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# THE MEASUREMENT OF BETATRON AND SYNCHROTRON TUNE SHIFTS IN AN INTENSE BUNCHED BEAM

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Abstract The transverse transfer function of a bunched beam has been measured in a wide intensity range at the U-70 injection flat bottom. The separation of synchrotron modes of oscillation allows one to determine the synchrotron and betatron tune shifts and to estimate the acting impedances from the well known theoretical models. The applicability of this method for prediction of beam instabilities and for adjusting the active feedback loops is discussed.

## 1. INTRODUCTION

The form of the transverse transfer function of a bunched beam (BBTF) has been obtained in[1] without the space charge effects taken into account. In this case, the BBTF is a number of equidistant spectral lines with the amplitude

$$A_m = N_b \xi (A)\ G_m(\omega_\xi, B) \tag{1}$$

and the distance between the neighbouring lines equals the synchrotron frequency $\omega_s$. Here $N_b$ is the number of particles in the bunch, the normalization coefficient $\xi(A)$ is dependent on the amplitudes of betatron oscillations and the formfactor $G_m$ is determined as

$$G_m = \int_0^\infty J_m(\lambda B) J_m(\lambda B - kB)\ F_{\shortparallel}(B) B\ dB.$$

In this integral, $J_m$ is a Bessel function, $\lambda = Q - \frac{\xi}{\alpha - \gamma^{-2}}$, $\xi$ is chromaticity, $F_{\shortparallel}(B)$ is the function of distribution over the amplitudes of synchrotron oscillations, Q is the betatron tune, $\alpha$ is the orbit compaction factor, $\gamma$ is the particle energy Note that in this case the satellites possessing the same number but opposite signs have equal amplitudes.

The space charge effects vary the form of the transfer function, frequency shifts depending on the number of the mode of syncrotron oscillation occur. The theoretical aspect of this shift has been treated in a number of papers[2]. As was shown in[3], not only mutual location of the syncrotron modes of oscillations but also their amplitudes vary with intensity.

This paper presents the results on measuring the BBTF within a wide range of the U-70 intensities. The separation of synchrotron modes of oscillations made it possible to determine the synchrotron and betatron frequency shifts. The resųlts are compared with these obtained using the known theoretical models.

## 2. MEASUREMENT RESULTS

The BBTF has been measured at the U-70 injection flat bottom for $E_p =$ $= 1.4$ GeV ($\gamma = 2.5$). Here the revolution frequency is $\omega_0 = 2\pi \cdot 1.83 \cdot 10^5\ s^{-1}$, the betatron tunes are $Q_x=9.88$, $Q_z=9.81$, the synchrotron frequency is $Q_s=9 \cdot 10^{-3}$ and the acceleration harmonic number is q=30. The octupole nonlinearity correction system was adjusted in such a way that the natural octupole nonlinearity could be compensated for and the maximum separation of synchrotron satellites maintained. The sextupole nonlinearity correction system allowed one to obtain the chromaticity $\xi = 15$ which, in accordance with formula (1), yields the following relationship between the satellites amplitudes for a bunch 80 ns long and zero beam current:

$$a_1/a_0=0.33, \quad a_2/a_0=0.08, \quad a_3/a_0=0.02.$$

The measurements were carried out at the lowest-frequency harmonic (k=10) of the revolution frequency with the help of standard hardware[1]. The frequency resolution of a two-channel spectrum analyzer was $2.5\ 10^{-4}\omega_0$. Note also that the feedback transverse system was switched on.

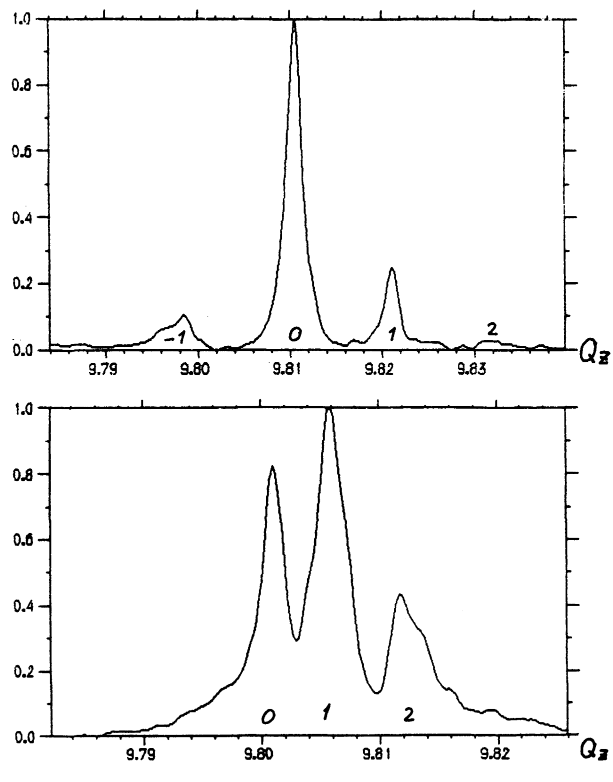


FIGURE 1 The beam transfer function for $N_b=2 \cdot 10^9$ p/b (fig.1a) and for $N_b=3 \cdot 10^{11}$ p/b (fig.1b).

