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# EFFECTS OF RADIATION DAMPING ON BEAM QUALITY IN THE INVERSE FREE-ELECTRON LASER ACCELERATOR

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The effect of synchrotron radiation damping on the transverse beam emittance for an inverse free-electron laser (IFEL) accelerator is studied. A beam envelope equation is derived and solved for an arbitrarily tapered wiggler field. The expression for the evolution of the normalized transverse beam emittance is derived and found to decrease exponentially with distance, due to radiation damping, until it is limited by quantum excitation. Our results show that for acceleration distances comparable to the radiation damping $e$-folding length, substantial improvements in the beam quality can be realized.

## 1. INTRODUCTION

Electron beam quality as measured by the transverse emittance is usually determined by the gun and propagation configurations in accelerators. Under idealized conditions, the transverse normalized beam emittance remains a constant of motion as the beam propagates through the accelerator. Therefore, to improve the quality of the beam, it is necessary to decrease the beam emittance at the injection point. However, since the normalized beam emittance is essentially the transverse area in phase space for the collection of beam particles, one can in principle reduce the emittance if a dissipative mechanism is introduced. A natural candidate for such a dissipation mechanism is the induced synchrotron radiation damping due to the transverse motion of the particles in an external periodic transverse magnetic field. It is on this mechanism that we will focus when the external magnetic field is chosen to be a helical wiggler field. Since this radiation damping effect is small at low energies, it is in the context of the recently proposed high energy IFEL accelerators[1–11] that we will concentrate in this paper.

We begin by obtaining the electron orbits in an IFEL accelerator. A fully relativistic formulation of the equations of motion that include radiation damping force is considered. The damping coefficients are obtained from the transverse dynamics of the particles, whereas the axial dynamics describe the acceleration of the particles. In Section 3, we derive a relativistic envelope equation for the average radius of the electron beam, assuming continuous emission of the synchrotron radiation. It is apparent from this envelope equation that the normalized transverse emittance decays exponentially at a rate given by the radiation damping coefficient. The envelope equation is solved using a WKB method in Section 4, and the spatial evolution of the beam radius is obtained. Quantum excitation sets a minimum value on the normalized transverse emittance in an IFEL accelerator, and it is derived in Section 5. Strong focusing is found to be necessary to reduce such a minimum to an acceptable value. An example is given in Section 6 for a set of proposed IFEL accelerator parameters.[2] It is found that radiation damping does reduce the emittance of the accelerated electron beam while resulting in an insignificant loss in particle energy.

## 2. SINGLE-PARTICLE DYNAMICS

We shall consider the motion of an electron under the influence of a helical wiggler field and a circularly polarized electromagnetic wave with the inclusion of the radiation reaction force. The fully relativistic equation of motion is[12]

$$\frac{d\mathbf{p}}{dt} = -|e|\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) + \mathbf{F}^R, \tag{1}$$

where

$$\mathbf{F}^R = \tau_R \left\{ \frac{\mathbf{p}}{m_0^2 c^2} \left[ \left( \frac{\gamma^2 - 1}{\gamma} \right) \left( \frac{d\,|\mathbf{p}|}{dt} \right)^2 - \gamma \left( \frac{d\mathbf{p}}{dt} \right)^2 \right] + \frac{d}{dt} \left( \gamma \frac{d\mathbf{p}}{dt} \right) \right\}$$

is the radiation damping force, $\tau_R = 2\,|e|^2/3m_0c^3$, and $\gamma^2 = 1 + |\mathbf{p}|^2/m_0^2c^2$. The radiation field is given by its vector potential $\mathbf{A}_L = A_L(\cos\phi\hat{e}_x - \sin\phi\hat{e}_y)$, where $\phi = kz - \omega t$. We shall assume $z$ dependence for both the magnitude and period of the wiggler field. The vector potential $\mathbf{A}_w$ for the helical wiggler field is given by $\mathbf{A}_w = A_w[\cos\theta\hat{e}_x + \sin\theta\hat{e}_y]$, where $A_w = A_w(z)$ and $\theta = \int_0^z k_w(z')\,dz'$. The requirement that the wiggler field satisfies both $\nabla \cdot \mathbf{B}_w = 0$ and $\nabla \times \mathbf{B}_w = 0$ introduces transverse variation as well as a nonzero $z$ component of the magnetic field.[13]

Since we shall be primarily interested in laser-driven accelerators, the normalized wigger field strength $a_w = |e|\,A_w/m_0c^2$ is assumed to be much greater than the corresponding quantity $a_L = |e|\,A_L/m_0c^2$ for the radiation; i.e., $a_w \gg a_L$. It can then be shown that the major contribution to the radiation damping is from the wiggler field.

