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THE EFFECT OF PERTURBATIONS ON THE LONGITUDINAL MOTION OF PARTICLES IN A STEPPED-PHASE-VELOCITY LINEAR ACCELERATOR

YU. V. SENICHEV and E. N. SHAPOSHNIKOVA

Institute for Nuclear Research of the Academy of Sciences of the USSR, Moscow, USSR

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We investigate the influence of the deviations from their design values of the electrical and geometrical parameters on the longitudinal motion of particles in the side-coupled linac. The behavior of the longitudinal acceptance (energy displacement, acceptance decrease, and the formation of breaks) for various types of perturbations is determined. This allows explanation of some results obtained during the tuning of the LAMPF accelerator. We suggest a method of mutual compensation of various errors in the accelerating modules. It is found that the definite spatial variation of the accelerating field within each module substantially increases the longitudinal acceptance bucket.

Construction of an intense 600-MeV linear accelerator of H^+ and H^- ions is under way at the Institute for Nuclear Research of the USSR Academy of Sciences in Moscow,[1] and the 800-MeV Los Alamos Meson Physics Facility (LAMPF) of the Los Alamos National Laboratory is in operation in the USA.[2] These accelerators are meson facilities, and special requirements regarding beam quality and radioactive purity are placed upon them. For example, the particle loss in the linear accelerator of the Moscow Meson Factory (MMF) at an average current of 1 mA must not exceed 6×10^{-5}, and the effective root-mean-square momentum spread of output particles must not exceed $\langle \Delta p/p \rangle = 1.25 \times 10^{-3}$.

To enhance the efficiency of the accelerator in the energy range of the order of 100 MeV one accelerating structure is changed to another with the maximum possible integer increase in the frequency of the accelerating field. In LAMPF the frequency increase is a factor of four; consequently the longitudinal phase acceptance in the second part of the accelerator is decreased by a factor of more than 4 so that particles appear near the boundary of the separatrix. (Since it was desired to accelerate particles of different signs, H^+ and H^-, in MMF, an odd multiplier, e.g., 5, was chosen). To simplify the technology the high-energy part consists of tanks with a constant phase velocity, which also creates some specific features.

Therefore, it is necessary to study thoroughly the peculiarities of the beam dynamics in such accelerators, to investigate possible causes of particle loss and to take additional steps to eliminate them, to ensure damping of coherent oscillations, to provide longitudinal and transverse filters, and to work out special procedures for tuning the accelerator.[3–5] Very valuable from this point of view are the experience and experimental results of the American workers obtained from tuning the LAMPF accelerator.[4,6,7] In particular, the ΔT procedure, developed by K. R. Crandall, forms the basis for the time-of-flight procedure for adjusting the amplitude and phase of the accelerating field to be used in tuning the MMF. The difficulties encountered, and time consumed (8 years from start up), in reaching the design parameters in the LAMPF,[7] e.g. average beam intensity of 1 mA, point to the difficulty of obtaining a low-radiation accelerator and a thorough theoretical preparation for starting.

One of the main causes of particle loss is connected with deviations from the design values of the accelerating structure parameters. These deviations are unavoidable. However, our problem here consists not only of determining tolerances but also of exploring the possibility of mutual compensation for various perturbations, since their independent decrease is sometimes difficult to attain technologically.

In this paper we investigate the effect of perturbations on longitudinal beam motion in the accelerating structure with a stepped change in the phase velocity (MMF, LAMPF). Based on this effect, one can explain some of the properties of the region of capture (energy displacement, acceptance decrease, and the formation of breaks) that were found in tuning the LAMPF. The use of the concept of the equivalent phase velocity of a cavity[8] makes it possible to account, in a unified manner, for the influence of perturbations of both the electrical and geometrical parameters of the accelerating cavity. In addition, such an approach made it possible to estimate in a new way the effect of deviations of some system parameters that were previously considered independently of the accelerator tuning procedure and were regarded as insignificant. The investigations provide a method of mutual compensation for various errors when tuning the cavities with no beam. An interesting result of the present work is the possibility of increasing the region of stability due to the spatial variation of the field in the cavity. The problem was solved within the framework of linear theory, which, after comparison with the results of numerical simulation, actually describes, also, the behavior of the stability region, which is indicative of its coherent properties.

The results obtained have direct practical application. Therefore, we begin with the classification of real errors and their causes.

1. THE MAIN TYPES OF ERRORS IN A MULTIPLE-CAVITY ION ACCELERATOR

The high-energy part of the MMF comprises 27 cavities. Each cavity consists of four tanks spaced by drift tubes and connected by coupling bridges. The tank incorporates from 19 to 28 accelerating cells of the same length, except for the first and last half-cells. The main type of oscillation of the accelerating structure with discs and diaphragms is the $\pi/2$ oscillation. The rf power to each cavity is fed from individual generators. The rf field in the cavities is stabilized by an automatic control system.

According to the effect on the beam, the errors of the accelerating channel are of two types: static and nonstatic. The former include deviations of the sizes of the accelerating structure elements from the design values and space distortions of the field along the cavity as well as the inaccuracies in the amplitude and phases of the accelerating rf field. They lead in each cavity to the constant (time-independent) momentum and phase displacement of the centers of particle bunches with respect to the design particle. This may degrade the conditions of particle capture and, at certain values of the error amplitudes, may lead to particle loss.

The instabilities of the amplitude and phase of the rf field belongs to the second (nonstatic) type of errors and leads to the time-dependent deviation of the parameters of the centers of bunches from the design values. These instabilities increase the effective beam sizes. Other nonstatic errors are space distortions of the field, associated with the excitation of nonoperating oscillations during the transient process. In the present paper the time-dependent deviations will be ignored.

In tuning the cavity the following main types of static errors are usually identified.

I. Deviations of the geometrical sizes of the cells, tanks, and cavities.

II. Deviations of the electrical parameters:

(1) the slope of the field amplitude along the tanks;

(2) nonuniformities of the field in the tank with respect to the sloping line;

(3) differences of the average amplitudes of the field in the adjacent tanks of the cavity; and

(4) deviations of the average amplitude and phase of the rf field in the cavity.

Variations in the lengths of the accelerating tanks arise from cell-fabrication errors, from assembly of the tanks from two or three separate sections, and variations in the cavity length also arise from the inaccuracy of setting of the tanks.

Fabrication errors in the tanks lead to excitation of nonoperating types of oscillations. The incomplete compensation in the tank for two symmetrical oscillations, nearest to the operating one, results in the formation of a field slope along the tank.[9] The excitation of other nonoperating oscillations creates nonuniformity of the field from cell to cell.

Owing to the unequality of the tanks, united in a single cavity, there is no compensation for oscillations with variations in the field level from tank to tank.[10]

The size of the deviations of the average amplitude and phase of the cavity rf accelerating field depends upon the accuracy of cavity tuning by the time-of-flight method.

It is generally conceded that information on the effect of these deviations on the accelerated beam can be obtained by processing statistically the results of numerical simulations of particle motion in various realizations of the accelerating channel with random errors of the indicated quantities. Thereby one can judge the degree of influence of the deviation of any parameter by the value of the momentum and phase spread of the centers of bunches at the accelerator output.[11]

Below, to study the effect of perturbations on the beam motion, we use an approach based on the notion of quasi-equilibrium motion in the nonideal accelerator. This method enables one to determine the degree of effect of a perturbation on the beam depending on its wavelength for any cavity of the accelerator. The results obtained are of major importance in setting the amplitude and phase of the accelerating field and in determining the requirements for tuning the cavities of the high-energy part of the MMF with no beam.

2. PERTURBATIONS OF THE PHASE MOTION OF PARTICLES IN THE TANK

First consider perturbations with a wavelength less than, or of the order of, the tank length.

