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ELECTRON-COOLING STORAGE RING FOR POSITRONS TO PRODUCE ANTIHYDROGEN

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The paper treats the possibility of using electron cooling to accumulate low-energy cold positrons and using them to produce antihydrogen. A model version of a storage facility with an intensity of a few thousand anti-atoms per second is considered. Ways for its further modification are discussed.

1. INTRODUCTION

At present, recombination of an antiproton p^- with a positron e^+ is the only conceivable way to form antihydrogen. This process is possible with sufficiently cold and intense p^- and e^+ beams that travel with equal velocities in storage rings.

The problem of antiproton accumulation may be considered to be settled with the aid of the electron-cooling method[1,2] from both the theoretical and experimental points of view. (In fact, the formulation itself of the antihydrogen-production problem has become possible after the invention of this method.[3] In order to accumulate and cool the positrons (and the antiprotons as well), we suggest using electron cooling, by means of a which a low-energy (roughly several hundred kilovolts) circulating beam with a temperature of the order of atomic energies (less than 1 eV) is assumed to be obtained. Radiative cooling, which is highly efficient in high-energy storage rings, is unacceptable for this purpose because of long damping times and thus high equilibrium temperatures of the positrons.†

In the first part of this paper, we briefly describe the basic properties of electron cooling of positrons in a homogeneous magnetic field and present the formulae for the characteristic times of positron relaxation in electron gas. This problem is discussed in detail in Ref. 4.

The second part of the paper is devoted to the most important problems associated with the creation of a low-energy storage ring for positrons that uses electron cooling.

We then estimate a possible rate of antihydrogen formation. For a facility with the parameters given in the paper, this rate proves to be of the order of several thousand anti-atoms per second (approximately the same as the rate of hydrogen

† At such low energies, positron diffusion will be determined not so much by quantum fluctuations of the radiation as by the noises in the power supply system, scattering from the residual gas, etc.

In conclusion, we analyse, the major reasons for the limitation of the rate of antihydrogen formation and possible ways of eliminating them.

formation in the experiments on electron cooling of the protons at the NAP-m device.[5,6]

2. ELECTRON COOLING OF POSITRONS

Electron cooling of positrons was studied comprehensively in Ref. 4. Here we dwell only on the basic specific features of this process and present the formulae necessary for further calculations.

When considering the main properties of electron cooling, we abstract ourselves from the cyclic effects of positron motion in a storage ring and consider the nonrelativistic positrons that travel in a homogeneous electron gas in a homogeneous longitudinal magnetic field. As a result of Coulomb interaction with electrons, the positrons will relax to a certain state determined by the statistical properties of the electron gas and, in particular, by the spread of electron velocities. A very important peculiarity of the electron beam, which makes it possible to substantially improve the efficiency of the cooling process, is the small range of the longitudinal velocity spread in comparison with the transverse spread. This anisotropy is formed in the course of acceleration of the electron beam and is maintained by a strong magnetic field that hinders the energy exchange between the longitudinal and transversal degrees of freedom of the electrons. In a real situation, the longitudinal temperature proves to be so low that it is limited, in the case of rapid acceleration of the electron beam, only by the energy fluctuations of the Coulomb interaction, i.e., by a quantity of about $e^2n^{1/3}$ where n is the electron-beam density in the co-moving system. When the beam is accelerated slowly and adiabatically,[2] the longitudinal temperature can be dropped further, and the crystalline-type correlation can appear in the electron gas as a result. At a small spread of the longitudinal velocities in the magnetized electron flux, it may turn out that the main contribution to the integral of collisions is due to the region of the impact parameters ρ satisfying the condition $\mu/\Omega < \rho < \rho_{max}$, where ρ_{max} is the maximum impact parameter at which the interaction is efficiently reduced, μ is the relative velocity of the electron and positron Larmor circles, and Ω is the cyclotron frequency. In this region the influence of the magnetic field on the collision process should be taken into account because the duration of interaction between the particles turns out to be larger than the Larmor period $T = 2\pi/\Omega$. An interesting peculiarity of the electron-positron interaction in this range of impact parameters is that, unlike the electron cooling of the ions, the intensity of interaction of the transverse degrees of freedom of the electrons with those of the cooled beam does not become weaker, as opposed to the case when the magnetic field is absent and the intensity of interaction becomes stronger. This is due to the resonant nature of interaction between particles of the same mass by virtue of the equality of their Larmor frequencies.†

† Strictly speaking, it is necessary to make allowance for the spread of Larmor frequencies, which is due to the energy spread in the electron and positron beams. All further conclusions remain the same, provided that $\Delta E/E \ll \mu/\Omega\rho_{max}$.

Let us now estimate the rate of *variation of* the longitudinal and transverse momenta of a positron in the region of the impact parameters $\rho > \mu/\Omega$. We confine ourselves to the calculation of the friction force since the diffusion terms need not be taken into account to determine the stationary distribution of the positron velocities. Because of the equality of the masses of the electrons and positrons, the latter will be the same as the velocity distribution in the electron flux.