We shall first look at the radiation damping term in Eq. (1). By neglecting the transverse dependence of the wiggler field for a beam that is confined sufficiently close to the axis, we have the immediate consequence that the canonical momenta in the $x$ and $y$ directions are constants of motion and may be chosen to be zero. The mechanical momenta are then given by $p_x = \frac{|e|}{c} \mathbf{A}_T \cdot \hat{e}_x$, $p_y = \frac{|e|}{c} \mathbf{A}_T \cdot \hat{e}_y$, where $\mathbf{A}_T = \mathbf{A}_w + \mathbf{A}_L$. Also, in the zeroth-order approximation, the total relativistic energy is conserved, which leads to $\dot{\gamma} = 0$ and $\dot{p}_z = 0$. Therefore, the only significant term remaining in the radiation reaction force is

$$\mathbf{F}^R = \tau_R \gamma \left[ \frac{d^2\mathbf{p}}{dt^2} - \frac{\mathbf{p}}{m_0^2 c^2} \left( \frac{d\mathbf{p}}{\mathrm{dt}} \right)^2 \right].$$

Neglecting terms of order $a_L/a_w \ll 1$, the components of the radiation reaction force are $F_x^R = -\nu_\perp c p_x$, $F_y^R = -\nu_\perp c p_y$, $F_z^R = -\nu_\parallel c p_z$, where

$$\nu_\perp = \tau_R \gamma k_w^2 c (a_w^2 + 1) \tag{2a}$$

and

$$\nu_\parallel = \tau_R \gamma k_w^2 c a_w^2 \tag{2b}$$

are, respectively, the spatial decay coefficients due to radiation damping in the transverse and axial directions. Note that $\nu_\perp \approx \nu_\parallel$ for $a_w^2 \gg 1$, which is the case in the IFEL accelerator.

The most significant feature of the transverse motions of the electrons is the betatron oscillation caused by either the inhomogeneity of the wiggler field in the transverse plane or other focusing mechanisms. It can be shown that, for small oscillations about the axis of the wiggler field, the transverse equations of motion are

$$\frac{d^2x}{dz^2} + K_B^2 x = -\left( \frac{\gamma'}{\gamma} + \nu_\perp \right) \frac{dx}{dz} \tag{3a}$$

and

$$\frac{d^2y}{dz^2} + K_B^2 y = -\left( \frac{\gamma'}{\gamma} + \nu_\perp \right) \frac{dy}{dz}, \tag{3b}$$

where $d/dt \simeq v_z \partial/\partial z$, $v_z \simeq c$, $\gamma' \equiv \partial\gamma/\partial z$ have been used, and $K_B$ is the wave number of the longitudinal betatron oscillation. For betatron oscillations that are originated from the $\mathbf{v} \times \mathbf{B}$ force due to the nonzero magnetic field in the $z$ direction of the realizable wiggler field,[13] $K_B = a_w k_w / (\sqrt{2}\gamma)$.