The accelerating tank of the MMF consists of identical cells, i.e., has a constant phase velocity β_p. In Ref. 8 the features of equilibrium motion, associated with the stepped change in the phase velocity from cell to cell, were investigated. In the given structure there is no equilibrium particle in the usual sense, i.e., no particle that moves along the whole length of the accelerator at the phase velocity of the wave and, consequently, performs no phase oscillations. However, if the tank is considered as one accelerating cell, it is possible to introduce the notion of a quasi-equilibrium particle that is analogous to the definition of an equilibrium particle in the accelerator with a smooth change in the accelerating periods. A particle is called quasi-equilibrium if during each macroperiod "tank + drift space," it has a velocity increase equal to the jump of the phase velocity from tank to tank.[8] Furthermore, the cavity parameters are calculated so that longitudinal phase oscillations of the quasi-equilibrium particle from tank to tank are maximally identical and are performed with the minimum amplitude with respect to the so-called average phase ϕ_a.

If the tank were infinitely long, then the quasi-equilibrium particle would perform phase oscillations with respect to the equilibrium phase of the tank. For the tank with a constant phase velocity, the phase $\phi_s = 0$ is equilibrium (the accelerating wave of the form $E_0 \sin wt$ is considered).

The equation of motion of the particle, having a velocity βc, in the field of a wave with amplitude E_0 and phase velocity β_p appears as

$$mc^2\gamma^3\beta \frac{d(\Delta\beta)}{dz} = eE_0 \sin \phi, \tag{1}$$

where z is the longitudinal coordinate, ϕ is the phase of the particle with respect to the rf field, $\Delta\beta = \beta - \beta_p$, and $\gamma = (1-\beta^2)^{-1/2}$.

The deviation of the particle phase from the equilibrium phase $\Delta\phi = \phi - \phi_s$ is related to the deviation of the velocity $\Delta\beta$ in the standing-wave field by the relation

$$\frac{d\,\Delta\phi}{dz} = -\frac{w\,\Delta\beta}{c\beta\beta_p}. \tag{2}$$

It is easy to show that for the particles to be accelerated after passing through the tank the average phase of the particles in the tank ϕ_a must be different from the equilibrium phase $\phi_s = 0$, which is achieved by choosing the tank length.

Therefore, when the motion of particles in the cavity is considered the average phase ϕ_a is usually called equilibrium or synchronous. Using the expression $\psi = \phi - \phi_a + \operatorname{tg} \phi_a$ from Eqs. (1) and (2) for small deviations[12] we have

$$\frac{d^2\psi}{dz^2} + K_0^2\psi = 0, \tag{3}$$

where

$$K_0^2 = \frac{eE_0 \cos \phi_a w}{mc^3\beta_p^3\gamma_p^3}.$$

In the MMF the value $\phi_a = 57°$ is chosen.

Using the definition of quasi-equilibrium motion, namely the equality of the input phases in each tank and the preservation of the velocity increase in the tank, it is possible to determine the changes in the parameters of the quasi-equilibrium particle caused by particular deviations in the tank parameters. In this case the remaining particles perform phase oscillations in the cavity with respect to a "new" quasi-equilibrium particle. By analyzing the solutions of Eqs. (2) and (3) we see that the change in the velocity of the quasi-equilibrium particle (or the virtually equivalent effective phase velocity), associated with deviations of the geometrical sizes of the cavity from the design values, takes the form[8]

$$\frac{\delta\beta_s}{\beta} = \frac{\Delta L^*}{L^*}, \tag{4}$$

where $L^* = L_t T(\mu_t/2) + L_d$ is the characteristic length required to ensure invariability of the phase velocity in ideal fields, L_t is the tank length, L_d is the length of the drift space between the tanks, $\mu_t = K_0 L_t$ is the phase advance of small longitudinal oscillations in the tank, and the function is $T(x) = \operatorname{tg} x/x$. When $\mu_t/2 \ll 1$, $T \sim 1$, and L^* is equal to the length of the macroperiod of the structure, $L = L_t + L_d$.

Figure 1 (curve I) presents the value of $\delta\beta_s/\beta$ as a function of the cavity Number $\mathcal{N}$ of the second part of the MMF with a change in the macroperiod length by $\Delta L = 100\,\mu$m.

Equation (4) yields an important conclusion, verified by numerical methods,[8] about the possibility of simple compensation in practice for tank fabrication errors by changing the lengths of the drift spaces between the tanks.

We will now analyze the influence of deviations of the electrical parameters of the system.

The motion of particles in the tank caused by perturbation of the field is described by Eqs. (2) and (3) with the corresponding replacement of the parameter K_0^2 by $K_0^2[1 + \varepsilon\mathcal{F}(z)]$, where $\varepsilon = \Delta E/E_0$ is the relative perturbation amplitude. For the perturbation of the field in the form of a slope, the function is $\mathcal{F}(z) = 1 - (2z/L_t)$, $0 \leqslant z \leqslant L_t$. In this case the slope sign is determined by the sign ε. The solutions of Eqs. (2) and (3) can be found as a series in the powers ε. The first order of perturbation theory, the matrix, which relates the input and

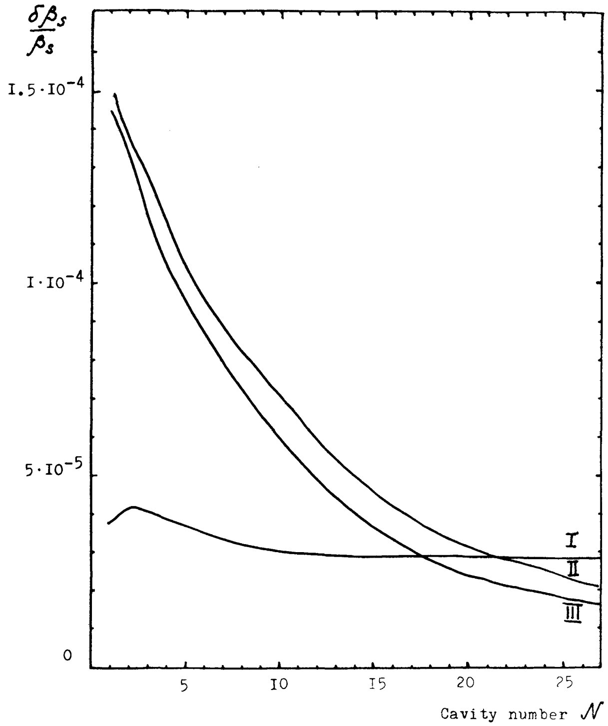


FIGURE 1 The quantity $\delta\beta_s/\beta$ as a function of the Cavity Number of the second part of the MMF in the case of the following perturbations: the change in the macroperiod length by $\Delta L = 100\ \mu$m (curve I), the amplitude of the field slope in the tank is $\varepsilon = 0.1$ (curve II), and the amplitude of the field slope in the cavity is $\varepsilon = 0.01$ (curve III).

output parameters of the particle in the tank ψ and $\Delta\beta/\beta$, is of the form

$$\begin{bmatrix} \left\{1 - \frac{\varepsilon}{2}[T(\mu_t) - 1]\right\} \cos \mu_t & -\frac{w}{K_0\beta_p c} \sin \mu_t \\ \frac{K_0\beta_p c}{w} \sin \mu_t & \left\{1 + \frac{\varepsilon}{2}[T(\mu_t) - 1]\right\} \cos \mu_t \end{bmatrix} \tag{5}$$

Using the conditions of quasi-equilibrium motion formulated above, we find that, for a tank field slope ε, the change in the quasi-equilibrium velocity is

$$\frac{\delta\beta_s}{\beta} = \varepsilon \frac{T(\mu_t) - 1}{\mu_t^2 T(\mu_t)} \cdot \frac{L_t}{L^*} \cdot \frac{\Delta\beta_0}{\beta}, \tag{6}$$

where $\Delta\beta_0/\beta$ is the design value of the deviation of the initial velocity of the quasi-equilibrium particle from the tank phase velocity. For the value of the phase advance $\mu_t \ll 1$, from Eq. (6) we have

$$\frac{\delta\beta_s}{\beta} \approx \frac{\varepsilon}{3} \frac{L_t}{L} \cdot \frac{\Delta\beta_0}{\beta}.$$

Under the same conditions the average phase is changed by

$$\delta\phi_a \approx \frac{\varepsilon}{12}\mu_t^2 \frac{L_d}{L}\psi_0, \tag{7}$$

where ψ_0 is the design deviation of the quasi-equilibrium particle phase at the tank input.

