Particle Accelerators, 1989, Vol. 24, pp. 177–185
Reprints available directly from the publisher
Photocopying permitted by license only

EFFECT OF ENERGY SPREAD ON THE SINGLE-BUNCH DIPOLE BEAM BREAK-UP INSTABILITY IN A HIGH-ENERGY RF LINAC†

D. CHERNIN and A. MONDELLI

Science Applications International Corporation, 1710 Goodridge Drive, McLean, Virginia 22102

(Received June 6, 1988)

It is shown that a small intrabunch energy spread in either a coasting or accelerated beam has a dramatic effect on the dynamics of the bunch when acted upon by the transverse dipole wake field and by a focusing system with nonzero chromaticity. In particular, for sufficiently large propagation distances we show that the oscillation amplitude of the bunch centroid stops increasing, attaining a fixed value that depends critically on the sign of the energy spread. This result has been obtained from an exact solution, expressed as a single proper integral over the betatron wave-number spread, of the equation of motion. We give explicitly the leading- and next-to-leading-order terms in the asymptotic expansion for large propagation distances. An approximate criterion for Landau damping (more accurately, "BNS damping") emerges from the solution and is compared with that obtained from the standard "two-particle" model of wake-field effects. Results of a numerical evaluation of the integral for intermediate propagation distances are given in an example.

1. INTRODUCTION

The effect of the transverse dipole wake field on single-bunch emittance growth is a matter of some concern in high-energy rf linacs, especially those presently being discussed for use in a very high energy linear collider. Since such a collider, in the ~TeV CM energy range, will probably operate at rf frequencies considerably higher than those used at the SLC (2.856 GHz), wake-field effects, which generally worsen as transverse structure dimensions are reduced, will likely play a significant role in limiting collider performance.

A considerable amount of work has been reported both on the calculation of wake fields for various structures[1,2] and on their dynamical effects on the beam.[3,4] Chao, Richter, and Yao[5] have analyzed single-bunch dynamics on the absence of energy spread, and Bane[6] has reported numerical results for a bunch with finite energy spread. Others,[7,8] including the present authors,[9] have produced computer programs, similar to Bane's, used to track a bunch through a linac, including the effects of longitudinal and transverse wake fields. Neil, Hall, and Cooper[10] have studied a monoenergetic beam pulse acted upon by a single deflecting cavity mode, whereas Gluckstern, Cooper, and Channell[11] have considered trains of bunches, each with a well-defined energy. Yokoya[12] has reported an analysis of

† Work supported by U.S. Department of Energy.

single-bunch beam break-up, without energy spread, using a Laplace transform method.

In this paper we present an analytical solution, which includes the effect of a small energy spread, to the equation of motion for the beam centroid of a single bunch. For a coasting beam our assumptions are that (1) it is sufficiently accurate to consider that the wake field, external focusing field, and accelerating field are all continuously applied, (2) the transverse delta-function wake field is well approximated as a linear function of its argument (a good assumption, at least for the SLAC structure out to distances of the order of 1 mm), (3) the bunch distribution is uniform, and (4) the energy of a particle is a linear function of its distance back from the head of the bunch (i.e. we do not include the nonlinear variation of energy due to the cosine shape of the rf accelerating wave or the energy variation due to the longitudinal wake field). We also obtain the accelerated beam solution, under the additional assumption that the beam energy does not change too much in a betatron wavelength. Several interesting features of the solution are illustrated by example.

2. MOTION OF THE BUNCH CENTROID

We denote by $x(s;\zeta)$ the displacement of a bunch from the machine axis, where s measures distance along the machine and ζ measures distance back from the head of the bunch. The equation governing the evolution of x is

$$\frac{\partial^2 x}{\partial s^2}+\frac{1}{\gamma}\frac{\partial \gamma}{\partial s}\frac{\partial x}{\partial s}+k_\beta^2 x=\frac{e^2}{m\gamma c^2}\int_0^\zeta d\zeta' f(\zeta')W(\zeta-\zeta')x(s;\zeta'), \tag{1}$$

where γ is the usual relativistic factor, k_β is the betatron wave number, f is the bunch distribution normalized to $\int_0^{l_b} d\zeta f(\zeta)=N$, the number of particles in the bunch, l_b is the bunch length, and $W(\zeta)$ is the transverse dipole delta-function wake—the wake produced at a distance ζ by the unit displacement of a single particle. In Eq. (1) e, m, and c have their usual meanings and mks units are used, in which the units of W are volts/coulomb-meter^2.