We shall consider the collision of two Larmor circles moving with a velocity μ relative to each other and whose Larmor radii are r_p and r_e (note that $r_p \gg r_e$). Let us ascertain the variation in the transverse energy of a positron by following the law of energy conservation. To do this, we shall analyse the motion of an electron in a magnetic field affected by the force $e^2/r_p^2 e^{i\Omega t}$. (At $\rho > r_p$ the electron-positron interaction is effectively decreased.) With the resonant character of the interaction taken into account, we obtain $p_{\perp e} \sim e^2 t/r_p^2 e^{i\Omega t}$. The number of electrons effectively involved in interactions with the positron is roughly nr_p^3 and, hence,

$$\frac{dp_{\perp p}^2}{dt} \cong -nr_p^3 \frac{d}{dt}\left(\frac{e^2 t}{r_p^2}\right)^2 \cong -\frac{ne^4 t}{r_p}. \tag{1}$$

In reality, the duration of an electron-positron interaction is limited either by the time of flight through the electron-cooling system $t_n \sim l/\beta\gamma c$, by the collision time $t_c \sim r_p/\mu$, or by the screening time of the interaction between the transverse degrees of freedom $t_a \sim \Omega/\omega_e^2$. (The quantity ω_e^2/Ω is the frequency shift of the Larmor oscillations caused by the Coulomb interaction of the electrons, and $\omega_e = (4\pi e^2 n/m_e)^{1/2}$ is the electron plasma frequency.) Thus,

$$dp_{\perp p}^2/dt \sim -ne^4 t/r_p,$$

where

$$t = \min\,(l/\beta\gamma c,\ r_p/\mu,\ \Omega/\omega_e^2).$$

An exact analysis demonstrates that under the condition

$$l/\beta\gamma c \gg r_p/\mu,\ \Omega/\omega_e^2,$$

an expression for the rate of change of the squared transverse momentum is of the form

$$\frac{dp_{\perp p}^2}{dt} = -8\pi e^4 n \begin{cases} \dfrac{\Omega}{\omega_e^2 r_p} & \text{at} \quad \dfrac{\omega_e^2 r_p}{\Omega\mu} \gg 1 \\[2ex] \dfrac{1}{2\mu} & \text{at} \quad \dfrac{\omega_e^2 r_p}{\Omega\mu} \ll 1 \end{cases} \tag{2}$$

It is worth noting that the finite duration of a positron-electron interaction is of particular concern in determining the rate of change of the longitudinal momentum of a positron. Indeed, if one considers the effect of collisions of Larmor circles upon their relative motion from minus infinity to plus infinity, then the integral change in the longitudinal momentum of each particle is zero by virtue of the collision symmetry. However, at any finite interaction time t there are always uncompleted collisions with impact parameters $\rho \sim t\mu$; these uncompleted collisions mainly contribute to the exchange of longitudinal momenta. Arguments similar to those mentioned above, when analysing the transverse degrees of freedom, show that the rate of change of the squared longitudinal momentum of a positron is determined by Eq. (1); the only difference is that the screening time turns out to be equal to the period of Langmuir oscillations, i.e.,

$$\frac{dp_{\parallel p}^2}{dt} = -4\pi e^4 n \begin{cases} \dfrac{1}{\pi\omega_e r_p} & \text{at} \quad \dfrac{\omega_e r_p}{\mu} \gg 1 \\ \dfrac{1}{\mu} & \text{at} \quad \dfrac{\omega_e r_p}{\mu} \ll 1 \end{cases} \tag{3}$$

The limit of applicability of Eqs. (2) and (3) is found from the relatively small variation in the longitudinal and transverse momenta during an electron-electron interaction for the characteristic times

$$\tau_{\parallel} \sim (\ell/\beta\gamma c,\ r_e\sqrt{m_e/T_{\parallel}},\ 1/\omega_e)$$

for longitudinal interaction, and

$$\tau_{\perp} \sim (\ell/\beta\gamma c,\ r_e\sqrt{m_e/T_{\parallel}},\ \Omega/\omega_e^2)$$

for transverse interaction.

It is clear from the above formulae that the rate of change of the longitudinal and transverse momenta of the positron at sufficiently strong magnetic fields proves to be inversely proportional to the relative velocity of the Larmor circles. The latter can turn out to be low because of the small variation in longitudinal temperature in the electron beam. This specific feature of electron cooling may seem to be useful to accelerate the positron cooling over the transverse degrees of freedom if the longitudinal spread of the positron velocities is previously deeply cooled.