Figure 1 (curve II) shows the dependence of the value $\delta\beta_s/\beta$ on the Cavity Number at the field-slope amplitude in the tank $\varepsilon = \Delta E/E = 0.1$.

Similarly, one can also consider the influence of the other field perturbations in the tank. With decrease in their wavelength Λ the deviation of the quasi-equilibrium velocity is decreased by $\sim\Lambda^2$. As is demonstrated by the results of numerical simulation of the particle motion, the maximum effect on the beam is caused by perturbations with a wavelength $\Lambda \sim 2L_c$, where L_c is the cavity length.

To answer the question about the influence of various structures of the field slopes in a tank (e.g., alternation of the sign of the slope ε from tank to tank), of the field slope in the cavity, and other perturbations with a wavelength greater than the macroperiod length, it is necessary to consider the particle motion in the cavity. Determination of the value $\delta\beta_s/\beta$ for each cavity is of great importance for cavity tuning and characterizing the degree of "nonideality" of the cavity.[5]

3. SMALL OSCILLATIONS OF PARTICLES IN THE CAVITY

In considering the particle motion in the cavity, one must account mathematically for the presence of drift spaces between the tanks.

For a quasi-equilibrium particle with energy W_s the equation of motion in the ideal cavity can be written as

$$\frac{dW_s}{dz} = eE_0\theta(z)\sin\phi_a, \tag{8}$$

where

$$\theta(z) = \begin{cases} 1 & z \in L_t \\ 0 & z \in L_d \end{cases}$$

is the function, periodic in the cavity, with the period $L = L_t + L_d$.

The equation of motion of the particle with energy W in the field $E = E_0(1 + \varepsilon\mathscr{F})$, where $\varepsilon = \Delta E/E$ is of the form

$$\frac{dW}{dz} = eE\theta(z)\sin\phi. \tag{9}$$

The problem consists of determining the change in the quasi-equilibrium velocity β_s (or the equivalent cavity phase velocity β_p), as well as in the synchronous phase ϕ_a, with the existence of perturbation $\varepsilon \neq 0$.

Using Eq. (2), which relates the deviations from the equilibrium values of the phase $\Delta\phi = \phi - \phi_a$ and the velocity $\Delta\beta = \beta - \beta_s$ from Eqs. (8) and (9) in the conservative approximation, we obtain the equation that describes small oscillations of the particle in the cavity with respect to the design quasi-equilibrium particle

$$\frac{d^2\,\Delta\phi}{dz^2}+K_0^2\theta(z)[\Delta\phi(1+\varepsilon\mathscr{F})+\varepsilon\mathscr{F}\,\mathrm{tg}\,\phi_a]=0. \tag{10}$$

Using perturbation theory, we write the solution of Eq. (10) as

$$\Delta\phi=\delta\phi+\varepsilon\delta\phi^{\mathrm{I}}+\varepsilon^2\delta\phi^{\mathrm{II}}+\cdots \tag{11}$$

and

$$\frac{\Delta\beta}{\beta}=\frac{\delta\beta}{\beta}+\varepsilon\frac{\delta\beta^{\mathrm{I}}}{\beta}+\varepsilon^2\frac{\delta\beta^{\mathrm{II}}}{\beta}+\cdots, \tag{12}$$

where $\delta\phi$, $\delta\beta/\beta$ are the solutions of Eq. (10) at $\varepsilon=0$ that describe oscillations of an arbitrary particle in the ideal cavity. We obtain them using the averaging method[12] or the so-called "smooth approximation" employed in some cases for the description of transverse oscillations of particles in a periodic focusing field. The smooth approximation gives sufficiently accurate results if the frequency of the periodic effect is much higher than the natural frequency of a dynamic system. It is easy to show that, in our case, this condition reduces to the following

$$\frac{2\pi}{L}\gg K_0\sqrt{\frac{L_t}{L}}. \tag{13}$$

Equation (10) at $\varepsilon=0$ will be written as

$$\frac{d^2\,\delta\phi}{dz^2}+K_0^2[\bar{\theta}+\tilde{\theta}(z)]\,\delta\phi=0, \tag{14}$$

where

$$\bar{\theta}=\frac{1}{L}\int_0^L\theta(z)\,dz=\frac{L_t}{L},\qquad \tilde{\theta}(z)=\theta(z)-\bar{\theta}.$$

Then the solution of Eq. (14) can be represented as oscillations with the modulated amplitude

$$\delta\phi=[1+q(z)][A\cos Kz+B\sin Kz], \tag{15}$$

where

$$K^2=K_0^2\left[\bar{\theta}+\frac{1}{L}\int_0^L q(z)\theta(z)\,dz\right]. \tag{16}$$

A and B are the constants determined by the initial conditions, and the function $q(z)$ satisfies the equation

$$\frac{d^2q}{dz^2}=-K_0^2\tilde{\theta}, \tag{17}$$

with the additional conditions

$$\overline{\frac{dq}{dz}}=0,\qquad \bar{q}=0. \tag{18}$$

The function $q(z)$, determined from Eqs. (17) and (18), is a sufficiently smooth periodic function with a period L

$$\frac{q(z)}{K_0^2L^2}=\begin{cases} Q_0+Q_1\dfrac{z}{L}+(\bar{\theta}-1)\dfrac{z^2}{2L^2}, & 0\leqslant z\leqslant L_t \\ Q_0+\dfrac{\bar{\theta}^2}{2}+(Q_1-\bar{\theta})\dfrac{z}{L}+\dfrac{\bar{\theta}}{2}\cdot\dfrac{z^2}{L^2}, & L_t\leqslant z\leqslant L, \end{cases} \tag{19}$$

where

$$Q_0=\frac{\bar{\theta}^3}{6}-\frac{\bar{\theta}^2}{7}+\frac{\bar{\theta}}{12}, \qquad Q_1=\frac{\bar{\theta}(1-\bar{\theta})}{2}, \quad \text{and} \quad \bar{\theta}=\frac{L_t}{L}.$$

The modulation amplitude is $q_{\max}\sim \mu_t^2/(4\pi^2\bar{\theta})$. So it follows from Eq. (13) that $q_{\max}\ll 1$. Thus, the occurrence of the term $\tilde{\theta}(z)$ in Eq. (14) leads in the solution of Eq. (15) to small corrections $q(z)\ll 1$.

The MMF value is $\bar{\theta}=(L_t/L)\approx\frac{2}{3}$. In this case it follows from Eq. (19) that $q_{\max}=1.5\times 10^{-2}\,K_0^2L^2=3.5\times 10^{-2}\,\mu_t^2$. The phase advance of longitudinal oscillations in the tank $\mu_t=K_0L_t$ in the first cavities of the second part of the MMF is 0.75 and in the last cavities 0.24. The condition of applicability of the smooth approximation, Eq. (13), can be rewritten as

$$\frac{\mu_t}{2\pi}\cdot\frac{1}{\sqrt{\bar{\theta}}}\ll 1.$$

Obviously, for the parameters of the second part of the MMF it is satisfied.