We will first study the coasting beam, for which γ does not depend on s, but *does* depend on ζ. We assume that the effect of external focusing may be approximated by using a "smooth approximation" for k_β so that it too depends only on ζ. A Laplace transform of Eq. (1) then gives, for the coasting beam,

$$(\kappa^2+k_\beta^2)\bar{x}(\kappa;\zeta)=\frac{e^2}{m\gamma c^2}\int_0^\zeta d\zeta' f(\zeta')W(\zeta-\zeta')\bar{x}(\kappa;\zeta')+\bar{h}(\kappa;\zeta), \tag{2}$$

where

$$\bar{x}(\kappa;\zeta)=\int_0^\infty ds e^{-\kappa s}x(s;\zeta), \tag{3}$$

$$\bar{h}(\kappa;\zeta)=\frac{\partial x}{\partial s}(0;\zeta)+\kappa x(0;\zeta). \tag{4}$$

If $W(\zeta)$ is a linear function of its argument

$$W(\zeta) = W' \cdot \zeta, \tag{5}$$

where W' is a constant, then the kernel in the integral in Eq. (2) is degenerate, and the equation may be transformed into a differential equation, which may be written

$$\frac{\partial^2 \bar{y}}{\partial \zeta^2} - \frac{\dfrac{e^2 W'}{m\gamma c^2} f}{\kappa^2 + k_\beta^2} \bar{y} = \frac{\dfrac{e^2 W'}{mc^2} f \bar{h}}{\kappa^2 + k_\beta^2}, \tag{6}$$

where

$$\bar{y}(\kappa; \zeta) \equiv \gamma[(\kappa^2 + k_\beta^2)\bar{x}(\kappa; \zeta) - \bar{h}(\kappa; \zeta)], \tag{7}$$

and where $\bar{y}$ satisfies the initial conditions

$$\bar{y}(\kappa; 0) = 0, \tag{8}$$

$$\frac{\partial \bar{y}}{\partial \zeta}(\kappa; 0) = 0. \tag{9}$$

Note that in Eq. (6) no assumptions are made about the ζ-dependence of γ, k_β, f, or $\bar{h}$.

Consider the case in which none of $\gamma \equiv \gamma_0$, $k_\beta \equiv k_{\beta 0}$, f, or $\bar{h}$ depends on ζ, that is, the case of a flat-top bunch with no spreads in energy or betatron wave number, the initial conditions of which are independent of ζ. In this case the solution to Eq. (6) is simple, and we obtain

$$\bar{x}(\kappa; \zeta) = \frac{\bar{h}}{\kappa^2 + k_{\beta 0}^2} \cosh\left[\frac{\alpha \zeta / l_b}{(\kappa^2/k_{\beta 0}^2 + 1)^{1/2}}\right], \tag{10}$$

where

$$\alpha = [e^2 W(l_b) N / m\gamma_0 c^2 k_{\beta 0}^2]^{1/2}, \tag{11}$$

and we have used $f = N/l_b$.

The function $\bar{x}(\kappa; \zeta)$ is seen to have essential singularities in the κ-plane at $\kappa = \pm i k_{\beta 0}$. If one expands the cosh function then $\bar{x}(\kappa; \zeta)$ may be inverted[13] term by term to give

$$x(s; \zeta) = \bar{s} \sum_{n=0}^{\infty} \left[x_0 j_{n-1}(\bar{s}) + \frac{x_0'}{k_{\beta 0}} j_n(\bar{s})\right] \frac{(\eta/2)^n}{n!\,(2n)!}, \tag{12}$$

where $\bar{s} = k_{\beta 0} s$, $\eta = \bar{s}(\alpha \zeta / l_b)^2$, j_n is the spherical Bessel function, $x_0 = x(0; \zeta)$, and $x_0' = \dfrac{\partial x}{\partial s}(0; \zeta)$. (Recall that x_0 and x_0' were assumed to be independent of ζ.) For $x_0' = 0$ and $\bar{s} \gg 1$ this reproduces the result of Ref. 5.