3. LOW-ENERGY POSITRON STORAGE RING USING ELECTRON COOLING

We shall consider the simplest realization of a storage ring comprising two solenoids. These solenoids are linked by achromatic sections that do not connect radial motion with vertical motion. The magnetic fields in the solenoids are assumed to be equal in magnitude but opposite in direction relative to the velocity of positron motion. Electron cooling is performed in both solenoids. Without going into details of the accelerator scheme, we shall treat the following basic problems:†

a. stability of positron motion in the storage ring with respect to small deviations from the equilibrium particle;

† When we discuss the installations that have electron cooling, we inevitably have to consider storage rings with a longitudinal magnetic field, which has strong influence on the particle dynamics. Similar storage rings have been discussed in Refs. 11 and 12 for electrons and antiprotons, respectively.

b. possible versions of the electron-cooling system; and
c. cooling times and the established phase volume of the cooled positron beam.

Each of these three problems is discussed below.

a. Stability of Positron Motion

The equilibrium orbit will be assumed to coincide with the symmetry axes of both solenoids. The necessary condition for stability of positron motion relative to small deviations from the equilibrium orbit is the equality of the moduli of all the eigenvalues of the transition matrix in a turn. A linear transformation of the variables $X + iY$ and $X' + iY'$ (where X and Y are the transverse displacements relative to the equilibrium orbit; $X' = dX/ds$; $Y' = dY/ds$; and ds is the differential of the arc along the equilibrium orbit) on passage through the sharp-edge solenoid, with adiabatically slow change of its magnetic field, is written as the following matrix:

$$M_{\text{sol}} = \begin{pmatrix} \dfrac{1+e^{-i\alpha}}{2}, & \dfrac{iE\beta(e^{-i\alpha}-1)}{eH_0} \\ \dfrac{-ieH_0(e^{-i\alpha}-1)}{4E\beta}, & \dfrac{1+e^{-i\alpha}}{2} \end{pmatrix}, \quad \alpha = \int \frac{eH(s)}{E\beta}\, ds, \tag{4}$$

where $H(s)$ is the value of the longitudinal magnetic field, E is the energy of a positron, and $\beta = V_{\|}/c$ and H_0 are the initial and final values of the magnetic field in the solenoid. For the storage ring, which consists of two equal solenoids linked to each other by equal sections that do not connect the vertical motion to the horizontal betatron motion, the single-turn transformation matrix is as follows:

$$\begin{aligned} M_{\text{tot}} &= \begin{pmatrix} a, & B \\ c, & d \end{pmatrix} M_{\text{sol}}(H) \begin{pmatrix} a, & B \\ c, & d \end{pmatrix} M_{\text{sol}}(-H) \\ &= \begin{pmatrix} |K|^2 + LM^*, & L^*K + LN^* \\ MK^* + NM^*, & ML^* + |N|^2 \end{pmatrix}, \end{aligned} \tag{5}$$

where

$$K = \frac{a}{2}(1+e^{-i\alpha}) - \frac{iB}{4K_0}(e^{-i\alpha}-1),$$

$$M = \frac{c}{2}(1+e^{-i\alpha}) - \frac{id}{4K_0}(e^{-i\alpha}-1),$$

$$L = iaK_0(e^{-i\alpha}-1) + \frac{B}{2}(1+e^{-i\alpha}),$$

$$N = icK_0(e^{-i\alpha}-1) + \frac{d}{2}(1+e^{-i\alpha}),$$

$K_0 = E\beta/eH_0$. The eigenvalues of Eq. (5) are found from the equation

$$\lambda_{1,2} = \frac{|K|^2 + |N|^2 + ML^* + LM^*}{2} \pm \sqrt{\frac{(|K|^2 + |N|^2 + ML^* + LM^*)^2}{4} - 1}. \quad (6)$$

From Eq. (6) the stability condition of positron motion in the storage ring may be obtained relative to the equilibrium orbit:

$$\left| \frac{a+d}{2} \cos\frac{\alpha}{2} + \left(cK_0 - \frac{B}{4K_0}\right) \sin\frac{\alpha}{2} \right| \leqslant 1. \quad (7)$$

Equation (7) shows that there exists a rather wide class of matrices $\begin{pmatrix} a, & B \\ c, & d \end{pmatrix}$, with $a = d$, $c = -B/4K_0^2$, $a^2 + 4c^2K_0^2 = 1$, at which the motion of positrons will be stable at any values of the parameter α. This choice of the magnetic structure of the storage ring can turn out to be very useful for cooling the positrons, with a large energy spread determining the dispersion of α. Such a consideration of the stability of positron motion does not take into account the perturbations of the storage ring's magnetic structure and the associated resonance effects. We shall dwell on this problem in somewhat greater detail after we present the approximate parameters of the electron-cooling system.

b. Possible Versions of the Electron-Cooling System

Let us consider a possible version of the electron-cooling system that can be easily realized and has a fairly high coefficient of recuperation. We can compare such systems to an electron system with axisymmetric beams with a current of several amperes and an energy of several hundred kilovolts whose perveance is roughly 0.1–1 mcA/V$^{3/2}$. In these systems the coefficient of recuperation can be made higher than 0.95[7] and, hence, the scattered power received by the collector is of the order of several tens of kilowatts.

