Particle Accelerators, 1990, Vol. 27, pp. 27–32
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LONGITUDINAL HIGHER ORDER MODES OF THE BOOSTER PROTON RF CAVITY LOADED WITH A LOSSY DISPERSIVE FERRITE*

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Abstract. The ferrite coupled longitudinal modes of the AGS Booster rf cavities are calculated and their effect on the beam is estimated.

RESULTS OF THE CALCULATIONS

The Booster is a fast cycling accelerator designed to inject high intensity beams of protons and heavy ions into the AGS. It uses 4 rf cavities loaded with ferrite. Using Superfish[1], we calculate the impedances of the resonant high order modes of these cavities taking the lossy and dispersive nature of the ferrite into account. Details of the calculations can be found in Ref.[2] For the geometry of the cavity, see Fig. 1a.

The resonant frequency $\omega_R \propto 1/\sqrt{\varepsilon\mu}$ is related to the magnetic permeability μ and to the electric permittivity ε of the ferrite. For μ constant, Superfish calculates the spectrum of frequencies for the cavity. However, problems arise for the case of a dispersive ferrite where μ is frequency dependent. The value of μ is determined by the requirements of the fundamental mode, and the higher order modes might occur at frequencies for which μ used in Superfish is different. Therefore, these frequencies are not correctly calculated, and to find the higher order modes, we need to use the curve of $\mu(\omega)$ for the ferrite.

These curves are not readily available and we had to refer to the general literature.[3] Fig. 2 shows the results of measurements on a ferrite of the NiZn type, with a NiO:ZnO ratio of 31.7:16.5, which corresponds to the biased ferrite used in our rf cavities. When μ is complex ($\mu = \mu' - j\mu''$), so is also ω_R, with a real part (ω'_R) corresponding to oscillations in time, and an imaginary part (ω''_R) corresponding to damping due to power dissipation in the ferrite given by

$$\begin{Bmatrix} \omega'_R \\ \omega''_R \end{Bmatrix} \propto \frac{1}{\sqrt{\varepsilon(\mu'^2 + \mu''^2)^{1/2}}} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \frac{\tan^{-1}(\mu''/\mu')}{2} \tag{1}$$