We now return to Eq. (6) and relax the assumptions made above about the ζ-dependence of γ and k_β. We will retain the assumptions that f and $\bar{h}$ are constants, independent of ζ for $0 \le \zeta \le l_b$, but will take

$$\gamma = \gamma_0(1 + \varepsilon \zeta / l_b) \tag{13}$$

$$k_\beta = k_{\beta 0}(1 + \xi \varepsilon \zeta / l_b), \tag{14}$$

where ε is a dimensionless measure of the energy spread, ξ is the lattice chromaticity, and γ_0 and $k_{\beta 0}$ are constants, the relativistic factor and betatron wave number of the *head* of the bunch. By measuring ζ in units of l_b and κ in units of $k_{\beta 0}$, we may cast Eq. (6) in dimensionless form as

$$\frac{\partial^2 \bar{y}}{\partial \zeta^2} - \frac{\alpha^2}{(1+\varepsilon\zeta)[\kappa^2 + (1+\xi\varepsilon\zeta)^2]}\bar{y} = \frac{\alpha^2 \gamma_0 \bar{h}}{\kappa^2 + (1+\xi\varepsilon\zeta)^2}. \tag{15}$$

Neglecting terms of order ε^2 in denominators of Eq. (15) gives

$$\frac{\partial^2 \bar{y}}{\partial \zeta^2} - \frac{\alpha^2}{\kappa^2 + 1 + \varepsilon\zeta(\kappa^2+1+2\xi)}\bar{y} = \frac{\alpha^2 \gamma_0 \bar{h}(1+\varepsilon\zeta)}{\kappa^2 + 1 + \varepsilon\zeta(\kappa^2+1+2\xi)}. \tag{16}$$

Equation (16) may be solved, subject to the initial conditions of Eqs. (8) and (9), in terms of Bessel functions. The result for $\bar{x}(\kappa;\zeta)$, after some calculation, is

$$k_{\beta 0}^2 \bar{x}(\kappa;\zeta) = \frac{\bar{h}\rho}{\kappa^2+1+\varepsilon\zeta(\kappa^2+1+2\xi)}\{I_1(\rho)K_0(\rho_0) + K_1(\rho)I_0(\rho_0) + \frac{\varepsilon}{\alpha}(\kappa^2+1)^{1/2}[I_1(\rho)K_1(\rho_0) - K_1(\rho)I_1(\rho_0)]\}, \tag{17}$$

where

$$\rho = 2\frac{\alpha}{\varepsilon}\frac{[\kappa^2+1+\varepsilon\zeta(\kappa^2+1+2\xi)]^{1/2}}{\kappa^2+1+2\xi}, \tag{18}$$

$$\rho_0 = 2\frac{\alpha}{\varepsilon}\frac{(\kappa^2+1)^{1/2}}{\kappa^2+1+2\xi}. \tag{19}$$

Equation (17) is easily shown to reduce to Eq. (10) as $\varepsilon \to 0$.

The function $\bar{x}(\kappa;\zeta)$ has six singularities in the κ-plane. These are branch points at

$$\kappa = \pm i\left[\frac{1+\varepsilon\zeta(1+2\xi)}{1+\varepsilon\zeta}\right]^{1/2} \equiv \pm i r_\varepsilon \tag{20}$$

and at $\kappa = \pm i$ and simple poles at $\kappa = \pm i r_\varepsilon$. The pole contribution to the

