HORIZONTAL FIELDS IN A RING WITH ONE SIBERIAN SNAKE AND A VERTICALLY POLARIZED BEAM

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(Received June 8, 1987)

We show that a single Type 1 Siberian snake, such as a solenoid, is able to eliminate the depolarization caused by certain harmonics of horizontal fields when a beam is vertically polarized. However, the corresponding depolarization due to the orthogonal set of harmonics cannot be eliminated by a single Type 1 snake. The two orthogonal sets of horizontal magnetic fields are respectively in phase with radial and longitudinal fields at the point in the ring opposite the snake.

1. INTRODUCTION

An important problem in accelerating polarized proton beams to high energies is the large number of depolarizing spin resonances that must be crossed. At the SSC, there will be about 36,000 depolarizing resonances; thus it appears impractical to use the jumping and correction techniques developed at the ZGS[1] and AGS.[2] A more promising method of overcoming so many depolarizing resonances is to use Siberian snakes.[3] Siberian snakes would, in principle, eliminate the effects of all depolarizing resonances by making the spin tune ν_0 independent of energy; specifically ν_0 would be 1/2. Therefore Siberian snakes have generated considerable theoretical interest, but, during the last 12 years, no experimental test of the snake concept has yet been performed.

The original analysis[3] and most subsequent theoretical work[4] on snakes assumes that the polarization vector $\mathbf{P}$ is periodic around the ring, i.e.,

$$\mathbf{P}(\theta + 2\pi) = \mathbf{P}(\theta), \tag{1}$$

where θ is the ring azimuth angle. For a ring with one snake, this periodicity can only occur when $\mathbf{P}$ is horizontal and can therefore precess in the ring bending magnets. However for some experiments vertical polarization is more desirable. By using pairs of snakes, it may be possible to keep $\mathbf{P}$ always vertical, while changing direction between up and down in traversing each snake. This two-snake scheme seems to be the most desirable solution for the SSC.

In this paper we study a ring with only one snake and a vertically polarized beam, so that $\mathbf{P}$ alternates between up and down upon consecutive turns. We are studying this case because we wish to consider possible near-term experimental tests of the snake concept[5,6] with $\mathbf{P}$ vertical. For various practical reasons one can

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use only a single Type 1 snake; thus we must ask if a single snake and a vertical **P** will allow valid tests of the Siberian snake concept. In this paper we shall study the effect of a single Type 1 snake upon vertical spin polarization. We find that the effect is very different depending upon whether the horizontal magnetic fields are in phase with a radial or longitudinal field at the position opposite the snake.

2. BASIC MODEL

We begin by defining a model of a horizontal storage ring. The ring contains one solenoid, which rotates a particle's spin by 180° around the longitudinal axis. This solenoid is equivalent to a Type 1 Siberian snake. In the absence of a snake, the spin of a particle on the closed orbit will precess around the vertical axis, $\hat{y}$, with a frequency $2\pi\nu_0$ radians per circumference. It is easily verified that the spin tune ν_0 is equal to $G\gamma$, where γ is the particle energy in units of rest mass and $G \equiv (g-2)/2$, where g is proportional to the particle's magnetic moment.

Some details will not be discussed in this paper. We will ignore the rotation and focusing of the particles' betatron orbits caused by the solenoid spin rotator. These effects can be compensated by a number of quadrupole magnets,[6] which will also affect the ring structure. Moreover, it may be difficult to ramp the solenoid magnetic field to exactly match the ramping of the beam energy; thus the solenoid spin rotation angle may not be exactly 180°.

We will use the coordinate system with the orbit frame $Oxyz$. As shown in Fig. 1, O is the location of a particle on the closed orbit, $\hat{x}$ is pointing radially inward, $\hat{y}$ is vertical, and $\hat{z}$ is longitudinal. We can solve the Thomas-BMT equation[7] for spin motion in this frame, without the snake and without depolarizing resonances. The resulting equation is

$$\frac{d\mathbf{s}}{d\theta} = \Omega\hat{y} \times \mathbf{s}, \tag{2}$$

where $\oint \Omega \, d\theta = 2\pi\nu_0$ is the total spin rotation caused by the vertical ring magnetic field in one turn around the orbit. Equation (2) has three linearly