The axial motion of the electron is governed by

$$\frac{dp_z}{dz} = \gamma m \frac{dv_z}{dz} + m v_z \frac{d\gamma}{dz} = -\frac{|e|}{c^2} (\mathbf{v} \times \mathbf{B})_z - \nu_\parallel p_z, \tag{4}$$

where

$$\frac{d\gamma}{dz} = \frac{-|e| \mathbf{v} \cdot \mathbf{E}}{m_0 c^3} - \frac{(\nu_\perp p_\perp^2 + \nu_\parallel p_\parallel^2)}{m_0^2 c^3 \gamma}.$$

It is straightforward to show that the axial electron acceleration is

$$\frac{dv_z}{dz} = -\frac{c}{2\gamma^2}\frac{\partial a_w^2}{\partial z} + \frac{2a_w a_L k_w c}{\gamma^2}\sin\psi - \frac{2k_w}{k}v_{\|}v_z + \frac{3v_{\perp}a_L a_w}{\gamma^2}\cos\psi, \tag{5}$$

where $\psi = \theta + \phi = \int_0^z [k + k_w(z') - \omega/v_z(z')]\,dz' + \psi_0$ is the phase between the electrons and the ponderomotive wave generated by the beating between the radiation and wiggler fields, and $\psi_0$ is the initial phase at the entrance of the interaction region. Equation (5) can be transformed into the following pendulum equation:

$$\frac{d^2\psi}{dz^2} = \frac{dk_w}{dz} - \frac{k}{2\gamma^2}\frac{\partial a_w^2}{\partial z} + \frac{2a_w a_L k k_w}{\gamma^2}\sin\psi - \frac{2v_{\|}k_w}{c} + \frac{3v_{\perp}a_L a_w k}{c\gamma^2}\cos\psi. \tag{6}$$

The rate of change of relativistic energy may be obtained from Eq. (4) and is

$$\frac{d\gamma}{dz} = \frac{a_L a_w k}{\gamma}\sin\psi - v_{\|}\gamma + \frac{v_{\perp}a_L a_w}{\gamma}\left(\frac{k}{k_w} - 2\right)\cos\psi - \frac{v_{\|}}{\gamma}(a_w^2 + a_L^2). \tag{7}$$

Equations (3), (6), and (7) will be the basic equations we shall use in studying the effects on beam quality due to radiation damping. The terms containing $\cos\psi$ in Eqs. (5), (6), and (7), as well as the last term in Eq. (7), may be neglected when the conditions $a_w^2 \gg a_L^2$, $a_w^2 \gg 1$, $k \gg k_w$, and $\gamma^2 \gg 1$ are satisfied. These conditions are easily achieved in high-energy IFEL accelerators.

## 3. DERIVATION OF ENVELOPE EQUATION WITH RADIATION DAMPING

The single-particle equations of motion that we have developed in the last section will enable us to study the macroscopic behavior of the beam. This is accomplished by considering the evolution of various averaged quantities associated with the single-particle variables.[14,15] We begin by multiplying Eq. (3a) by $x'$ and $x$, and Eq. (3b) by $y'$ and $y$, where the prime denotes $\partial/\partial z$. Combining the resulting equations yields the following set of equations:

$$\frac{1}{2}\frac{d}{dz}\beta_{\perp}^2 + \frac{K_B^2}{2}\frac{d}{dz}r^2 = -\mu\beta_{\perp}^2, \tag{8a}$$

$$\frac{1}{2}\frac{d^2}{dz^2}r^2 - \beta_{\perp}^2 + K_B^2 r^2 = -\frac{\mu}{2}\frac{d}{dz}r^2, \tag{8b}$$

$$\frac{dl}{dz} = -\mu l \tag{8c}$$

where $r^2 = x^2 + y^2$, $\beta_{\perp}^2 = x'^2 + y'^2$, $\mu = \gamma'/\gamma + v_{\perp}$, and $l = (x'y - y'x)$ is the normalized angular momentum. We eliminated $\beta_{\perp}^2$ by substituting Eq. (8b) into Eq. (8a). By taking transverse ensemble averages over beam particles in Eq. (8), and denoting the ensemble average of $r^2$ by $a^2 = \langle r^2 \rangle$, we obtain an equation that governs the evolution of the root-mean-square radius of the electron beam:

$$\mu^2 \frac{d}{dz} a^2 + \mu \frac{d^2}{dz^2} a^2 + 2\mu K_B^2 a^2 + \frac{d}{dz}\left(\frac{\mu}{2}\frac{d}{dz} a^2\right) + \frac{1}{2}\frac{d^3 a^2}{dz^3} + \frac{d}{dz}(K_B^2 a^2) + K_B^2 \frac{d}{dz} a^2 = 0. \tag{9}$$