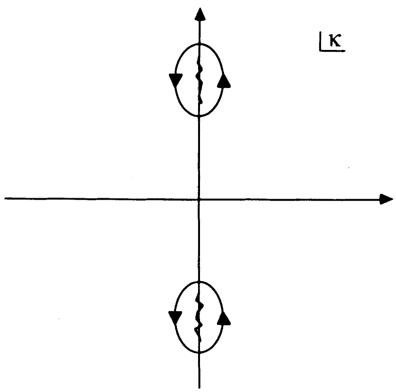

FIGURE 1 Inversion contour in the κ-plane. The branch points are at $\kappa = \pm i$ and $\pm i r_\varepsilon$.

integral is easily split off, and one is left with an integral around the cuts, as illustrated in Fig. 1. The final result may be expressed as

$$x(s;\zeta)=\left(x_0\frac{\partial}{\partial s}+\frac{x_0'}{k_{\beta 0}}\right)F(s;\zeta), \tag{21}$$

where

$$F(s;\zeta)=\left[I_0(u)+\frac{2\varepsilon\zeta}{u}I_1(u)\right]\frac{\sin(r_\varepsilon s)}{(1+\varepsilon\zeta)r_\varepsilon}-\frac{1}{1+\varepsilon\zeta}\int_{r_\varepsilon}^{1}dr\,\frac{\sigma J_1(\sigma)}{r^2-r_\varepsilon^2}$$
$$\times\left[I_0(\sigma_0)+\frac{\varepsilon}{\alpha}(1-r^2)^{1/2}I_1(\sigma_0)\right]\sin(rs), \tag{22}$$

and where

$$u=2\alpha\left[\frac{\zeta(1+\varepsilon\zeta)}{-2\varepsilon\xi}\right]^{1/2}, \tag{23}$$

$$\sigma=2\frac{\alpha}{\varepsilon}(1+\varepsilon\zeta)^{1/2}\frac{(r^2-r_\varepsilon^2)^{1/2}}{r^2-1-2\xi}, \tag{24}$$

$$\sigma_0=2\frac{\alpha}{\varepsilon}\frac{(1-r^2)^{1/2}}{r^2-1-2\xi}. \tag{25}$$

In Eqs. (21) and (22) s is a dimensionless distance measured in units of $k_{\beta 0}^{-1}$.

Equations (21) and (22) have several interesting features. First, the form of Eq. (22) shows that a particle at ζ generally oscillates at its own frequency as well as at a distribution of the frequencies of the particles *ahead* of it, as expected on physical grounds. In the limit $\alpha\to 0$ (no wake fields) the integral vanishes, and each particle oscillates independently; the head particle ($\zeta=0$) always oscillates at $k_{\beta 0}$, at constant amplitude, for all α and s.

For sufficiently large values of s the integral vanishes (by the Riemann–Lebesgue lemma), and one has the interesting result that, asymptotically, each particle at ζ oscillates independently, at its own free betatron frequency, with a finite, constant amplitude; this amplitude depends rather sensitively on the values of the energy spread, wake-field strength, and external focusing strength. This says that for large s the wake fields produced by the betatron oscillations of all particles ahead of a given particle have destructively interfered or become "phase mixed" and give no additional amplitude growth. However, this asymptotic regime is reached only for rather large values of s for practical parameters, as may be shown by evaluating the leading-order asymptotic term of the integral. One finds by integration by parts that, as $s\to\infty$,

$$F(s;\zeta)\sim\left[I_0(u)+\frac{2\varepsilon\zeta}{u}I_1(u)\right]\frac{\sin(r_\varepsilon s)}{(1+\varepsilon\zeta)r_\varepsilon}$$
$$+\frac{1}{s}\left\{\frac{2\alpha}{\zeta^{1/2}}\frac{1}{(-2\varepsilon\xi)^{3/2}}J_1\left(\frac{u}{(1+\varepsilon\zeta)^{1/2}}\right)\cos(s)\right.$$
$$\left.-\frac{\alpha^2}{2\varepsilon^2\xi^2}(1+\varepsilon\zeta)^2\left[I_0(u)+\frac{2\varepsilon\zeta}{u}I_1(u)\right]\cos(r_\varepsilon s)\right\}+O(1/s^2). \tag{26}$$