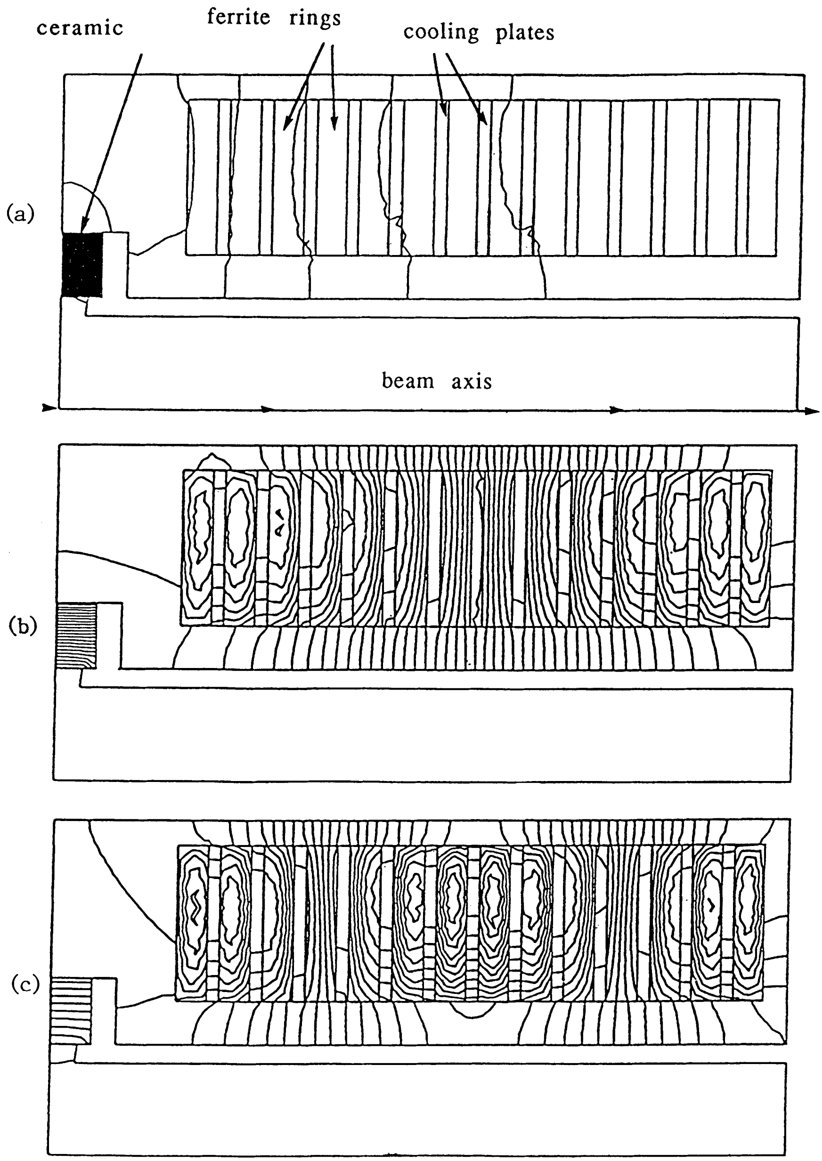

FIGURE 1 Field lines of the (a) fundamental and (b,c) first two higher modes, in a quarter of the rf cavity. ($\varepsilon_{ceramic} = 32\ \varepsilon_0$, $\varepsilon_{ferrite} = 71\ \varepsilon_0$, $\varepsilon_{Cu} = \varepsilon_0$)

The effective μ (μ_{eff}) used in Eq. (1) will be

$$\mu_{eff} = \frac{\sqrt{\mu'^2 + \mu''^2}}{\cos^2\left(\frac{\tan^{-1}(\mu''/\mu')}{2}\right)} \tag{2}$$

It is approximately constant up to about 20 MHz and changes therefrom with frequency (Fig. 3). In Superfish, the cavity modes with frenquencies higher than 20 MHz were individually calculated by matching the value of μ to the effective μ at the mode's eigenfrequency. This was done by finding the intersection point of the curve $\mu_{eff}(\omega)$ for the ferrite and the curve of eigenfrequency versus μ for a given mode calculated by Superfish. This is illustratded in Fig. 3 for some higher order modes at the beginning, middle and end of the Booster acceleration cycle.

To find the modes at the middle and end of the cycle, we assumed that the d.c. bias field, needed to achieve the 2.5--4.1 MHz frequency swing during acceleration[4], does not affect the frequency dependence of μ' and μ'' but only affects their relative values. Fig. 1 shows plots of the field lines for the fundamental and the next two higher order modes.

Superfish also calculates for each mode the quality factor, Q, and the shunt impedance $R_{sh} = E_0^2 L^2/\wp$, where $\wp$ represents the power losses in the cavity, L the cavity length, and E_0 is the average accelerating electric field along the axis. For a gap voltage of 22.5 kV (E_0 = 18.75 kV/m), $f \equiv \omega/2\pi$, Q, and R_{sh} of the fundamental and some of the higher order modes are listed in Table I.

So far we only considered losses in the cavity walls, since Superfish does not have an option to calculate losses in the ferrite. We estimated these losses using Fig. 2 to find $\mu'$and μ'' corresponding to a given mode eigenfrequency. The loss angle δ is related to the quality factor Q by

$$\tan\delta = \frac{\mu''}{\mu'} = \frac{1}{Q} \tag{3}$$

and the magnetic losses in the ferrite are given by

$$\wp \approx 2.2\pi f\, \mu'' V < |H|^2 >_W \tag{4}$$

where V is the volume of the ferrite ($\approx$ .103 m^3) and H_W is the magnetic field at the cavity wall. The shunt impedance of a given mode is calculated upon substitution of the total magnetic losses in the ferrite from Eq. 4. This is done for the fundamental and two

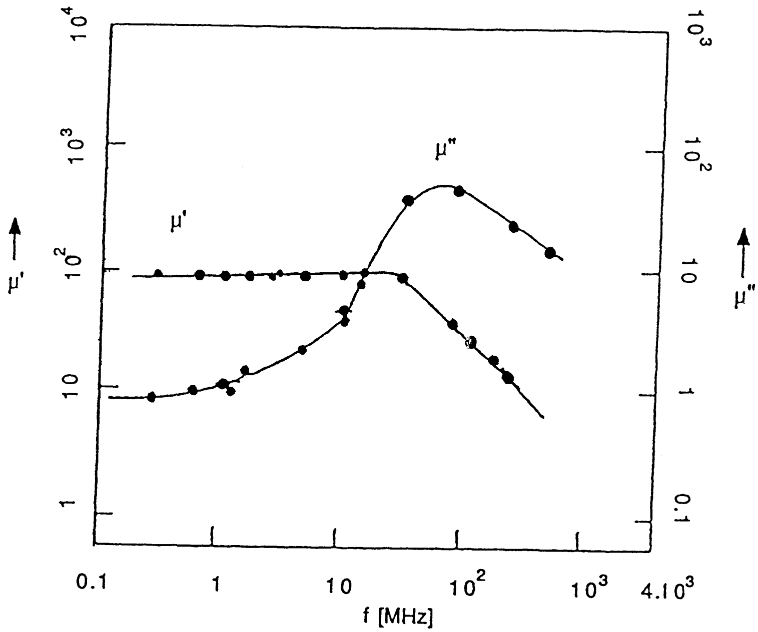

FIGURE 2 Permeability spectrum for unbiased NiZn ferrite.