It is easy to show that the integration factor for Eq. (9) is $g^2 a^2$, where $g^2 = \gamma^2 \exp(2 \int_0^z \nu_\perp \, dz')$. Equation (9) can now be put into the form $d/dz[g^2(a^3 a'' + \mu a^3 a' + a^4 K_B^2)] = 0$ and can be integrated to give $g^2(a^3 a'' + \mu a^3 a' + a^4 K_B^2) = H^2$, where $H^2$ is a constant of motion associated with the beam. It can be shown, using the following representation for the particles' normalized transverse velocities,[14]

$$\boldsymbol{\beta}_\perp = \frac{a'}{a} r \hat{e}_r + \frac{Lr}{a^2} \hat{e}_\theta + \delta \boldsymbol{\beta}_\perp,$$

where $\delta\boldsymbol{\beta}_\perp$ is the normalized transverse velocity spread and $L = \langle l \rangle$ from Eq. (8c), that the constant $H^2$ is given by

$$H^2 = \gamma^2(0) L^2(0) + \gamma^2 a^2 \langle \delta \boldsymbol{\beta}_\perp|^2 \rangle \exp\left(2 \int_0^z \nu_\perp \, dz'\right),$$

where $\gamma(0) = \gamma(z=0)$ and $L(0) = L(z=0)$. We may therefore define the squared normalized beam emittance[14,16] as $\varepsilon_n^2(z) = \gamma^2 a^2 \langle |\delta \boldsymbol{\beta}_\perp^2|^2 \rangle$ and arrive at the following envelope equation

$$\frac{d^2 a}{dz^2} + \left(\frac{1}{\gamma}\frac{d\gamma}{dz} + \nu_\perp\right)\frac{da}{dz} + K_B^2 a - \frac{[\varepsilon_n^2(z) + \gamma^2 L^2(z)]}{\gamma^2 a^3} = 0. \tag{10}$$

The spatial dependence of the normalized emittance and average angular momentum, respectively, are given by

$$\varepsilon_n(z) = \varepsilon_n(0) \exp\left(-\int_0^z \nu_\perp \, dz'\right), \tag{11a}$$

$$L(z) = (\gamma(0)/\gamma) L(0) \exp\left(-\int_0^z \nu_\perp \, dz'\right), \tag{11b}$$

where $\varepsilon_n(0) = \varepsilon_n(z=0)$. Equation (10), together with Eqs. (11a,b), constitute the beam envelope equation with radiation damping terms included.

One can see that when $\nu_\perp = 0$ Eq. (11a) shows that $\varepsilon_n$ remains constant, and Eq. (10) reduces to the usual relativistic beam-envelope equation, where $\varepsilon_n$ is the familiar normalized beam emittance.[14,16] Therefore, in the presence of radiation damping, the root-mean-square beam radius is still described by an envelope equation, but the normalized beam emittance is no longer constant but decays exponentially according to Eq. (11a). However, the decay of the normalized beam emittance will eventually be limited by quantum excitation due to the discrete nature of the synchrotron radiation. It is shown in a later section that when an equilibrium is reached between these two competing processes, the minimum normalized emittance achievable through radiation damping in the IFEL accelerator is given by $(\varepsilon_n)_{\min} \approx 3\hbar a_w^3 k_w/(\sqrt{2} m_0 c K_B)$.

In the presence of radiation damping, the average angular momentum also decays exponentially, as given by Eq. (11b). However, one may choose $L(0) = 0$ for beam-generation schemes that do not impart an average angular momentum to the electron beam, i.e., zero magnetic field at the cathode. We shall assume that this is the case in our study of beam quality. We shall also not distinguish between $\nu_\perp$ and $\nu_\parallel$ and will denote both by $\nu$.