The optical properties of the electron-beam transport system are unambiguously determined by the requirement of small transverse and longitudinal velocity scatter in the electron flux. Indeed, as follows from the results of Ref. 8, beam transport in a longitudinal magnetic field with the electron source plunged into it seems to be most suitable from this point of view. The electron-velocity scatter in this system will be determined by the cathode temperature, space charge of the beam, and imperfection of the optics. The action of the space charge of the electron beam should have no significant influence on the kinetics of the electron cooling of positrons, because this influence would lead to the same drift velocity for both electrons and positrons.† As for the longitudinal velocity scatter, the contribution of the initial temperature decreases sharply in the case of potential acceleration.[6]

† The energy scatter, which is associated with the presence of the space charge, proves to be negligibly small for the dimension of the positron beam in our case.

An important question among the electron-cooled positron problems is the coincidence of the magnetized electron and positron beams. One method for its solution is the use of an electric field, varying slowly and adiabatically, that is directed perpendicular to a magnetic field. (The authors are indebted to I. N. Meshkov, who pointed out this possibility.) The electrons will drift in the crossed electric and magnetic fields with the same velocity $\vec{V}_{dr} \sim \vec{E}_\perp \times \vec{H}/H^2$, and, therefore, in order for the beams to coincide, the longitudinal velocities of the electrons and positrons must be different. Then the difference in the transverse shifts on the length L_{dr} is $\Delta = L_{dr}E(1/\beta - 1/\beta_e)/H$. At $E_\perp = 15\,\mathrm{kV/cm}$, $H = 3\,\mathrm{kG}$, $L_{dr} = 200\,\mathrm{cm}$. $E_p = 200\,\mathrm{keV}$, and $E_e = 50\,\mathrm{keV}$, we obtain $\Delta = 3.5\,\mathrm{cm}$. The presence of the transverse electric field will result in an energy gradient over the electron-beam cross section and, hence, its deformation. Assuming the parameters indicated above, the difference in the shifts of the diametrically opposite points of the electron beam of 2-mm diameter is about 2 mm. (Note that the diameter of the positron beam in the solenoid is assumed to be less than one mm.) It is noteworthy that the electron beam density remains the same in this case.

One possible realizations of the electron-cooling system can look as follows (see Fig. 1).

In the first section of the system, the electrons are accelerated to several tens of kilovolts and the positrons reduce their energy by the same quantity. In the next section, the beams that have different longitudinal velocities coincide. After this section, the energies of the electrons and positrons become equal and the electron-cooling section follows. Then the electrons decelerate again, and the positrons are accelerated. Further along, the beams are separated in the transverse electric field; the electrons go to the collector, and the positrons restore their energy and leave the solenoid. The transverse electric field, which is employed for beam convergence and separation, gives rise, generally speaking, to the coherent Larmor twisting of both the electrons and positrons. However, the action of this effect proves to be extremely weak for an electric field that varies

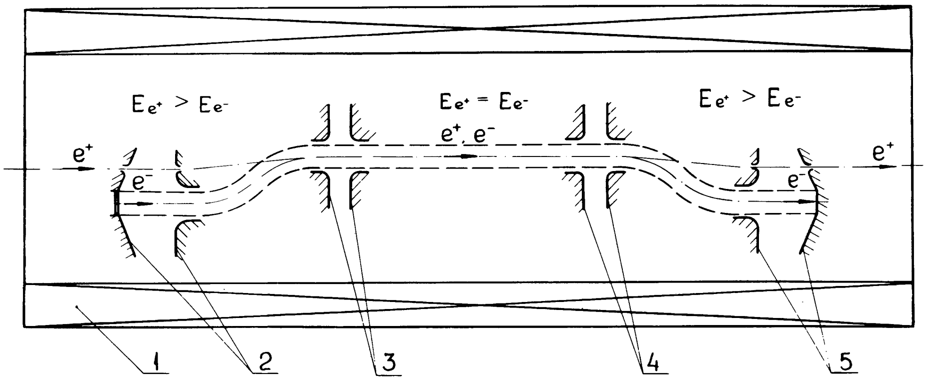

FIGURE 1 Schematic of a possible electron-cooling system. (1) solenoid; (2) electron gun; (2–3) coincidence of e^+ and e^- beams; (3–4) electron cooling of positrons; (4–5) separation of e^+ and e^- beams; (5) recuperator.

slowly and adiabatically with respect to the Larmor rotation, i.e., at $\ell_c \gg \lambda$, where ℓ_c is the characteristic length of variation of the electric field and $\bar{\lambda} = p_{\parallel} c/eH$. So, for $E = 200$ kV and $H = 3$ kG, we obtain $\ell_c \gg 1$ cm.