The longitudinal oscillation frequency, calculated from Eq. (16), is

$$K^2=K_0^2\cdot\frac{2}{3}\left(1+\frac{\mu_t^2}{72}\right)\simeq\tfrac{2}{3}K_0^2.$$

Using Eqs. (2) and (15) the deviations of the phase and velocity from the parameters of the design particle at the cavity output will be written as $\delta\phi_{\text{out}}$ and $\delta\beta_{\text{out}}/\beta$ if at the input of the cavity they are equal, respectively, to $\delta\phi_{\text{in}}$ and $\delta\beta_{\text{in}}/\beta$

$$\begin{Vmatrix} \delta\phi_{\text{out}} \\ \dfrac{\delta\beta_{\text{out}}}{\beta} \end{Vmatrix} = \begin{Vmatrix} \cos\mu-\alpha\sin\mu & -\dfrac{w}{c\beta_sK}\sin\mu \\ \dfrac{c\beta_sK}{w}\sin\mu & \cos\mu+\alpha\sin\mu \end{Vmatrix} \begin{Vmatrix} \delta\phi_{\text{in}} \\ \dfrac{\delta\beta_{\text{in}}}{\beta} \end{Vmatrix}. \tag{20}$$

Here $\mu=KL_c$ is the phase advance of small longitudinal oscillations in the cavity, and

$$\alpha=\frac{(1-\bar{\theta})}{2\bar{\theta}^{1/2}}\,\mu_t.$$

In obtaining the solution to Eq. (20), only the terms $\sim(dq/dz)$ were taken into account, whereas the small corrections of the order of $q<q_{\max}\ll 1$ were neglected.

Now consider the motion in the cavity with errors. Find corrections to the solution of Eq. (15) $\delta\phi^{\mathrm{I}}$ and $\delta\beta^{\mathrm{I}}/\beta$ that arise when $\varepsilon\neq 0$. In Eq. (10) we omit the term $\tilde{\theta}$; i.e., we replace the quantity θ by its mean value $\bar{\theta}$. In fact, the corrections associated with the presence of the term $\tilde{\theta}$ correspond in size to the corrections considered below and are taken into account in the first approximation of the solutions $\delta\phi$ and $\delta\beta/\beta$ [Eqs. (11) and (12)] for the ideal accelerator.

Then for $\delta\phi^{\mathrm{I}}$ from Eq. (10) we have

$$\frac{d^2(\delta\phi^{\mathrm{I}})}{dz^2} + K^2[\delta\phi^{\mathrm{I}} + \mathscr{F}\delta\phi + \mathscr{F}\,\mathrm{tg}\,\phi_a] = 0, \tag{21}$$

with the initial conditions

$$\delta\phi^{\mathrm{I}}|_{z=0} = 0,$$
$$\left.\frac{d(\delta\phi^{\mathrm{I}})}{dz}\right|_{z=0} = 0.$$

Hence, at the output of the cavity $z = L_c$ we have

$$\delta\phi^{\mathrm{I}}_{\mathrm{out}} = a_n \cos\mu - \frac{w}{K\beta_s c} b_n \sin\mu + c_n \delta\phi_{\mathrm{in}} \cos\mu - \frac{w}{K\beta_s c} d_n \frac{\delta\beta_{\mathrm{in}}}{\beta} \sin\mu \tag{22}$$

and

$$\frac{\delta\beta^{\mathrm{I}}_{\mathrm{out}}}{\beta} = \frac{K\beta_s c}{w} a_n \sin\mu + b_n \cos\mu - \frac{K\beta_s c}{w} d_n \delta\phi_{\mathrm{in}} \sin\mu - c_n \frac{\delta\beta_{\mathrm{in}}}{\beta} \cos\mu, \tag{23}$$

where the coefficients a_n, b_n, c_n, and d_n depend on the value of μ and on the form of perturbation $\mathscr{F}(z)$.

Any perturbation of the field in the cavity can be represented as the superposition of harmonic perturbations. For linear oscillations the solution is the sum of solutions found for the harmonics of the Fourier expansion. Consider the effect of separate harmonic perturbations on the particle motion.

At $\mathscr{F}_n = \cos\dfrac{\pi n z}{L_c}$, where $n = 1, 2, \ldots$, we have

$$a_n = -\mathrm{tg}\,\phi_a \frac{(-1)^{n+1}\cos\mu + 1}{\left(\dfrac{\pi n}{\mu}\right)^2 - 1}, \tag{24}$$

$$b_n = \frac{K\beta_s c}{w}\,\mathrm{tg}\,\phi_a \frac{(-1)^{n+1}\sin\mu}{\left(\dfrac{\pi n}{\mu}\right)^2 - 1}, \tag{25}$$

$$c_n = -\frac{1-(-1)^n}{\left(\dfrac{\pi n}{\mu}\right)^2 - 4}, \tag{26}$$

$$d_n = \frac{1+(-1)^n}{\left(\dfrac{\pi n}{\mu}\right)^2 - 4}. \tag{27}$$

The first two terms in Eqs. (22) and (23) are non-zero, also, under the zero initial conditions ($\delta\beta_{\text{in}}/\beta=0$, $\delta\phi_{\text{in}}=0$) and correspond to forced oscillations. The last two terms describe the parametric effect of perturbation on the beam with the existence of initial deviations of velocity and phase from the design values.

Operating within the framework of representations of the change in the parameters of the quasi-equilibrium particle with varying cavity parameters, we write the solutions to Eqs. (11) and (12) in the following form

$$\begin{Vmatrix} \Delta\phi_{\text{out}}-\delta\phi_a \\ \dfrac{\Delta\beta_{\text{out}}}{\beta}-\dfrac{\delta\beta_s}{\beta} \end{Vmatrix} = \begin{Vmatrix} \cos\mu & -\dfrac{w}{k\beta_s c}\sin\mu \\ \dfrac{K\beta_s c}{w}\sin\mu & \cos\mu \end{Vmatrix} \begin{Vmatrix} \Delta\phi_{\text{in}}-\delta\phi_a \\ \dfrac{\Delta\beta_{\text{in}}}{\beta}-\dfrac{\delta\beta_s}{\beta} \end{Vmatrix}. \tag{28}$$

The solution to Eq. (28) virtually represents the particle motion in the nonideal cavity as oscillations with respect to a "new" quasi-equilibrium particle. Its parameters can be found from Eq. (28) using Eqs. (22) and (25).

We note that in the solution to Eq. (28), the last two terms of Eqs. (22) and (23) are neglected; they lead to insignificant corrections [a change in the coefficients of the matrix Eq. (28), of the type $(1\pm\varepsilon c_n)\cos\mu$, $(1\pm\varepsilon d_n)\sin\mu$].

Then for $\mathscr{F}_n=\cos\pi nz/L_c$ the deviation of the synchronous phase is

$$\delta\phi_a=-\varepsilon\frac{\operatorname{tg}\phi_a}{\left(\dfrac{\pi n}{\mu}\right)^2-1}\cdot\frac{[1+(-1)^n]}{2}, \tag{29}$$

and the change in the quasi-equilibrium velocity is determined by

$$\frac{\delta\beta_s}{\beta}=-\frac{\varepsilon\operatorname{tg}\phi_a(1-(-1)^n]}{\left(\dfrac{\pi n}{\mu}\right)^2-1}\cdot\frac{K\beta_s c}{2w}\cdot\frac{\sin\mu}{1-\cos\mu}. \tag{30}$$

Hence at $n=1,3$ $\delta\phi_a=0$, and $\delta\beta_s/\beta=0$ at $n=2,4$. On the whole with increasing number n the effect of perturbation on the quasi-equilibrium motion markedly decreases ($\sim n^{-2}$).

Since in reality the number of elements (tanks) in which field deviations occur is four, the interesting (practical) cases are reduced to perturbations with $n=1,2,4$. We shall deal with them in more detail.