## 4. EVOLUTION OF BEAM RADIUS

The equation for the root-mean-square radius $a$ in Eq. (10) is nonlinear. It is found, however, that the mean-square radius $a^2$ satisfies Eq. (9), which is a linear differential equation. For beam focusing provided by the wiggler, Eq. (9) may be solved exactly for untapered wiggler fields when $\gamma' = 0$. If $\gamma' \neq 0$ or when the tapering is known, it can be solved using a WKB method if we assume the coefficients are slowly varying. Equation (9) can be simplified in certain limits of accelerator designs to facilitate analytical study. It can be shown that $\gamma'/\gamma \ll K_B$ and $\nu \lesssim K_B$, which allow us to arrive at the following appropriate equation:

$$S''' + 3\mu S'' + 4K_B^2 S' + [4\mu K_B^2 + 2(K_B^2)']S = 0, \tag{12}$$

where $S = a^2$.

In order to obtain net acceleration of the electrons trapped in the ponderomotive potential, the wiggler field must be spatially tapered. In such a case, the envelope equation, Eq. (12), is a linear differential equation with spatially dependent coefficients. We solved it by using the WKB method, which assumes these coefficients to be slowly varying functions of longitudinal distance. By assuming both $K_B'/K_B$ and $\mu \ll K_B$, the general solution to Eq. (12) is found to be

$$S = e^{-M} \frac{K_B(0)}{K_B(z)} (A + B \cos 2\Sigma + C \sin 2\Sigma),$$

where $M = \int_0^z \mu(z')\, dz'$, and $\Sigma = \int_0^z K_B(z')\, dz'$. The coefficients $A$, $B$, and $C$ can be found by using the initial conditions for a matched beam, $a(z=0) = a_0$, $a'(z=0) = 0$, $a''(z=0) = 0$. The matched-beam radius $a_0$ is related to the initial transverse emittance $a_0^4 = \varepsilon_n^2(0)/[K_B^2(0)\gamma^2(0)]$. Using the initial conditions, we arrive at the following expression for the root-mean-square beam radius:

$$a = a_0 e^{-M/2} \left[\frac{K_B(0)}{K_B(z)}\right]^{1/2} \left[1 + \frac{\mu(0) + K_B'(0)/K_B(0)}{2K_B(0)} \sin 2\Sigma\right]^{1/2}. \tag{13}$$

Equation (13) shows that the beam radius does not remain constant even when the beam is matched at injection. In addition to the exponential decay from the radiation damping, the beam envelope develops an induced betatron oscillation. However, the normalized emittance is just an exponential decay given by Eq. (11a).

We may gain some insight into the general effect of radiation damping on the transverse emittance by studying Eq. (12) in the case of untapered wiggler field. We shall first consider the case where $\gamma' = 0$. This could be the situation when the acceleration mechanism is saturated by the radiation damping, and the beam energy is constant. The evolution of the beam radius is then given by the appropriate limit of Eq. (13). Since there is no tapering of the wiggler, the solution is exact and given by

$$a = a_0 e^{-\nu z/2}\left(1 + \frac{\nu}{2K_B}\sin 2K_B z\right)^{1/2}.$$

The beam radius again exponentially decays with an induced betatron oscillation. Since $\gamma$ is constant, the damping rate $\nu$ is constant, and the normalized emittance $\varepsilon_n$ is given by $\varepsilon_n(z) = \varepsilon_n(0)\exp(-\nu z)$.

Next, we consider the situation when an accelerated beam is cooled by passing it through an untapered external wiggler field. Since the beam decelerates due to the synchrotron radiation damping, we have $\gamma'/\gamma = -\nu$. This gives $\mu = 0$, and, since $K_B = a_w k_w/(\sqrt{2}\gamma)$, the betatron wave number $K_B$ is a function of $z$. The spatial dependence of $\gamma$ can be evaluated using $\gamma'/\gamma = -\nu$, and Eq. (13) reduces to $a = a_0(1 + \nu_0^2 z^2)$, where $\nu_0 = \tau_R a_w^2 k_w^2 \gamma_0 c$. Although the beam radius remains constant up to order of $z^2$, the normalized beam emittance decreases algebraically: $\varepsilon_n = \varepsilon_n(0)/(1 + \nu_0 z)$.