The availability of kilogauss solenoids of several meters in length leads to high (of the order of several hundreds) effective betatron frequencies, $\nu \sim HL_s/2\pi p_{\parallel}$, where L_s is the length of the solenoid. These high effective frequencies impose rather stringent tolerance requirements on the displacement of some elements of the storage ring, the stability of the power supply system, and the parameters of the positron beam. Indeed, at a certain given value of the nominal $\Delta\nu$, which is much less than unity (the distance between the parametric resonances is equal to 0.5), the relative tolerance $\Delta\nu/\nu$ changes proportionally to ν^{-1}. This change requires an increase in the relative accuracy of all the parameters. It is worth mentioning, however, that the magnetic-field peturbation harmonics of such high order ($n \sim \nu \sim 100$) for the storage ring under consideration, which comprises a relatively small number N of elements ($N \ll 100$), seem to be very weak. This small number must substantially simplify the creation of a similar installation. In principle, this situation is likely to be possible when the power of electron friction will be comparable to even exceed the power of machine resonances.

Let us evaluate the requirements for the stability of the power-supply system of the solenoids and for the parameters of the positron beam at a reasonably permitted shift of betatron frequencies, $\Delta\nu \sim 10^{-1}$. For a storage ring of positrons with energy $E_p \sim 200$ keV, consisting of two solenoids each having field $H = 3$ kG and length $L_s = 5$ m,

$$\sqrt{\left(\frac{\Delta H}{H}\right)^2 + \left(\frac{\Delta p_{\parallel}}{p_{\parallel}}\right)^2} \sim 10^{-3}. \tag{8}$$

This condition is most likely to impose a limitation on the rate of positron accumulation (the magnitude of the captured phase volume). In fact, the stability of the magnetic field can be made better than 10^{-4}, and the relative longitudinal-momentum scatter in the cooled positron beam will be determined, in the case under consideration, by the longitudinal temperature of the electron beam, i.e.,

$$\frac{\Delta p_{\parallel}}{p_{\parallel}} \sim \sqrt{\frac{T_{\parallel}}{m_e c^2 \beta_e^2}}. \tag{9}$$

At $T_{\parallel} \sim e^2 n^{1/3} \sim 10^{-4}$ eV ($n \sim 10^9$ cm^{-3}) this will be roughly equal to 2×10^{-5}.

c. *Cooling Times and Established Phase Volume*

The established phase volume of the cooled positron beam will be determined by the relationship between friction and diffusion. Friction in the storage ring under consideration is due to an interaction of the positrons with the electron beam. Meanwhile, there are a few processes that heat the beam and thereby increase its phase volume. Among them, there is, first of all, an interaction of the positrons with the electron beam itself; in addition, there are scattering and energy losses (fluctuations of losses) caused by interactions with the atoms of a residual gas, noises of various kinds that have a purely external origin, and, finally, a positron-positron interaction inside the stored positron beam itself: incoherent shift of betatron oscillation frequencies and internal scattering. As the estimates show, it is easy to realize the situation where the phase volume of the cooled positron beam of *low intensity* is determined only by the interaction with the electron beam. This means that in the absence of the connections between the longitudinal and transverse motion ($\Psi = 0$) on the cooling section, the longitudinal and transverse temperatures of the cooled positron beam will prove to be equal, corresponding to the longitudinal and transverse temperatures of the electron beam. Here the transverse phase volume† for large cooling times (much larger than the revolution period in the storage ring) will be of the order of $T_\perp/eH$, where H is the longitudinal magnetic field at the electron-cooling section. The relaxation times for this state can be estimated according to the formulae presented in Section 3. For example, for the storage ring and electron-cooling system with parameters $E_p = 200$ keV ($E_p = 100$ keV on the cooling section), $n = 10^9\,\text{cm}^{-3}$, L_c (length of the electron-cooling section) = 6 m, p (perimeter of the storage ring) = 30 m, and $H = 3$ kG, the cooling time of the positron beam with the relative longitudinal-velocity scatter $\mu/\beta c \sim 10^{-3}$ and with angular spread $\theta^2 \sim 10^{-3}$ will be equal to

$$t = \frac{(\gamma\beta)^3\theta^2\mu p}{4\pi\beta c^2 n r_e^2 L_c} \simeq 0.1\text{ sec}, \quad \left(\frac{\omega_e^2 r_p}{\Omega\mu} \cong 10^{-2} \ll 1\right). \tag{9}$$

After cooling, the transverse phase volume of the positrons will reduce by three orders of magnitude to 10^{-6} cm rad. The cooling time, determined by a large transverse phase volume in this case, can be considerably decreased if the damping decrements are redistributed by using the connections between the longitudinal and transverse degrees of freedom. Here it is required, first, to have the dependence of the positron energy losses (longitudinal friction force) on the transverse shift and, second, the availability of the function ψ at the location of the solenoid with electron cooling. The damping times of the transverse (betatron) and longitudinal velocities can then be estimated from the formulae

$$\gamma\frac{dv_\perp^2}{dt} \sim -\frac{4\pi e^4 n}{v_\parallel m_e^2}\left(1 - 2\frac{\psi}{\beta_p c}\frac{\partial v^h}{\partial r}\frac{v_\perp^2}{v_\parallel^2}\right), \tag{10a}$$