At $n=1$ we have the so-called field slope in the cavity. Figure 1 (curve III) presents the dependence of the value of $\delta\beta_s/\beta$ on the Cavity Number of the second part of the MMF at the relative amplitude of the field slope in the cavity $\varepsilon=0.01$. It appears from the figure that, regarding the effect on the beam, the effect of field slope in the cavity is approximately one order of magnitude higher than the effect of the field slope in the tank. And this is in the case where the field slopes in all the tanks of one cavity have the same sign. At $n=4$ Eqs. (29) and (30) describe the effect of the harmonic that has the maximum amplitude with the alternation of signs of the tank field slopes from tank to tank within one cavity. In this case $\delta\beta_s/\beta=0$.

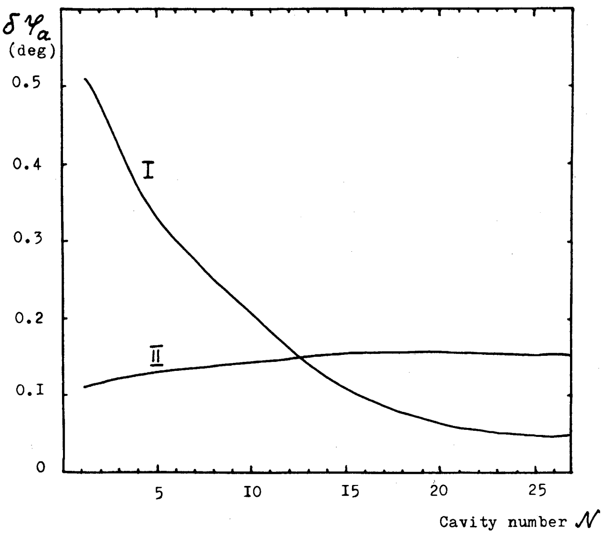


FIGURE 2 The change in the synchronous phase $\delta\phi_a$ in the case of perturbation of the field in the cavity of the type $\varepsilon\cos(2\pi z/L_c)$ at $\varepsilon = 0.01$ (curve I) and the deviation of the synchronous phase, compensated for in the process of setting the amplitude by the time-of-flight method (curve II).

The case $n = 2$ corresponds to perturbation with the equal deviations of the average amplitudes of the tank fields with the alternation of sign of the type $(+--+)$ or $(-++-)$. In this case $\delta\beta_s/\beta = 0$, and the change in the synchronous phase at $\varepsilon = 0.01$ is presented in Fig. 2 (curve I). The sign of the quantity $\delta\phi_a$ depends on the sign of ε and, correspondingly, may lead to an increase or a decrease in the phase width of the capture region of the second part of the MMF [which is approximately equal to $3\times(\pi/2-\phi_a)$].

Similar calculations can also be made for $\mathcal{F}_n = \sin\pi n z/L_c$. In this case

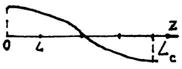
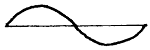
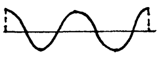
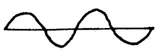
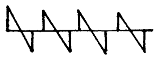
$$\delta\phi_a = -\varepsilon\,\mathrm{tg}\,\phi_a \frac{1-(-1)^n}{2} \frac{\dfrac{\pi n}{\mu}}{\left(\dfrac{\pi n}{\mu}\right)^2 - 1} \cdot \frac{\sin\mu}{(1-\cos\mu)}, \tag{31}$$

$$\frac{\delta\beta_s}{\beta} = -\varepsilon \frac{K\beta_s c}{w}\,\mathrm{tg}\,\phi_a \cdot \frac{1+(-1)^n}{2} \cdot \frac{\dfrac{\pi n}{\mu}}{\left(\dfrac{\pi n}{\mu}\right)^2 - 1}. \tag{32}$$

For perturbations, which do not change the average field amplitude in the cavity, $n = 2, 4$ and, as follows from Eq. (31), $\delta\phi_a = 0$. The case $n = 2$ corresponds to the opposite deviations of the average field amplitudes in the cavity halves and the case $n = 4$ to the alternation of signs of deviations of the average field amplitudes in the cavity tanks. In the latter case the effect of

TABLE I

Effect of Perturbations of the Accelerating Field Amplitude in the Cavity on the Change in the Parameters of the Quasi-equilibrium Particle ($\varepsilon = 0.01$)

			$\frac{\delta\beta_s}{\beta}$		$\delta\phi_a$	
	Type of perturbation	n	$N=1$	$n=27$	$N=1$	$N=27$
1	 $\cos\frac{\pi z}{L_c}$	1	$-1.5 \cdot 10^{-4}$	$-1.6 \cdot 10^{-5}$	0	0
2	 $\cos\frac{2\pi z}{L_c}$	2	0	0	$-0.53°$	$-0.04°$
3	 $\sin\frac{2\pi z}{L_c}$	2	$-1.3 \cdot 10^{-4}$	$-1.3 \cdot 10^{-5}$	0	0
4	 $\cos\frac{4\pi z}{L_c}$	4	0	0	$-0.09°$	$-0.01°$
5	 $\sin\frac{4\pi z}{L_c}$	4	$-4.4 \cdot 10^{-5}$	$-6.2 \cdot 10^{-6}$	0	0
6	 $1 - 2z/L_t$, $0 < z < L_t$; 0, $L_t < z < L$		$-1.5 \cdot 10^{-5}$	$-2.0 \cdot 10^{-6}$	$0.013°$	$0.002°$

perturbation on the change in the quasi-equilibrium velocity is approximately a factor of 2.5 in the first cavities and by a factor of 2 in the last cavities of the second part of the accelerator—smaller than in the former case.

All the situations outlined above are presented in Table I. Estimates for harmonics with the large number n on the one hand show an abrupt decrease in the effect of perturbation on the quasiequilibrium parameters and on the other hand become in themselves less accurate because they do not account for the "fine" structure of the cavity—the presence of tanks with drift spaces.

4. THE EFFECT OF CHANGE IN THE QUASI-EQUILIBRIUM PARAMETERS ON THE CAPTURE REGION

As already mentioned, the change in the parameters of a quasi-equilibrium particle describes the coherent behavior of the capture region. The results obtained make it possible, for example, to calculate the change in the level of the synchronous energy at the accelerator output in the presence of perturbations that lead to the deviation $\delta\beta_s/\beta \neq 0$ of the same sign in different cavities. This argument was verified during the numerical simulation of the capture region of the second part of the MMF for the changed lengths of the accelerator macroperiods.[8]

All the numerical calculations were made with the aid of a program employing the "rectangular-field" model (for details, see, for example, Ref. 12).

Similar results have also been obtained for the deviations of the electrical parameters of the system; in the case where the field slopes have the same sign in the tanks or cavities on the phase plane (ϕ, β), the capture region is lifted or lowered coherently (depending upon the sign of ε) relative to the design energy level.

At the same time the alternation of the deviation sign $\delta\beta_s/\beta$ from cavity to cavity in the case of phase advance of small longitudinal oscillations $\mu \sim \pi$ leads to the resonance build-up of coherent phase oscillations, with a decrease of the capture region. Depending upon the initial perturbation phase (e.g. the sign of slope in the first cavity) characteristic "breaks" appear on one or the other side of the separatrix. The field slope in the cavity leads to a partial disappearance of the capture region even at $\varepsilon = 0.01$. For comparison, Fig. 3 shows the design capture region at the input of the second part of the MMF. Figures 4a and 4b present the capture region at the input of the second part of the MMF with (4a) the field slopes in the cavity with the amplitude $\varepsilon = 0.02$ and alternation of signs according to the law $\Delta E/E = \varepsilon(-1)^N$ and with (4b) the deviations of the lengths of the drift spaces between the tanks $\Delta L_d = 600 \cdot (-1)^{N+1}\,\mu$m, where N is the Cavity Number.