The relevance of the above analysis depends on the magnitude of the damping rate $\nu_0$. For the following set of accelerator parameters,[2] $E_L = 1.5 \times 10^9$ V/cm, $B_w = 50$ kG, $\lambda_w = 1$ m, it is estimated that the $e$ fold length, $1/\nu_0$, could be as short as 600 m for $\gamma_0 = 10^5$. Therefore, our results show that one can improve, by induced synchrotron radiation, the quality of an electron beam by passing it through an external wiggler field.

## 5. QUANTUM EXCITATION

An estimate for the minimum transverse normalized beam emittance due to quantum excitation in an IFEL accelerator can be obtained from the following qualitative treatment. Similar arguments can be made for electron beams in storage rings.[17,18] The normalized transverse velocity and radial displacement of an electron in a wiggler field are given by $\beta_w = a_w/\gamma$ and $r_w = a_w\lambda_w/(2\pi\gamma)$. For a fluctuation $\delta E$ in the energy of the electron, the corresponding fluctuations in $r_w$ and $\beta_w$ are $\delta r_w = \eta\,\delta E/e$ and $\delta\beta_w = \xi\,\delta E/E$, where $\eta = a_w\lambda_w/(2\pi\gamma)$ and $\xi = a_w/\gamma$. The increase in normalized emittance due to such fluctuations is[17,19] $\Delta\varepsilon_n = \gamma(K_B\langle\delta r_w^2\rangle + \langle\delta\beta_w^2\rangle/K_B)$, which for a weakly focusing channel, $K_B \ll k_w$, can be approximated by $\Delta\varepsilon_n \approx \gamma\langle\delta\beta_w^2\rangle/K_B = (\gamma\xi^2/K_B)\langle\delta E^2\rangle/E^2$. Due to the discrete nature of the synchrotron radiation, $\langle\delta E^2\rangle$ is given by $N(\hbar\omega_c)^2$, where $N = Pz/(c\hbar\omega_c)$ is the number of photons emitted in a distance $z$, $P$ is the synchrotron radiation power, and $\hbar\omega_c$ is the energy associated with a quantum of synchrotron radiation. We can therefore obtain the rate of change of $\varepsilon_n$ due to quantum excitation,

$$\left(\frac{d\varepsilon_n}{dz}\right)_{\text{Q.E.}} = \frac{\gamma\xi^2}{K_B}\frac{P\hbar\omega_c}{cE^2}.$$

However, with radiation damping, the total change in $\varepsilon_n$ is given by

$$\left(\frac{d\varepsilon_n}{dz}\right)_{\text{Q.E.}} = -\nu\varepsilon_n + \left(\frac{d\varepsilon_n}{dz}\right)_{\text{Q.E.}}$$

The normalized emittance $\varepsilon_n$ reaches a minimum, $d\varepsilon_n/dz = 0$, when the two effects are balanced. This gives $\varepsilon_n = \gamma\xi^2\hbar\omega_c/(K_B E)$ for the minimum normalized emittance, where we have used $\nu c \approx P/E$. For synchrotron radiation, $\hbar\omega_c = 3\hbar c\gamma^3/(2\rho)$, where $\rho = \gamma/(a_w k_w)$ is the radius of curvature of the electron orbit in the wiggler. The minimum transverse normalized beam emittance is then approximately given by

$$(\varepsilon_n)_{\min} \approx 3\hbar a_w^3 k_w/(2m_0 c K_B). \tag{14}$$

In the case of weak focusing due to wiggler transverse gradients, $K_B = a_w k_w/(\sqrt{2}\gamma)$, and the minimum normalized emittance is