$$\gamma\frac{dv_\parallel^2}{dt} \sim -\frac{4\pi e^4 n}{v_\parallel m_e^2}\left(1 + 2\frac{\psi}{\beta_p c}\frac{\partial v^h}{\partial r}\right) \tag{10b}$$

where $v_\perp$ and $v_\parallel$ are the amplitudes of oscillations of the transverse and longitudinal velocities of positrons, respectively, in the co-moving system at the cooling section, ψ is a function averaged over the period of Larmor oscillations, and $\partial v^h/\partial r$ is the gradient of the longitudinal hydrodynamical velocity of the electron beam. Then, at $\psi = 10$ cm, $1/\beta_p c \cdot \partial v^h/\partial r \sim 10^{-3}\,\text{cm}^{-1}$, and at the same

† As for the longitudinal phase volume, we shall consider the nonbunched positron beam. The radiation losses in the storage ring under consideration will be so small that they can be filled in (compensated) by the electron beam.

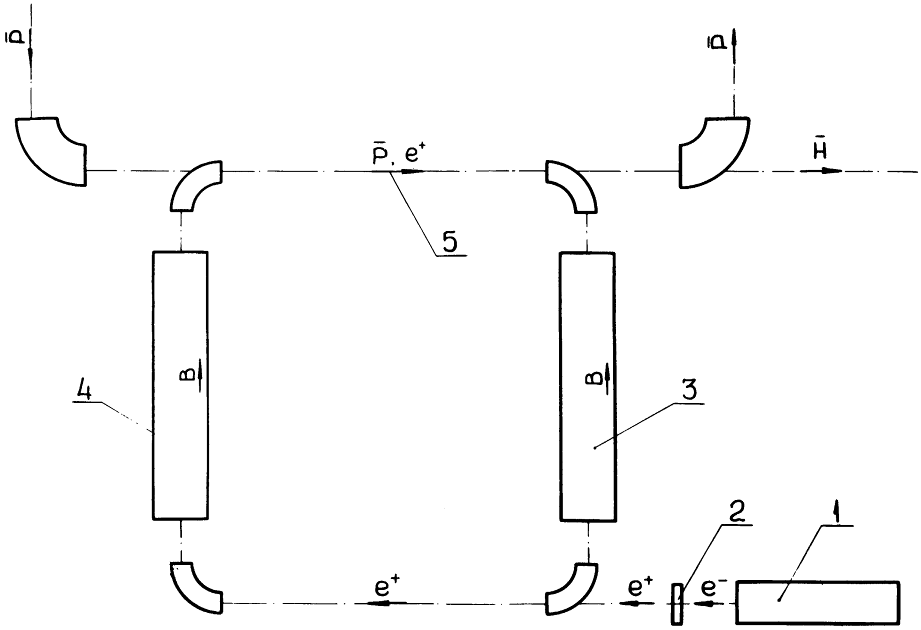

FIGURE 2 Possible scheme for positron storage and antihydrogen production: (1) pulse electron accelerator; (2) converter; (3, 4) solenoids with electron cooling; (5) area for antihydrogen production.

values of the phase volume of the injected positron beam, the damping time will be one order of magnitude smaller.

Figure 2 demonstrates a possible version of the installation scheme intended for positron storage and production of antihydrogen. The positron beam, obtained after conversion, is injected into the storage ring, where the positrons are stored using electron cooling in solenoids 3 and 4. The injection frequency is determined by the damping time and can be about 10 Hz for the above example (without the decrement redistribution). Antihydrogen formation occurs in the straight section 5 of the storage ring where the positron and antiproton beams converge.

4. ANTIHYDROGEN PRODUCTION

Antihydrogen is formed in the straight section of the positron storage ring as a result of the recombination of the e^+ and p^- beams. In our case, the energy of relative positron and and antiproton motion in the co-moving reference system turns out to be much lower than the ionization potential. In this energy range of relative motion, the recombination cross-section is given by the formula

$$\sigma_A \simeq \frac{16\pi\alpha r_e^2 c^2}{3\sqrt{3}v^2}\ln\left(\frac{2I}{m_e v^2}\right).$$

Here r_e is the classical radius of an electron, α is the fine-structure constant, v is the relative velocities in the co-moving reference system, and $I = 13.6\,\text{eV}$ is the ionization potential. Assuming that the e^+ and p^- beams are not bunched, we obtain the following estimate for the rate of antihydrogen production:

$$\frac{dN_A}{dt} = \frac{N_p N_a \ell_r \cdot \min(S_p, S_a)}{\gamma^2 p_p p_a S_p S_a} \sigma_A v, \tag{11}$$

where N_p and N_a are the total number of positrons and antiprotons, respectively, S_p and S_a are the transverse dimensions of the beams, p_p and p_a are the perimeters of the storage rings, and l_r is the length of the recombination section.