Figure 1 presents the BBTF measured in the Z-direction for different beam intensities: a) $N_b=2 \cdot 10^9$ p/b, b) $N_b=3 \cdot 10^{11}$ p/b. The numerals in the lower part of the figure correspond to the numbers of synchrotron modes. With this chromaticity, the modes m=-1,0,1,2 are identified without difficulty. Figure 2 shows the tune location of these modes versus the beam intensity within the range from $N_b=1.5\cdot10^9$ p/b to $N_b=3\cdot10^{11}$ p/b while fig.3 gives the relative amplitude of synchrotron satellites within the same intensity range.

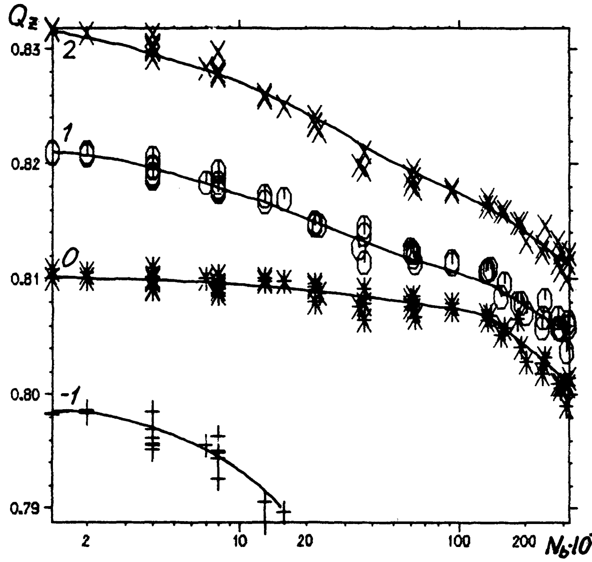


FIGURE 2. The tunes of synchrotron modes of oscillations versus bunch current.

Let us note some qualitative features of these dependences:
a) the satellites numbered +1, -1 are shifted almost concurrently;
b) the relative amplitude of these satellites varies essentially with intensity;
c) the shift of mode tune increases with its number.

It was also noted that with the intensity varying from $7\cdot10^9$ p/b to $3\cdot10^{11}$ p/b the bunch length also varies from 80 to 120 ns. This is accompanied by the variation of the momentum bunch spread from $(\Delta p/p)_{min}=3.65\cdot10^{-3}$ to $(\Delta p/p)_{max}=5.5\cdot10^{-3}$. This means that an increse of the intensity leads to a 2.3 factor of growth of the phase beam volume rather than to a deformation of the potential well due to the space charge effects. In our opinion, this is caused by the coherent synchrotron oscillations at the injection flat bottom.

In addition, the frequency spectrum of the longitudinal signal in the neighbourhood of the RF harmonic n=3 has also been measured (see fig.4). The numerals in the figure show the numbers of synchrotron harmonics. As is seen from these measurements, the coherent shift of synchrotron frequency does not exceed $\Delta Q_s < 10^{-3}$, which corresponds to a longitudinal impedance of $Z_n/n < 100i$ Ohm. Taking into account the known from the theory[2] relation between coherent and incoherent shifts one may conclude that in the betatron spectrum the convergence of the satellites having the same number but opposite signs should not exceed $\Delta Q_c=2\cdot10^{-3}$, which is just confirmed in fig.2.

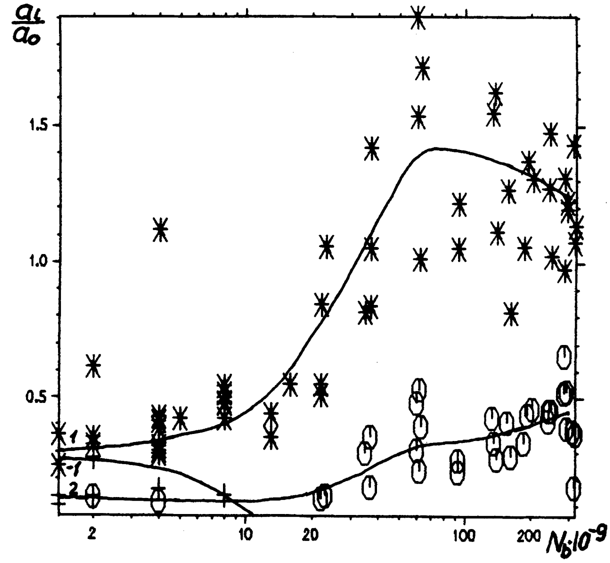


FIGURE 3. The relative amplitudes of synchrotron satellites versus bunch intensity.