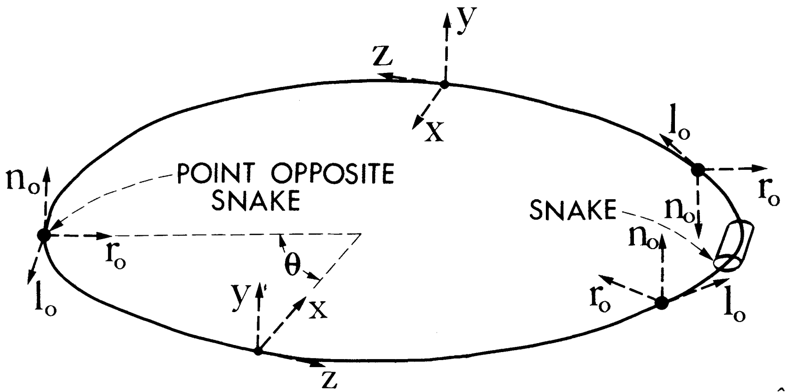

FIGURE 1 Horizontal ring with a Siberian snake at $\theta = \pi$ and the rotating frame $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$, which is defined at the point opposite the snake ($\theta = 0$) to be $\hat{r}_0 = \hat{x}$ (radial), $\hat{n}_0 = \hat{y}$ (normal), and $\hat{l}_0 = \hat{z}$ (longitudinal). Typical orientations of the rotating frame also shown just before and just after passing through the snake at the end of the first turn.

independent solutions, such as $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$, which is a right-handed orthonormal triad that precesses along with the spinning particle. If we choose $\hat{n}_0$ to point along the vertical (normal) direction $\hat{y}$, then $\hat{l}_0$ and $\hat{r}_0$ clearly precess in the horizontal plane with spin tune frequency $\nu_0 = G\gamma$. We will define the frame by choosing the initial conditions, $\hat{r}_0 = \hat{x}$ (radial) and $\hat{l}_0 = \hat{z}$ (longitudinal) at $\theta = 0$, which is the point opposite the snake. The snake produces a spin rotation of 180° around $\hat{z}$ at $\theta = \pi$ as shown in Fig. 1. With these initial conditions, it can be shown that $\hat{l}_0$ points along the stable polarization direction at every point around a ring with a Type 1 Siberian snake at $\theta = \pi$.

Notice that the snake rotates both the spin and the $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$ frame by 180° about the longitudinal direction. Thus, on the second turn around the ring $(\pi < \theta < 3\pi)$, $\hat{r}_0$ and $\hat{n}_0$ will both point opposite to their direction on the first turn. The third turn will be identical to the first, and the fourth turn will be identical to the second. Thus the Siberian snake changes the ring's spin periodicity from 2π to 4π, both by making the spin tune 1/2 and by making the periodicity of the $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$ frame 4π. Let us denote the horizontal imperfection fields by $\boldsymbol{\epsilon}(\theta)$. Then in the rotating frame $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$, the Thomas-BMT equation has the simple form

$$\frac{d\mathbf{s}}{d\theta} = \boldsymbol{\epsilon}(\theta) \times \mathbf{s}. \tag{3}$$

In this frame the spin will precess around the horizontal fields.

3. INSTABILITIES

If we assume that the spin is initially vertical, we can study the spin evolution caused by the horizontal imperfection field

$$\boldsymbol{\epsilon}(\theta) = \epsilon_x(\theta)\hat{x} + \epsilon_z(\theta)\hat{z}. \tag{4}$$

On the first turn around the ring, $\boldsymbol{\epsilon}(\theta)$ has components in the rotating frame $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$ given by

$$\boldsymbol{\epsilon}(\theta) \equiv (\epsilon_r^1, 0, \epsilon_l^1) = [\epsilon_x \cos(G\gamma\theta) - \epsilon_z \sin(G\gamma\theta), 0, \epsilon_x \sin(G\gamma\theta) + \epsilon_z \cos(G\gamma\theta)], \tag{5}$$

while on the second turn $\boldsymbol{\epsilon}(\theta)$ is given by

$$\boldsymbol{\epsilon}(\theta) \equiv (\epsilon_r^2, 0, \epsilon_l^2) = (-\epsilon_r^1, 0, \epsilon_l^1). \tag{6}$$