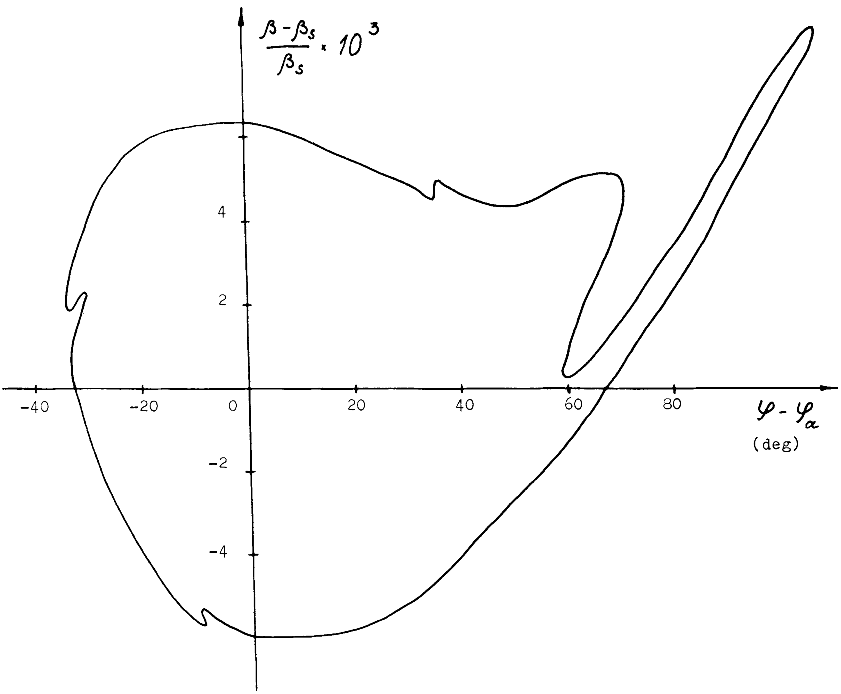


FIGURE 3 The design region of longitudinal capture at the input of the second part of the MMF.

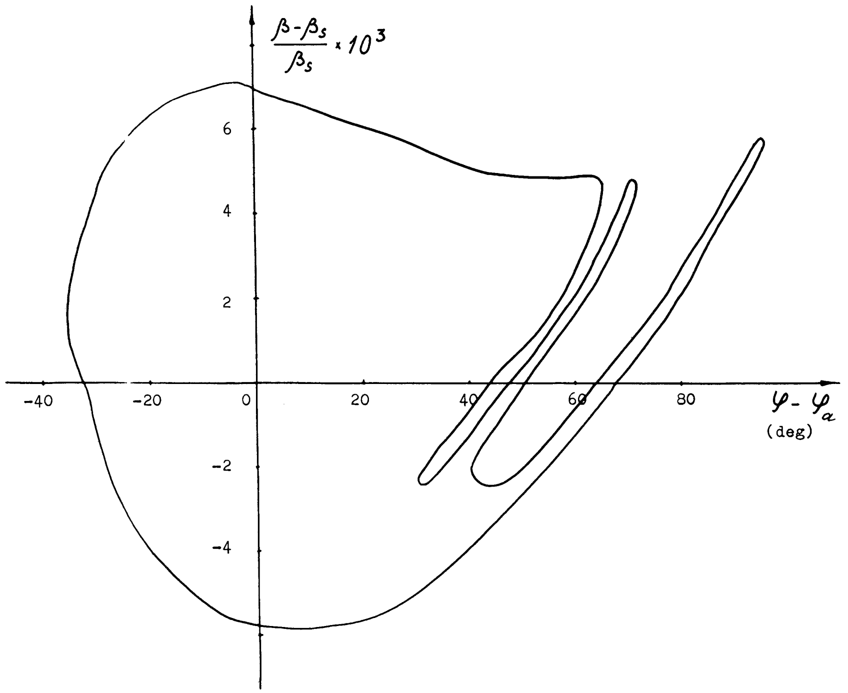


FIGURE 4a The region of capture at the input of the second part of the MMF in the case of field slopes in the cavity with the amplitude $\varepsilon = 0.02$ and with the alternation of signs according to the law $\Delta E/E = \varepsilon(-1)^N$.

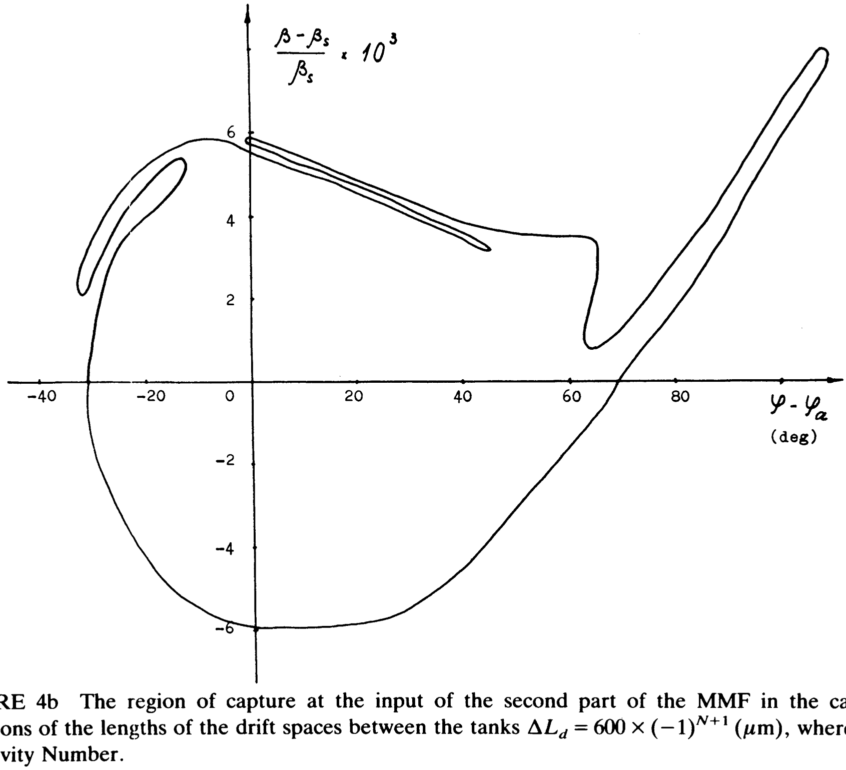


FIGURE 4b The region of capture at the input of the second part of the MMF in the case of deviations of the lengths of the drift spaces between the tanks $\Delta L_d = 600 \times (-1)^{N+1}$ (μm), where N is the Cavity Number.

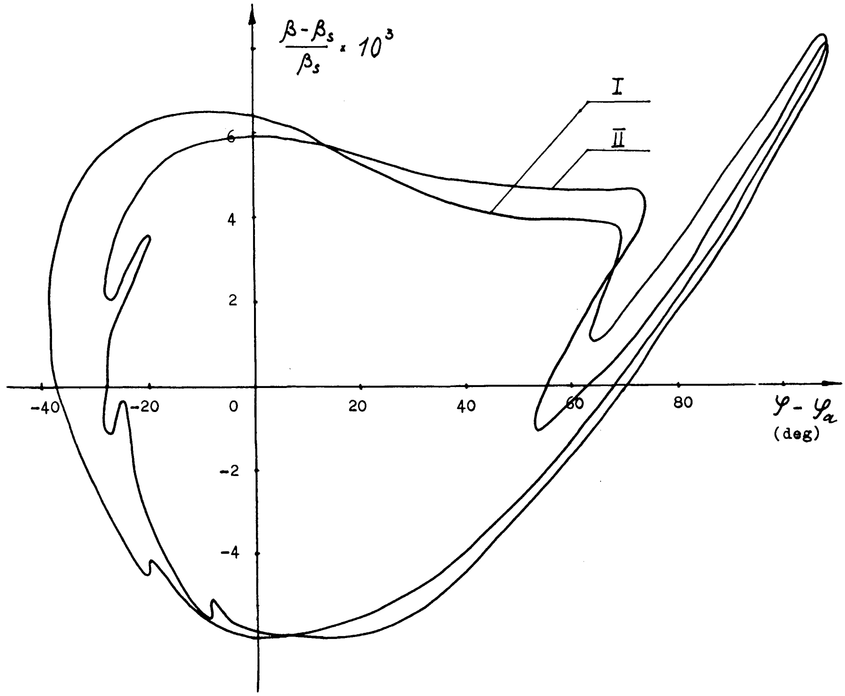


FIGURE 5 The capture region of the second part of the MMF in the case of field perturbations in the cavity of the type $\varepsilon \cos(2\pi z/L_c)$ at $\varepsilon = -0.05$ (curve I) and $\varepsilon = 0.05$ (curve II).