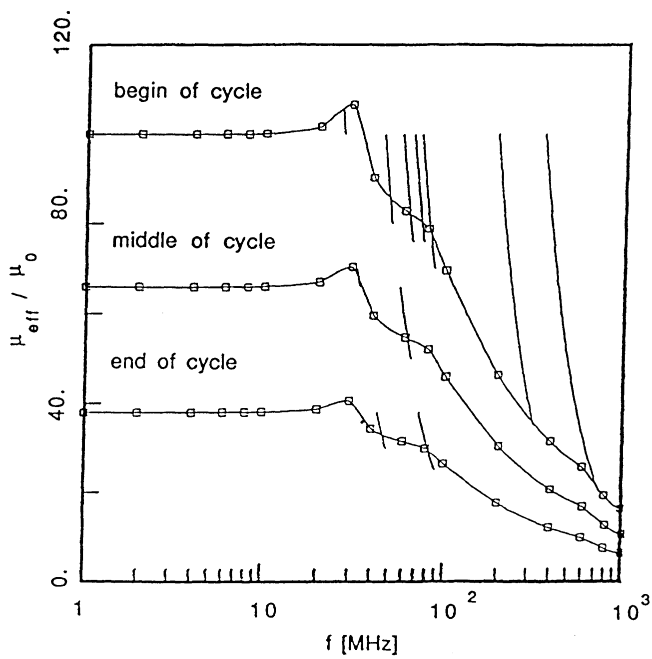

FIGURE 3 Determination of resonance frequencies and μ_{eff} of the cavity modes

higher order modes, with results shown in Table I. The magnetic losses predominate and are the only ones taken into account.

TABLE I Calculation of shunt impedance and quality factor for the case of ferrite losses (no ferrite losses) .

	f'_R[MHz]	$R_{sh}[\Omega]$		Q	
	2.5	76.10^3	(148.10^6)	61.	(119.10^3)
beginning of cycle	26.5	4.	(724.10^3)	4·	(415.10^3)
	28.8	0.4	(196.10^3)	1.4	(870.10^3)
	3.1	56.10^3	(88.10^6)	56.	(88.10^3)
middle of cycle	32.9	2.4	(440.10^3)	2.8	(299.10^3)
	61.7	0.24	(116.10^3)	1.	(606.10^3)
	4.1	38.10^3	(44.10^6)	49.	(59.10^3)
end of cycle	45.9	1.2	(196.10^3)	1.6	(179.10^3)
	84.2	0.2	(60.10^3)	0.8	(372.10^3)

CONCLUSIONS

We calculated the frequencies of the fundamental and some of the higer order modes of the Booster proton rf cavities and found approximate values of their shunt impedances and quality factors. We studied many higher order modes but we list only some of them. Moreover, if we interpolate the values of μ' and μ'' in Fig. 2 up to 1 GHz (the pipe cutoff frequency), we see that there are no high Q longitudinal ferrite modes that will affect the beam.

Our values of Q and R_{sh} for the fundamental mode, at the beginning of the cycle, are in good agreement with the measured values on a sample ferrite ring with a very low bias field.[5] At the middle and end of the cycle, when the bias field is sizable, our values of Q and R_{sh} for the fundamental mode are comparable to the measured ones.[5]

The effect of the parasitic modes on the beam is estimated by approximating their impedances with those of an equivalentf parallel RLC circuit

$$Z^R(\omega) = \frac{R_{sh}}{1 + jQ(\omega/\omega_R - \omega_R/\omega)} \qquad (5)$$

In a previous simulation[6], it was found that the effect on the rf capture of a broad band wall impedance of 200 Ω was negligibly small compared to the effect of the space charge impedance. From Table I, we see that the maximum (total) R_{sh} encountered by the beam in one turn is about 4 Ω. We, therefore, expect the single bunch effects of the parasitic higher order modes to be negligible compared to the space charge effects. We recall that $|Z^{sc}/n| \approx 700\,\Omega$ at injection, and $|Z^{sc}/n| \approx 100\,\Omega$ at extraction (sc stands for space charge). To estimate the coupled bunch effects of the rf parasitic modes, we use Eqs. (4) and (5) to calculate the time τ_e it takes the amplitude of a given higher order mode to decrease to $1/e$ of its initial value and compare it to the time ($\tau_b = T_0/3$, T_0 = revolution period) between the centers of two consecutive proton bunches in the Booster ring. The fields are damped in the ferrite as $\exp(-\omega''_R t)$. The time in question therefore is

$$\tau_e = \frac{1}{\omega''_R} = \frac{1}{\omega'_R\left(\sqrt{1+Q^2} - Q\right)} \tag{6}$$

We found that $\tau_e << \tau_b$ at all times and, therefore, we do not expect the ferrite coupled higher order modes to induce logitudinal coupled bunch instabilities.

ACKNOWLEDGEMENTS

We gratefully aknowledge discussions with A. Ratti and M. Goldman.

FOOTNOTES AND REFERENCES

* Work performed under the auspices of the US Department of Energy.
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