The N_p and N_a terms are, in turn, determined by the betatron-frequency shift because of the Coulomb interaction.† In a smooth approximation of the storage-ring magnetic structure, the number of the stored particles is equal to

$$N_{p,a} \sim \frac{2\,|\delta v|\,(\gamma^2-1)\gamma}{r_{p,a}} \varepsilon_{p,a}, \tag{12}$$

where $\delta v \sim 10^{-1}$ is the permitted betatron-frequency shift, γ is a relativistic factor, ε_p and ε_a are the beam emittances, and $r_p(r_a)$ is the classical radius of a positron (antiproton). The maximum possible emittances of the positron and antiproton beams are determined by various aperture limitations (dimension of the storage-ring vacuum chamber and nonlinear machine resonances). These emittances should exceed the values of the emittances established as a result of electron cooling in order that the utilization of cooling make sense, i.e.,

$$\varepsilon_p^{\max} \gg \frac{T_\perp \beta_p^c}{m_p c^2(\gamma^2-1)}, \quad \varepsilon_a^{\max} \gg \frac{T_\parallel \beta_a^c}{m_a c^2(\gamma^2-1)}, \tag{13}$$

where $T_\perp$ and $T_\parallel$ are the effective transverse and longitudinal temperatures of the electron beam, β_p^c and β_a^c are the beta-functions of the positron and antiproton storage rings, respectively, at the locations of electron cooling. Therefore, substituting ε_p, ε_a, the emittance values of the positron and antiproton beams in Eqs. (11) and (12), (these emittances have been established as a result of electron cooling, and the action of the spatial charge of the stored beam is not taken into account), we are likely to obtain an underestimated value of the rate of antihydrogen production. A considerably higher production rate cannot be ruled out.

In the preceding section, the emittance of the positron beam, $\varepsilon_p \sim 10^{-6}$ cm·rad ($N_p \sim 10^6$), has been estimated. For the emittance of antiprotons, we shall use the value $\varepsilon_a \sim 10^{-6}$ cm·rad ($N_a \sim 2 \times 10^9$) obtained in experiments on electron cooling of protons.[4,5]

Then, when the $\beta_p^s = \beta_a^s$-functions at the site of antihydrogen production are 100 cm, the perimeters of storage rings are $p_p = p_a = 30$ m, and the length of the antihydrogen production section is 3 m, we obtain:

$$\frac{dN_A}{dt} \sim 3 \times 10^3 \text{ sec}^{-1} \qquad (\sigma_A \simeq 5 \times 10^{-18} \text{ cm}^2).$$

† This consideration holds for weakly relativistic beams; for ultrarelativistic beams, the maximum possible number of the stored particles is determined by coherent instabilities.

In addition to antihydrogen production, two more processes leading to the loss of positrons occur. These are the production of a positronium with the rate

$$\frac{dN_{ps}}{dt} \sim 5 \times 10^2 \,\text{sec}^{-1} \qquad (\sigma_{ps} = 4\sigma_A)$$

and an annihilation of electron-positron pairs, which takes place mainly via positronium production ($I \gg m_e v^2/2$).

Thus, to produce antihydrogen at the highest possible ($\sim 3 \times 10^3 \,\text{sec}^{-1}$), a $3 \times 10^3 \,\text{sec}^{-1}$ rate of positron storage accumulation should be provided.

The rate of positron accumulation is†

$$N_p = 6 \times 10^{18} J_e K_1 K_2$$

where J_e is the mean current of a linear electron accelerator (A), K_1 is the coefficient of electron-to-positron conversion at an energy of about 0.2 MeV ($\text{MeV}^{-1}\,\text{ster}^{-1}$), K_2 is the capture factor of positrons into a storage ring ($K_2 \simeq \Delta E_p(\text{MeV})\varepsilon_{st}/\varepsilon_{cp}$, where ΔE_p is in the 10^{-3} MeV range of energies covered by this storage ring of positrons), ε_{st} is a two-dimensional transverse phase volume, accepted by the storage ring, of about $10^{-5}\,\text{rad}^2\,\text{cm}^2$, and ε_{cp} is a phase volume of about $1\,\text{rad}^2\,\text{cm}^2$ of the positrons produced as a result of the conversion.

The experimental data available on the electron-to-low-energy-positron conversion[9] make it possible to accept a conversion factor of roughly 2×10^{-3} ($\text{MeV}^{-1}\,\text{ster}^{-1}$) for electrons with an energy of 35 MeV to positrons with an energy of 0.2 MeV.

The estimates indicated above show that a pulsed electron accelerator with the parameters $E_e = 35$ MeV, $J_e = 50$ A, pulse duration of about 100 ns at an injection frequency of about 5 Hz, and damping time of about 0.1 sec, ensure the required rate of positron accumulation.

Thus, the scheme presented here of a positron storage ring enables several thousand antihydrogen atoms to be produced per second. Note that our estimates do not suggest the use of any components of the cooling system with parameters that deviate significantly from those already realized. In view of this, the estimate given of the rate of antihydrogen yield proves to be rather realistic. The development of systems with more stressed parameters and the use of dedicated techniques whose realization will call for additional experimental studies offer the prospect of increasing the antihydrogen yield up to the existing antiproton yield.

