentum space or in betatron amplitude space are equivalently effective in producing Landau damping of the transverse instability. One should, therefore, obtain equivalent stability diagrams in both[3,5], where different types of distribution functions have been used.

Damping Via Momentum and Betatron Spaces

In most of the low energy synchrotrons, the frequency spreads due to momentum space and betatron space are large. One should, therefore, treat the dispersion integrals for two/three dimensions. We shall solve Eq. (3) for the model which is described in examples 2 and 3 above. The dispersion integral can be integrated to give,

$$\frac{U+V+iV}{\Delta Q\omega_o}\left(\frac{A+\Delta S}{2\Delta S}\,ln\,\frac{A+\Delta S}{A-\Delta S} - \frac{A+\Delta S-\Delta Q\omega_o+\eta\Delta Q\omega_o}{2(\Delta S+\eta\Delta Q\omega_o)}\,ln\,\frac{A+\Delta S-\Delta Q\omega_o+\eta Q\omega_o}{A-\Delta S-\Delta Q\omega_o-\eta\Delta Q\omega_o}\right) = 1 \quad (13)$$

where $A = \Omega - (n-Q_o)\ \omega_o$. At this point, notice that the frequency spreads due to the betatron motion and momentum do not simply add.

In the limit that U>>V, we expect that $A \gg \Delta S$ and $A \gg \Delta Q\omega_o$. Expanding Eq. (13) up to the second order in A^{-2}, we obtain the following,

$$\Omega_i = \text{Im}\ [-\Delta S(U+V+iV)]^{\frac{1}{2}}$$

Note here that in the limit of a large coherent shift, we do not obtain the same limit of Ω_i=V. The result indicates that the effect of the Landau damping may become more significant.

Conclusion

When the dispersion integral is evaluated in a higher dimensional space, the Landau damping mechanism becomes more effective. The growth rate of the transverse instability can be reduced. Our analysis is preliminary due to the following simplifications of the model: 1) The nonrealistic particle distribution, 2) missing synchrotron motion, and 3) only one transverse dimension. The physics of the model should, however, be correct. The results for a single dimensional analysis does agree with the numerical solution of the dispersion integral. This points out that a realistic treatment of the transverse instability should include the tune spread due to the longitudinal as well as the transverse spaces.

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