In discussing the behavior for large s we need to distinguish between two distinctly different cases: $\varepsilon > 0$, for which the head particle has lower energy than the tail, and $\varepsilon < 0$ for which the head particle has higher energy than the tail. In the first case each particle is slightly less strongly focused than particles ahead of it (we assume $\xi < 0$), and the wake fields can drive the oscillations to very large asymptotic amplitudes $[\sim I_0(u)]$, which are reached, according to Eq. (26) for

$$s > \frac{\alpha^2}{2\varepsilon^2\xi^2}. \tag{27}$$

If, on the other hand, each particle is slightly more strongly focused than those ahead of it ($\varepsilon < 0$; u is imaginary), the asymptotic amplitude is much reduced $[\sim J_0(|u|)]$, but it is reached only for

$$s > \frac{2\alpha}{\zeta^{1/2}} \frac{1}{(-2\,|\varepsilon|\xi)^{3/2}} I_1\left(\frac{|u|}{(1+\varepsilon\zeta)^{1/2}}\right), \tag{28}$$

which may be very large for realistic parameters. We defer consideration of an actual numerical example until after the accelerated-beam solution is discussed in the next section.

When $\varepsilon < 0$ the asymptotic amplitude may be made small by choosing ε so that

$$\frac{2\alpha}{(2\varepsilon\xi)^{1/2}} = j_{0,n}, \tag{29}$$

where $j_{0,n}$ is the nth zero of J_0. For this choice the competing effects of wake-field deflection and energy-spread detuning are nearly in complete cancellation. This criterion is identical in form, differing only by a numerical coefficient, to the standard criterion for Landau (or BNS[14]) damping derived from a two-particle model.[6,14] Perhaps a significant difference is that Eq. (29) predicts a discrete set of energy spreads for which this damping is effective. One should bear in mind, however, that this small asymptotic value is reached only when Eq. (28) is satisfied; oscillation amplitudes for intermediate values of s, not satisfying Eq. (28), may or may not be small.

3. ACCELERATED BEAM SOLUTION

We return to consideration of Eq. (1) (and to dimensional quantities) to study the effect of acceleration. Assume that $\gamma(s; \zeta)$ is now given by

$$\gamma(s; \zeta) = \gamma_0(1 + Gs)(1 + \varepsilon\zeta/l_b), \tag{30}$$

where γ_0 is independent of s and ζ, and G is an accelerating gradient. Equation (30) corresponds to acceleration with fixed fractional energy spread.

Two types of acceleration are commonly considered: type 1, in which the magnet strengths in the linac are increased in proportion to energy and the focusing cell size remains fixed, and type 2 in which the magnet strengths are fixed and the cell size increases as $\gamma^{1/2}$ in order to keep the phase shift per period fixed. For these two types

$$k_\beta^{(1)} = k_{\beta 0}(1 + \xi\varepsilon\zeta/l_b), \tag{31}$$

$$k_\beta^{(2)} = k_{\beta 0}\frac{(1 + \xi\varepsilon\zeta/l_b)}{(1 + Gs)^{1/2}}, \tag{32}$$

respectively.

Type 2 is easier to treat and perhaps of more interest for high-energy linear colliders than type 1. By changing both dependent and independent variables in Eq. (1), one may directly show that for type 2 acceleration the solution is given by the unaccelerated solution under the replacements

$$s \to \frac{2}{G}[(1 + Gs)^{1/2} - 1], \tag{33}$$

$$x \to x(1 + Gs)^{1/4}, \tag{34}$$

subject only to the assumption that $G \ll k_{\beta 0}$, that is, the beam energy does not change much in a betatron wavelength. The adiabatic damping exponent of 1/4 in Eq. (34), instead of the familiar 1/2 for type 1 acceleration, is due to the adiabatically weakening focusing strength.

For type 1 acceleration it has been shown in Ref. 5 that, for $\varepsilon = 0$, the analogous replacements are

$$s \to \frac{1}{G}\ln(1 + Gs), \tag{35}$$

$$x \to x(1 + Gs)^{1/2}, \tag{36}$$

where Eq. (35) applies everywhere *except in the betatron phase.* It does not seem straightforward to generalize this result to the case $\varepsilon \neq 0$.