$$(\varepsilon_n)_{\min} \approx 3\hbar\gamma a_w^2/(\sqrt{2}m_0 c). \tag{15}$$

Using the accelerator parameters at the end of Section 4, Eq. (15) gives the value of the minimum normalized emittance to be ~1.8 cm-rad. Such a large value of the minimum emittance indicates the inadequacy of the weak focusing from the wiggler transverse gradients. Strong focusing from, for example, a rotating quadrupole field produced by a pair of (or four) helical current windings[20,21] may be required. The betatron wave number for such a focusing mechanism[22] is given by $K_B^2 = |e|\,(\partial B/\partial r)/\gamma m_0 c^2$, where $\partial B/\partial r$ is the magnetic field gradient of the quadrupole field on axis. For $\partial B/\partial r \cong 250$ G/cm, $a_w = 600$, $\lambda_w = 10$ m, and $\gamma = 4\times 10^5$, Eq. (14) gives a minimum normalized emittance of $\varepsilon_n \cong 0.13$ cm-rad. Another possible strong-focusing force could be the electrostatic radial electric field of an ion column. Such a column could be created by the ionization of the residual gas by a low-energy, high-current electron beam pulse preceding the main accelerating beam pulse.[23–25] The betatron wave number for such a focusing mechanism can be easily shown to be $K_B^2 = \omega_{\text{pi}}^2(m_i/m_0)/(2\gamma c^2)$, where $\omega_{\text{pi}}$ is the ion plasma frequency and $(m_i/m_0)$ is the mass ratio between the ions and the electrons. For $n_i = 10^{12}/\text{cm}^3$, $a_w = 600$, $\lambda_w = 10$ m, and $\gamma = 4\times 10^5$, Eq. (14) gives a minimum normalized emittance of $\varepsilon_n \cong 0.04$ cm-rad. An additional benefit of having ion focusing in the IFEL accelerator is that the radial plasma electron density profile in an ion column can also be a focusing medium for the laser beam.

## 6. NUMERICAL EXAMPLE

We shall consider only resonant particles whose phase $\psi$ satisfies the conditions $d\psi/dz = 0$ and $d^2\psi/dz^2 = 0$. The first condition gives

$$\gamma_R = \frac{|e|\sqrt{k}}{\sqrt{2}m_0c^2} B_w k_w^{-3/2}, \tag{16a}$$

$$\gamma_R' = R_1 k_w^{1/2} - R_2 B_w^4 k_w^{-3}, \tag{16b}$$

$$\nu = \frac{\sqrt{2}}{3}\frac{|e|^5}{m_0^4c^8} B_w^3 k_w^{-3/2}\sqrt{k}, \tag{16c}$$

where $R_1 = \sqrt{2}\,|e|\,E_L \sin\psi_R/(m_0c^2\sqrt{k})$, $R_2 = |e|^6 k/(3m_0^5c^{10})$, $\psi_R$ is the resonance phase, $E_L$ the laser electric field strength, and $k$ the laser wave number. The second condition, together with the pendulum equation, Eq. (6), provide the spatial dependences of $k_w$ and $B_w$:

$$3k_w' - \frac{2k_w}{B_w}B_w' + \frac{4E_L}{k}\sin\psi_R\frac{k_w^3}{B_w} - \frac{2\sqrt{2}mc^2}{|e|\sqrt{k}}R_2\frac{B_w^3}{\sqrt{k_w}} = 0. \tag{17}$$

Equation (17) shows that the required tapering of the wiggler field may be obtained by prescribing $\psi_R$ and a relationship between $k_w$ and $B_w$ in Eq. (17). As an example, we assume the tapering of the wiggler field to be that of a maximum-rate IFEL accelerator.[2] For such a case, the wiggler strength and the wiggler period are related by the following power law:

$$B_w = (R_1/6R_2)^{1/4} k_w^{7/8}.$$

Equation (17) may then be solved to give

$$B_w = B_w(0)[1 + R_4 z]^{-7/9}, \tag{18a}$$

$$k_w = k_w(0)[1 + R_4 z]^{-8/9}, \tag{18b}$$

where

$$R_4 = \frac{9\sqrt{2}mc^2}{|e|\sqrt{k}} R_2(R_1/6R_2)^{3/7} B_w(0)^{9/7}.$$

Evaluating Eqs. (11) and (16a) with (16c) and (18a,b) gives the normalized transverse emittance and the resonant energy of the beam as functions of the propagation distances.

For our example, we will consider the following set of accelerator parameters[2]: $E_L = 1.5\times10^9$ V/cm, $B_w(0) = 50$ kG, $\lambda_w(0) = 100$ cm, and $\lambda = 10.6\,\mu$m with a resonance phase of $\sin\psi_R = 0.6$. The initial conditions are for a matched beam with a radius of 1 mm and a normalized emittance of $\varepsilon_0 = 0.205$ cm-rad, and the required beam injection energy is ~52 GeV. The beam is allowed to propagate for 1 km without depleting the laser radiation. We repeated the calculation by assuming there is no radiation damping but with the same power law tapering of the wiggler field.