We will now calculate the spin evolution in the $\{\hat{r}_0, \hat{n}_0, \hat{l}_0\}$ frame as a particle makes two turns around the ring. Assume that the spin is initially vertical ($\mathbf{s} = \hat{y}$) at $\theta = -\pi$. For the first turn around the ring we then have, using Eq. (3) and keeping only first-order terms in $\boldsymbol{\epsilon}$,

$$\begin{aligned} \mathbf{s}(\theta = \pi) &= (0, 1, 0) + \int_{-\pi}^{\pi} \frac{d\mathbf{s}}{d\theta} d\theta \\ &\simeq (0, 1, 0) + \left[\int_{-\pi}^{\pi} \epsilon_l^1(\theta)\, d\theta, 0, -\int_{-\pi}^{\pi} \epsilon_r^1(\theta)\, d\theta\right]. \end{aligned} \tag{7}$$

The spin motion in the second turn around the ring is given by

$$
\begin{aligned}
\mathbf{s}(\theta = 3\pi) &= \mathbf{s}(\theta = \pi) + \int_{\pi}^{3\pi} \frac{d\mathbf{s}}{d\theta}\, d\theta \\
&= \left[\int_{-\pi}^{\pi} \epsilon_l^1(\theta)\, d\theta,\ 1,\ -\int_{-\pi}^{\pi} \epsilon_r^1(\theta)\, d\theta\right] \\
&\quad + \left[\int_{-\pi}^{\pi} \epsilon_l^1(\theta)\, d\theta,\ 0,\ \int_{-\pi}^{\pi} \epsilon_r^1(\theta)\, d\theta\right] \\
&= \left[2\int_{-\pi}^{\pi} \epsilon_l^1(\theta)\, d\theta,\ 1,\ 0\right].
\end{aligned} \tag{8}
$$

In general, successive pairs of turns will tilt the spin increasingly away from the vertical direction. However, it is important to notice that only the integral of the ϵ_l component of the imperfection field,

$$E_l \equiv \int_{-\pi}^{\pi} \epsilon_l^1(\theta)\, d\theta = \int_{-\pi}^{\pi} [\epsilon_x(\theta) \sin(G\gamma\theta) + \epsilon_z(\theta) \cos(G\gamma\theta)]\, d\theta, \tag{9}$$

contributes to this depolarization, whereas the integral of the ϵ_r component,

$$E_r \equiv \int_{-\pi}^{\pi} \epsilon_r^1(\theta)\, d\theta = \int_{-\pi}^{\pi} [\epsilon_x(\theta) \cos(G\gamma\theta) - \epsilon_z(\theta) \sin(G\gamma\theta)]\, d\theta, \tag{10}$$

is cancelled by successive turns. Note that ϵ_l is in phase with a longitudinal field at the position opposite the snake while ϵ_r is in phase with a radial field at the same $\theta = 0$ position opposite the snake. A similar analysis can be done for a single Type 2 snake, which rotates the spin by 180° around the radial direction $\hat{x}$ rather than the longitudinal direction $\hat{z}$. The effects are then reversed: E_r causes depolarization, whereas E_l does not.

Let us now evaluate the integral in Eq. (9) for the simple case of a localized imperfection $\boldsymbol{\epsilon} = (\alpha\hat{x} + \beta\hat{z})\delta(\theta - \theta_0)$ at $\theta = \theta_0$, where α and β are constants. We obtain

$$E_l \equiv \int_{-\pi}^{\pi} \epsilon_l^1(\theta)\, d\theta = \alpha \sin(G\gamma\theta_0) + \beta \cos(G\gamma\theta_0). \tag{11}$$

Depending on the exact value of $G\gamma\theta_0$, the magnitude of E_l is typically in the range $|E_l|^2 \simeq \frac{1}{2}(\alpha^2 + \beta^2)$. Thus the tilt of the spin away from the vertical increases by approximately $[2(\alpha^2 + \beta^2)]^{1/2}$ radians every two turns.

4. CONCLUSION

In a ring with a vertically polarized beam and a single Siberian snake, depolarization will in general occur if the imperfections are distributed randomly around the ring. However, it may be possible to maintain vertical polarization if E_l, which is the integral of the horizontal fields in phase with a longitudinal field at $\theta = 0$, can be made to vanish. A Type 1 snake could then eliminate the depolarization due to the remaining horizontal fields in phase with a radial field at the $\theta = 0$ position opposite the snake.

This work was supported by a research grant from the U.S. Department of Energy. We would like to thank E. D. Courant, S. Y. Lee, R. E. Pollock, L. G. Ratner, L. C. Teng, and K. M. Terwilliger for their comments.

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