We note that the mathematical effect of acceleration is to shorten the "effective length" of the machine, making it more difficult to reach the asymptotic regime of Eq. (26).

ɩ. APPLICATION TO A 500-GeV LINAC

As a numerical example we consider a 500-GeV linac, 3 km in length, with an injection energy of 5 GeV. The linac is assumed to operate at 4×2.856 GHz, using a scaled SLAC structure. Let the initial betatron wavelength be 5 m, and let the betatron wave number vary with energy as in Eq. (32); it follows that $G = 0.033\ \mathrm{m}^{-1}$, and $G/k_{\beta 0} = 0.026$, so that the acceleration is adiabatic, and the assumptions of the theory apply. Let us further assume a bunch population of 10^{10} and a bunch length of 1 mm. The delta function wake field at the end of the bunch may be found using the scaling law

$$W(\zeta) = \nu^3 W_S(\nu\zeta), \tag{37}$$

where $\nu = f_{rf}$ (GHz)/2.856 and W_S denotes the SLAC wake field.[15] For $\zeta = 1$ mm and $\nu = 4$, one finds W (1 mm) $= 2.36 \times 10^{17}$ V/(C-m^2). It follows the parameter α, Eq. (11), is 0.22.

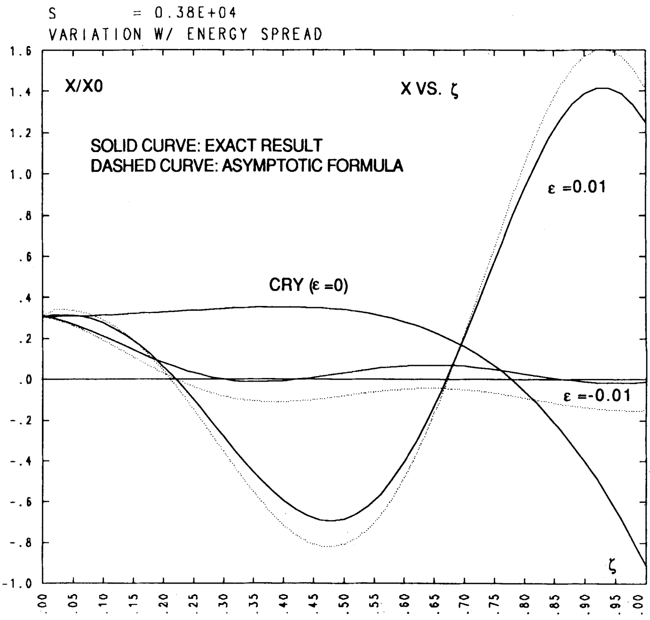

FIGURE 2 Bunch centroid location versus ζ at the end of a 500-GeV linac for three values of energy spread, $\varepsilon = -0.01$, 0.0, and +0.01. Solid lines indicate exact solution; dotted lines, for $\varepsilon = \pm 0.01$, are calculated from Eq. (26).

Figure 2 shows the bunch displacement x/x_0 versus ζ at the end of the linac for total energy spreads, ε, of -0.01, 0.0, and +0.01 for a chromaticity $\xi = -1$. (Note that the rms energy spread is given by $\varepsilon/(2\sqrt{3})$ or $0.289\,\varepsilon$.) Initial conditions are x_0 fixed and $x_0' = 0$. The solid lines in the figure are from a numerical evaluation of the integral in Eq. (22), and the dotted lines, for the $\varepsilon \neq 0$ curves, are from the two-term asymptotic formula, Eq. (26); the conditions of Eqs. (27) and (28) are moderately well satisfied for $\varepsilon = \pm 0.01$. The $\varepsilon = 0$ case is plotted using Eq. (12).[5] One confirms that a small negative energy spread can be quite effective in reducing the defocusing effect of the transverse wake field, whereas a positive energy spread enhances the wake-field effect.