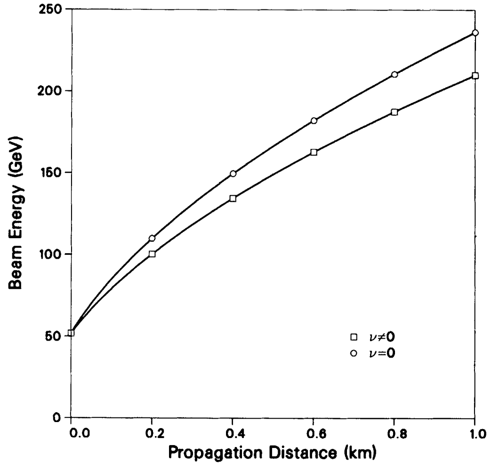



FIGURE 1 Beam energy as a function of propagation distance, with (□) and without (○) radiation damping.

The results are represented in Figs. 1, 2, and 3. The open squares denote the presence of radiation damping, while open circles denote its absence. From Fig. 1, we can see that the final energy is not significantly reduced by the radiation damping. Figure 2 shows the exponential decay of the normalized emittance. At the end of the one-kilometer accelerator, the normalized emittance is reduced to 0.05 cm-rad, which is very close to the minimum normalized emittance of ~0.04 cm-rad at that point if ion-column focusing is assumed in the accelerator. In Fig. 3, the appropriate tapering of $k_w$ and $B_w$ for the two cases is shown.

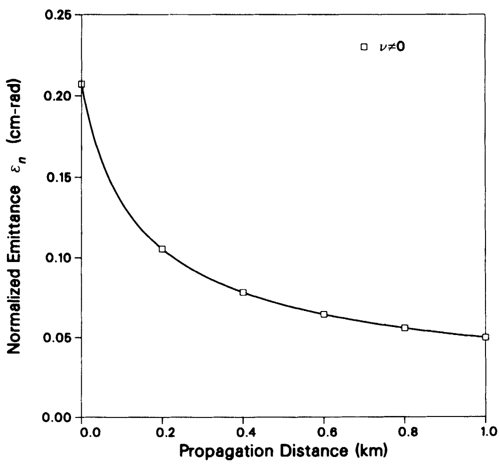
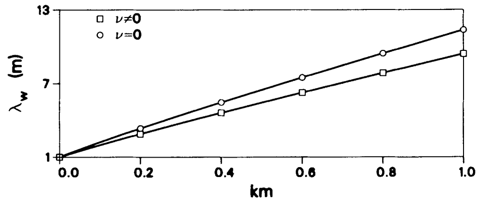



FIGURE 2 Normalized emittance as a function of propagation distance.



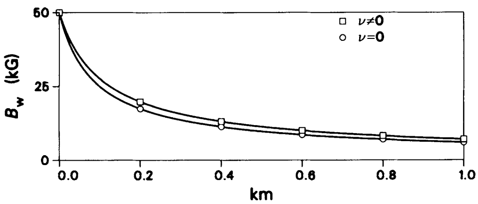




FIGURE 3 Appropriate tapering of the wiggler period length and magnetic field, with (□) and without (○) radiation damping.

## 7. CONCLUSION

We have studied the evolution of transverse emittance and the beam radius due to the radiation damping effect in an IFEL accelerator. We derived the beam envelope equation, Eq. (10), which includes the effects of radiation damping, and have demonstrated that the normalized transverse emittance decreases exponentially with a damping rate given by the radiation damping coefficient $\nu$ until it reaches a minimum value due to quantum excitation. The beam envelope equation was solved analytically for a slowly varying wiggler field. We have derived an expression for the minimum normalized emittance in the IFEL accelerator and showed that strong focusing is essential in reducing this minimum emittance due to quantum excitation. We have shown that radiation damping can play an important role in improving beam quality without a significant sacrifice in beam energy.

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