5. CONCLUSION

The Coulomb limit on the stored current of positrons and antiprotons is likely to be the strongest limitation on the rate of antihydrogen production. There are two ways to increase the production rate: compensate for the spatial charge and

† The injection scheme presented (see also the Conclusion) is not, of course, the only possible scheme, but it is quite adequate to the formulated problem. The design with a high-energy intermediate storage ring first, seems to be more cumbersome and, second, cannot substantially increase the rate of accumulation, because the peak current in this storage ring will be limited by a Coulomb interaction at a low energy (injection energy to the major storage ring).

increase the kinetic energy of the beams. The current that is ultimately stored will be determined in the first case by various coherent instabilities and, in the second case, by the possibilities of the electron-cooling system. (Creation of this system at comparatively high energies presents known difficulties). So, when the positron energy is increased to 1–2 MeV, the rate of antihydrogen production will rise by two to three orders of magnitude because of an increase of the positron and antiproton currents. As the simplest estimates show, making use of the stimulated recombination permits the rate of antihydrogen production to be increased additionally by one order of magnitude.

To provide such a high rate of positron accumulation ($\sim 10^7$), there are at least two possibilities: using preliminary gas cooling of positrons, and reducing the damping time of positrons during their injection into a storage ring. The operational capability of the gas cooling of positrons has been experimentally proved in Ref. 10; according to the preliminary estimates, its employment in the scheme under consideration can increase the rate of positron accumulation by 3–4 orders of magnitude.

To reduce the damping time (and, hence, to increase the injection frequency), two methods are possible: redistribution of the decrements (this method has been evaluated above), and "sweeping."[2]

"Sweeping" turns out to be very expeditious by virtue of one specific feature of positron cooling in a magnetic field. The cooling rate of both the longitudinal and transverse phase volumes in a rather strong magnetic field proves to be inversely proportional to the relative longitudinal velocity of electrons and positrons. With the use of this method, injection of the positrons into a storage ring may be arranged as follows.† The reinjected hot electrons are cooled very fast (possibly for a few tens of turns) by sweeping over the longitudinal degree of freedom, ultimately down to temperatures of the order of $e^2n^{1/3}$. After this sweeping, an intense cooling of the transverse phase volume occurs determined by very small relative longitudinal velocities of electrons and positrons. Furthermore, this cooling is intensified by the decrement redistribution. With this method, one can achieve the damping of injected positrons with an energy spread $\Delta E/E \sim 10^{-3}$ and one-dimensional transverse phase volume $\varepsilon_p \sim 3 \times 10^{-3}$ cm rad in a few hundred turns and thus achieve a multifold increase of the injection frequency and the rate of positron accumulation.

Although the above arguments are in need of further thorough analysis, then allow one to hope for the creation of a device whose antihydrogen output is 10^7–10^8 atoms per second. It cannot be ruled out that a similar source of the monochromatic positron beam could be employed for the solution of a number of research problems.

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† We would like to note that "sweeping" is worth performing only when the oscillations of the longitudinal positron velocity, which are associated with a finite transverse phase volume, are very small, $\Delta v_{\parallel} \sim (e^2n^{1/3}/m_e)^{1/2}$. This nature of particle motion in a storage ring is possible (within the known accuracy) if the matrix of transition between solenoids is unitary.

REFERENCES

1. G. I. Budker, *At. Energ.* **22,** (1967).
2. Ya. S. Derbenev and A. N. Skrinsky, *Sov. Phys. Rev.* **3,** 165.
3. G. I. Budker and A. N. Skrinsky, *Sov. Phys. Usp.* **124,** 561 (1978).
4. A. S. Artamonov and Ya. S. Derbenev, *Electron Cooling of Positrons* (in press).
5. G. I. Budker *et al., Particle Accelerators* **7,** 197 (1976).
6. G. I. Budker, *et al.* in Proc. 5th All-Union Conf. on Charged-Particle Accelerators, Vol. 1, p. 236 (Dubna, 1977).
7. A. N. Sharapa, author's abstract of dissertation, Novosibirsk (1982).
8. V. I. Kudelainen, I. N. Meshkov and R. A. Salimov, *Sov. Phys. Tech. Phys.* **41,** 2294 (1971).
9. R. E. Sund and R. B. Walton, *Nucl. Instrum. Methods* **27,** 109 (1964).
10. B. P. Sannikov, preprint IYaF SO AN SSSR, No. 72-33, Novosibirsk.
11. Ya. S. Derbenev, in Proc. of Workshop on the Cooling of High Energy Beams, (University of Wisconsin, 1978).
12. T. A. Vsevologskaya *et al.,* in Proc. 7th All-Union Conf. on Charged-Particle Accelerators, Vol. 1, p. 229 (Dubna, 1981).