5. CONCLUSIONS

The calculations presented here describe the evolution of the beam centroid displacement as a function of position in the linac, s, and position in the beam bunch, ζ. The spread in centroid position, Δx, and centroid slope, $\Delta x'$, over the bunch length at the end of the linac is one component of the final beam emittance. The other component is given by the evolution of the internal degrees of freedom of the bunch. We have not computed these here. For the parameters assumed in the previous section, the beam radius will be $\sigma_\perp \simeq 1\ \mu$m at 5 GeV if the normalized beam emittance is $\varepsilon_n \simeq 10^{-8}$ m, and it will vary only as $\gamma^{-1/4}$. If the injection error is $x_0 \simeq 1\ \mu\text{m}$, as is believed to be required for the linear collider, then the internal degrees of freedom will be significant whenever $\Delta x/x_0 \lesssim 1$. The negative energy spread result in Fig. 2 has this property. When $\Delta x/x_0 \approx 1$, the spread in the beam centroid will be comparable to the beam radius (for $x_0 \approx \sigma_\perp \approx 1\ \mu\text{m}$), and the beam emittance will approximately quadruple. This is the situation for the positive-energy-spread result in Fig. 2.

Finally, we note that mechanisms other than chromatic effects in the focusing system can give rise to a spread in betatron frequencies and so lead to a finite asymptotic amplitude of oscillation. Nonlinearities, for example, can induce an amplitude dependence of the betatron frequency. We have seen cases in numerical simulations in which sextupole fields act effectively to saturate bbu growth, though at fairly large amplitudes for the parameters we chose.

REFERENCES

1. K. L. F. Bane, T. Weiland, and P. Wilson in "Physics of Particle Accelerators," *AIP Conference Proceedings No 127* (BNL/SUNY Summer School) pp. 875 ff. (1985) and references cited therein.
2. B. Zotter in "Physics of Particle Accelerators," *AIP Conference Proceedings No. 153* (SLAC/Fermilab Summer Schools) pp. 663 ff. (1987).
3. A. W. Chao in "Physics of Particle Accelerators," *AIP Conference Proceedings No. 105* (SLAC Summer School) pp. 353 ff. (1983).
4. K. L. F. Bane in "Physics of Particle Accelerators," *AIP Conference Proceedings No. 153* (SLAC/Fermilab Summer Schools) pp. 971 ff. (1987).
5. A. W. Chao, B. Richter, and C-Y Yao, *Nucl. Instr. and Methods* **178,** 1 (1980).
6. K. L. F. Bane, *Proceedings of the 1985 Particle Accelerator Conference,* Vancouver, B.C. [*IEEE Trans. Nucl. Sci.* **NS-32,** 2389 (1985)].
7. H. Henke, *Proceedings of the ICFA Workshop on Low Emittance Beams,* Brookhaven, March 1987.
8. F. Selph and A. Sessler, Lawrence Berkeley Laboratory TBA Note 23, (July 1985).
9. A. Mondelli, D. Chernin, A. Drobot, M. Reiser, and V. Granatstein, *Proceedings of the 1986 Linear Accelerator Conference*, SLAC June 1986, pp. 531 ff.
10. V. K. Neill, L. S. Hall, and R. K. Cooper, *Particle Accelerators* **9,** 213 (1979).
11. R. L. Gluckstern, R. K. Cooper, and P. J. Channell, *Particle Accelerators* **16,** 125 (1985).
12. K. Yokoya, DESY 86-084 (August 1986).
13. A certain contour integral representation of the Bessel function $J_\nu(z)$ is helpful. See, for example, G. Carrier, M. Krook, and C. Pearson, *Functions of a Complex Variable,* McGraw-Hill, New York (1966), p. 234, Eq. (5-142).
14. This "cancellation" effect in accelerators was first discussed by V. Balakin, A. Novokhatsky, and V. Smirnov, *Proceedings of the 12th International Conference on High Energy Accelerators* (Fermilab, 1983) p. 119.
15. *SLAC Linear Collider Conceptual Design Report,* SLAC-229 (1980), Appendix A.