To estimate the coherent betatron shift let us used the well-known formula obtained by F.Sacherer for a wide-band low-frequency impedance[2]:

$$(\omega_{cm} - m\omega_s) = \frac{eI}{4\pi m_0 c\gamma QB} C_m(iZ(\omega_\xi)) \tag{2}$$

Here $\omega_{cm}$ is the coherent frequency of the m-th synchrotron mode, $C_m$ is the formfactor of the m-th mode, $Z(\omega_\xi)$ is the value of the transverse wide-band impedance at the chromatic shift frequency

$$\omega_\xi = \frac{\omega_0 \xi}{\eta}, \quad \eta = \alpha - \gamma^{-2}, \quad \xi = \frac{dQ}{dp/p},$$

where $\omega_0$ is the revolution frequency, B is the bunching factor. The wall impedance of an elliptic vacuum chamber with semiaxes 5x10 cm is Z=2.94i MOhm/m (inductive wide-band impedance) at a chromatic shift frequency of $\omega_\xi = 2\pi \cdot 1.3 \cdot 10^7\ s^{-1}$ For a beam intensity of $N_b = 3 \cdot 10^{11}$ p/b the coherent zero-mode tune shift caused by this impedance is $\Delta Q_{0c} = (6-8) \cdot 10^{-3}$, i.e. it is in a good agreement with the experimental value.

The following values of the coefficients $C_m$ have been obtained in paper[2]:

$$C_0 = 1.1, \quad C_1 = 0.46, \quad C_2 = 0.3 \text{ etc},$$

which means a decrease of the shift with number m as $(|m|+1)^{-1}$. However, our experimental data contradict this model. The shift of the 1st mode is

$\Delta Q_{1c}=1.9\cdot\Delta Q_{0c}$, that of the 2nd one is $\Delta Q_{2c}=2.6\cdot\Delta Q_{0c}$. Such a shift can be interpreted within the frames of the above model only by assuming the presence of a frequency-dependent inductive impedance having the following values:

$Z_1=12.1$ MOhm/m at $\omega_{\xi 1}=2\cdot\pi\, 1.2\cdot 10^7\ s^{-1}$ and

$Z_2=24.1$ MOhm/m at $\omega_{\xi 2}=2\cdot\pi\, 2.4\cdot 10^7\ s^{-1}$.

However, this does not seem realistic.

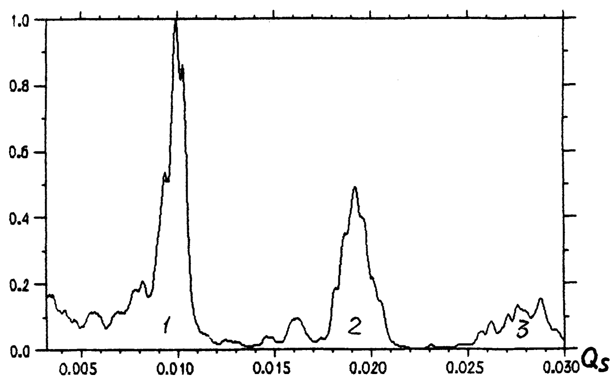


FIGURE 4. The longitudinal signal spectrum in the neighbourhood of the n=3 RF harmonic.

In our opinion, to interpret this effect it is necessary to take into account the incoherent space charge shift effect. The following formula for the synchrotron satellite frequency has been obtained in paper[4]:

$$\omega_m - m\omega_s = \omega_o \cdot \Delta Q_{io}\,\varkappa\cdot \chi_m.$$

Here $\Delta Q_{io}$ is the incoherent space charge betatron tune shift, $\varkappa\sim 1$ is the normalization factor, $\chi_m$ is the formfactor of distribution over the amplitudes of synchrotron oscillations. According to our calculations, $\Delta Q_{io}=3.4\cdot 10^{-2}$, $\chi_o=0.3$, $\chi_1=0.82$. Unfortunately, the coefficient $\chi_2$ has not been calculated in[4], neither have the amplitudes of satellites. We have obtained the following tune shifts:

$\Delta Q_0=1.1\cdot 10^{-2}$, $\Delta Q_1=2.8\cdot 10^{-2}$.

This indicates that the model under discussion yields the results which agree qualitatively with the experimental ones.

The intensity-dependent behaviour of the amplitudes of synchrotron modes can be explained only with their mutual influence taken into account. This follows from the theory of coupled mode instability (see, for example[3]). The amplitudes of synchrotron modes are enhanced mutually with the distance between them decreased due to the coupling beam impedances. The experimental data available do not contradict this theory qualitatively though it does not take into account the incoherent space charge shift.

To the best of our knowledge, the experimental results presented above are the first measurements which cannot be explained within the frames of the presently available models of transverse bunched beam instability and therefore this problem requires a further study.

## 3. CONCLUSION

The authors have failed to find a fairly complete model explaining the experimental data. But still, we hope that these data will be helpful for theorists displaying during the recent years a keen interest to these problems, in particular, to the ones related to the development of the theory of coupled modes instability.

Simultaneously we see that the obtained results can find direct applications for choosing the working point of the accelerator and for damping transverse instabilities. For example, as is evident from the BBTF shown in fig.1b, the description of this function with the help of only one parameter, betatron tune, is not only conditional, but also in many cases illegal. Therefore it is necessary to take into account, firstly, its complicated nature and, secondly, the degree of excitation of higher oscillation modes when measuring betatron frequencies.

Since the beam stability is directly related to the BBTF it is necessary to construct a sound technique of adjusting the damping mode (for sextupoles, octupoles, feedback loops).

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