If the field slopes along the tanks in the cavity have the same sign, with $\varepsilon \sim 0.1$, there is a marked effect on the stability region.

The nonuniformity of the accelerating field from cell to cell up to $\langle \Delta E/E \rangle \sim 20\%$ does not lead to any changes in the capture region, as compared with the ideal case (Fig. 3).

Thus the results of numerical simulation of the particle motion in the nonideal accelerating channel show that the maximum effect on the beam is caused by perturbations with a wavelength $\Lambda \sim 2L_c$. It should be noted that perturbations of this type include deviations of the average amplitude and phase of the rf field in the cavity, which result from tuning errors as well as instabilities of the system.

We have found previously the form of field perturbation that leads, according to Eq. (29), to a change in the phase of the quasi-equilibrium particle $\delta\phi_a$ with no change in the average amplitude of the cavity field. During the numerical simulation of the effect of perturbation $\mathcal{F}_n = \cos(\pi n z/L_c)$ at $n = 2$, the corresponding increase in the capture region in comparison with the ideal case for $\varepsilon < 0$ (see Fig. 5, curve I) and its decrease for $\varepsilon > 0$ (curve II) were revealed.

The unified approach to the description of the effect of various perturbations enables one to consider the possibility of compensating for the deviations in some parameters by changing the others. In fact, as has been checked numerically, the synchronous energy level has the design value if, as a result of the effect of

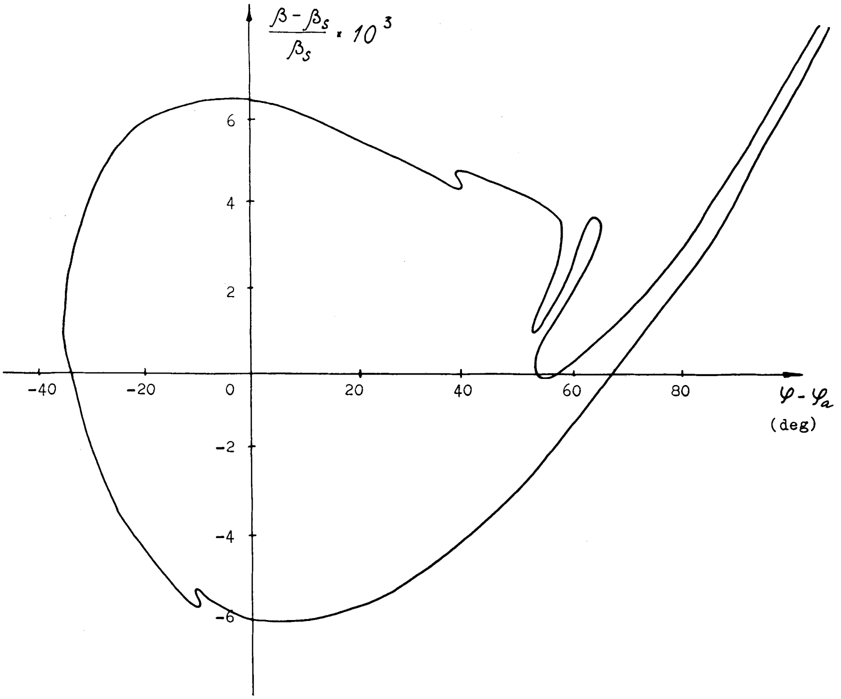


FIGURE 6 The capture region of the second part of the MMF, obtained from the simultaneous effect of perturbations of two types: field slopes in the cavity with the amplitude $\Delta E/E = 0.02 \times (-1)^N$ and deviations of the lengths of the drift spaces between the tanks $\Delta L_d = (-1)^{N+1} \cdot 600$ (μm), where N is the Cavity Number.

various perturbations, $\delta\beta_s/\beta = 0$ in each of the cavities. The possibility of restoring the capture region, which disappeared as a result of the effect of any perturbation, with the aid of another perturbation seems to be even more interesting. Figure 6 presents the capture region of the second part of the MMF, which arose from the simultaneous effect of perturbations of two types—the alternating-sign field slopes in the cavities and the deviations of the lengths of the drift spaces between the tanks ΔL_d with the corresponding law of alternating the sign of deviation from cavity to cavity. The perturbation amplitudes ($\varepsilon = 0.02$ and $\Delta L_d = 600$ μm) are chosen so that in the first few cavities the resulting deviation is $\delta\beta_s/\beta \sim 0$ (see Fig. 1). In comparing the capture region thus obtained with the capture regions displayed in Figs. 4a and 4b and obtained from the effect of each of the perturbations separately, one can see that, on the whole, there occurs restoration of the capture region, which is similar to the capture region of the ideal accelerator (Fig. 3).

In tuning the accelerator in the LAMPF, initial attempts to obtain the design capture region failed due to the deviations of the lengths of the accelerating tanks.[7] In that case an attempt was made to preserve the capture region by creating slopes of the field amplitude in the cavities. However, the searches were random and have not led to the desirable results,[6] because, apparently, compensation for the perturbation is only possible at the point of its appearance. It was demonstrated in Ref. 5 that tuning the accelerator using the ΔT procedure makes it possible to determine the value of the deviation $\delta\beta_s/\beta$ in the cavity tuned and, consequently, to compensate for perturbations directly.

At present all the tanks of the MMF are assembled from modules, and tuned radiotechnically. Thus the deviations of the tank lengths from the design values can be measured to high accuracy. The values of the tank field slopes are also known. The next stage is tuning the cavities with no beam. Thus, the variable parameters are the lengths of the drift spaces between the tanks and the average tank field amplitudes. The tanks are set up so as to allow compensation for deviations in tank lengths, which arise in the process of fabrication and assembly, by changes in the lengths of the drift spaces. In the future there is a possibility of compensating by the changes in the lengths of macroperiods, introducing field slopes, and by the deviations of the average field amplitudes in the tanks.

5. THE SPECIFIC FEATURES OF THE TIME-OF-FLIGHT PROCEDURE FOR TUNING THE NONIDEAL CAVITY

The cavity is tuned by the time-of-flight method is to set the design value of the amplitude of the rf field in the cavity E_0 (i.e. to form the capture region of the given sizes) and to place the centers of bunches in the determined (synchronous) phase with respect to the rf field. The ΔT procedure developed[3] is based on the time of flight of the design quasi-equilibrium particle through the cavity. The change in the cavity parameters leads to a change in the time of flight of the design particle. The cavity nonideality can be allowed for during tuning if the influence of various errors is reduced to a change in the parameters of the quasi-equilibrium particle.

In tuning the cavity relative to the beam, the following characteristics are investigated:

$$\Delta t_1 = \Delta\phi_{\text{out}} - \Delta\phi_{\text{in}} + \alpha_1 \frac{\delta\beta_{\text{in}}}{\beta} \tag{33}$$

$$\Delta t_2 = -\alpha_2\left(\frac{\delta\beta_{\text{out}}}{\beta} - \frac{\delta\beta_{\text{in}}}{\beta}\cdot\frac{\beta_{\text{out}}}{\beta_{\text{in}}}\right), \tag{34}$$

where

$$\alpha_1 = \frac{L_{c_1} w}{c\beta_{\text{in}}}, \qquad \alpha_2 = \frac{L_{c_2} w}{c\beta_{\text{out}}}, \quad \text{and} \quad L_{c_1} \text{ and } L_{c_2}$$

are the lengths of the cavity tuned and the one next to it.

The dependences $\Delta t_1(\delta\phi_{\text{in}})$ and $\Delta t_2(\delta\phi_{\text{in}})$ are experimentally measured at the fixed field amplitude value $E = E_0 \pm \Delta E_0$. The curves, corresponding to the dependence $\Delta t_2(\Delta t_1)$ at different E, are called the variable-phase curves and intersect at one point with the coordinates[5]

$$\Delta\tilde{t}_1 = \alpha_1 \frac{\delta\beta_{\text{in}}}{\beta}, \qquad \Delta\tilde{t}_2 = 2\alpha_2\left(\frac{\delta\beta_{\text{in}}}{\beta} - \frac{\delta\beta_s}{\beta}\right). \tag{35}$$

The relations in Eq. (35) were obtained in Ref. 13 for $\delta\beta_s/\beta = \text{const}$, which are thus independent of the amplitude E (or μ). This condition is fulfilled for $\delta\beta_s/\beta$, which arise from the deviation of the geometrical sizes from the design values. In the case of deviation of the electrical parameters, the value of $\delta\beta_s/\beta$, as follows from Eqs. (30) and (32), is a function of the value of μ. Thus in principle the coordinate $\Delta\tilde{t}_1$ of the point of intersection of the variable-phase curves with the various amplitudes $E = E_0 \pm \Delta E_0$ can be nonzero at $\delta\beta_{\text{in}}/\beta = 0$. However, it follows from the analysis of Eqs. (30) and (32) that for $\Delta E/E \ll 1$ at $\delta\beta_{\text{in}}/\beta = 0$ we have $\Delta\tilde{t}_1/\alpha_1 \ll \Delta\tilde{t}_2/\alpha_2$, and in a first approximation we may put $\delta\beta_s/\beta(E) = \delta\beta_s/\beta(E_0)$.

The numerical simulation of the tuning process, in particular the plotting of the variable-phase curves, enables one to determine the numerical value $\delta\beta_s/\beta$ for various perturbations in the cavity.

For all the cases listed in Table I the numerical and analytical results were compared. In fact, for the perturbations of the second and fourth types from Table I the point of intersection of the variable-phase curves lies at zero; i.e., $\delta\beta_s/\beta = 0$. The values of $\delta\beta_s/\beta$, obtained numerically and by Eqs. (30) and (32), also turn out to be similar. For example, for the third cavity of the MMF, with the slope in the cavity with the relative amplitude $\Delta E/E = 1\%$, the value $\delta\beta_s/\beta = 1.1 \times 10^{-4}$ has been obtained numerically, and from Eq. (30) we have $\delta\beta_s/\beta = 1.2 \times 10^{-4}$.

The deviation of the quasi-equilibrium velocity from the design value $\delta\beta_s/\beta$ leads to oscillations of the particle bunch in the cavity and to the energy mismatch of the tuned cavity and the cavity next to it. However, as discussed above, compensation is possible for the effects of various perturbations. During the numerical simulation of the cavity tuning process by the time-of-flight method it was found that with the appropriate selection of perturbation amplitudes [according to Eqs. (4), (6), (30), and (32)] the point of intersection of the variable-phase curves lies at zero; i.e., $\delta\beta_s/\beta = 0$. The cavity, despite the presence of perturbations, behaves as the "ideal" one with respect to tuning with the beam.

With increasing Cavity Number the sensitivity of the time-of-flight method of tuning gradually decreases, and the coordinates of the point of intersection of the variable-phase curves can be determined only with a large error (comparable with the value of $\delta\beta_s/\beta$).

In contrast to the measurement of the change in the quasi-equilibrium velocity, it has not been possible to determine the change in the synchronous phase in the process of tuning, which may lead to a decrease (or an increase) in the phase width of the capture region by the value $\sim 3\delta\phi_a$.

At the same time analysis shows that the occurrence of the last two terms in the solutions of Eqs. (22) and (23) leads at $\delta\phi_a \neq 0$ to a change in the slope of the variable-phase curves, which is equal in the "ideal" cavity to

$$f = \frac{K\beta_s c}{w} \cdot \frac{\sin\mu}{1 - \cos\mu}.$$

If we assume that the field amplitude will be set by using the value of f, it appears that in this case there corresponds to the design value of f a change in the average field amplitude in the cavity ΔE_0, which partly compensates for the deviation of the synchronous phase $\delta\phi_a$. In fact, in the presence of a perturbation with even n, so that $d_n \neq 0$ and $\delta\phi_a \neq 0$

$$f_{\text{pert}} = (1 - \varepsilon d_n) f. \tag{36}$$

Taking into account the relation between the deviations of the synchronous phase and the average field amplitude in the cavity

$$\Delta\phi_a = -\text{tg}\, \phi_a \frac{\Delta E_0}{E_0}, \tag{37}$$

we find that when the field amplitude in the cavity is set by using the value of f the following quantity will be compensated for

$$\Delta\phi_a = \frac{\varepsilon \sin 2\phi_a}{1 - \dfrac{\mu}{\sin\mu}} \cdot \frac{2}{\left[4 - \left(\dfrac{\pi n}{\mu}\right)^2\right]}. \tag{38}$$

Figure 2 (curve II) presents the dependence of the quantity $\delta\phi_a$ (at $n = 2$ and $\varepsilon = 0.01$) that is compensated for in the process of setting the given value of f, on the Cavity Number N.

We note that the deviation of the synchronous phase from the given value of $\delta\phi_a$ does not necessarily lead to an increase in the amplitude of phase oscillations since, in the course of tuning, the input phase of the beam with respect to the phase of the rf field can easily be changed. At $\delta\phi_{\text{in}} = \delta\phi_a$ and $\delta\beta_{\text{in}}/\beta = \delta\beta_s/\beta$ the particle in the given cavity is quasi-equilibrium and performs minimum phase oscillations.

6. SUMMARY

In this paper we have investigated the effect of an extensive class of perturbations of the electrical and geometrical parameters of the accelerating channel on the longitudinal motion of particles in a stepped-phase-velocity linear accelerator. It is shown that the effect of perturbations in the frequency values in the range from the frequency of small longitudinal oscillations K_0 to much-higher values can be reduced to a change in the quasi-equilibrium velocity and synchronous phase, which in turn determine the capture region of the accelerator.

Owing to the unified approach to the description of various perturbations by introducing quasi-equilibrium characteristics, we have found a method of mutual compensation for electrical and geometrical errors in the cavity. The method of determining the change in the parameters of a quasi-equilibrium particle is based on the solution of the equation of oscillations in the nonideal system within the framework of perturbation theory. The condition, under which the approximation used is valid, is fulfilled for all the deviations considered of the cavity parameters. However, as the perturbation frequency decreases, the nonlinear properties of the separatrix become more pronounced. Nevertheless, the qualitative description of the behavior of the system on a large number of cavities and the exact description of one or two cavities give an efficient method for correcting stability region.

Using the results of this work it is possible to explain the distortions of the longitudinal stability region that were revealed in tuning the LAMPF accelerator. For example, installation of tanks relative to each other with the aid of bridges of the design length leads to an uncontrolled change in the equivalent phase velocity of the cavity and, as a result, to a decrease in the stability region.[7] And, attempts to correct these distortions by slopes in a cavity[6] can be efficient only under certain conditions that have just been determined in the present paper.

The perturbation class considered encompasses all most important static distortions that are found in practice and that significantly affect the accuracy of carrying out the ΔT-procedure for setting the amplitude and phase of the accelerating field. In this paper some of the peculiarities of this procedure in the "nonideal" accelerator have been analyzed. In particular, it was shown how one can determine the inaccuracies of tuning the cavity with no beam when carrying out the ΔT procedure.

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