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SPIN-FREQUENCY SPREAD MEASUREMENTS IN A STORAGE RING

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The energy of electrons or positrons circulating in a storage ring is often calibrated by measuring the frequency at which resonance depolarization occurs. We report experimental measurements, made in the VEPP-2M storage ring, of the width of the resonance resulting from frequency spread caused by nonlinearities.

1. INTRODUCTION

In recent years, the method of particle energy calibration using resonance depolarization has been, extensively used in experiments on various electron-positron storage rings.[1] This method is based on the relation between the energy E and the spin precession frequency Ω of a relativistic electron:

$$\Omega = \left(1 + \frac{E}{mc^2}\frac{q'}{q_0}\right)\omega = (1 + \nu)\omega, \tag{1}$$

where ω is the revolution frequency of the particle in a guiding magnetic field, and q' and q_0 are the anomalous and normal parts of the gyromagnetic ratio.

For a particle with $E = E_s + \Delta E$, different from the equilibrium value of E_s, the revolution frequency is

$$\omega = \omega_s\left(1 + \alpha\frac{\Delta E}{E_s}\right),$$

where α is the momentum compaction factor, and the precession frequency $\Omega = \Omega_s + \delta\Omega$ differs from that of the equilibrium particle

$$\Omega_s = \omega_s\left(1 + \gamma\frac{q'}{q_0}\right)$$

by the quantity

$$\delta\Omega = \omega_s\left[(\alpha + \alpha\nu_s + \nu_s)\frac{\Delta E}{E_s} + \alpha\nu_s\left(\frac{\Delta E}{E_s}\right)^2\right]. \tag{2}$$

Synchrotron (energy) oscillations given by

$$E = E_s\left(1 + \frac{\Delta E}{E}\sin \nu_\gamma\omega_s t\right)$$

modulate the precession frequency and lead to a spectrum of spin frequencies consisting of the lines spaced by $\nu_\gamma \omega_s$. A central spectral line is the precession frequency averaged over synchrotron oscillations:

$$\langle \Omega \rangle = \Omega_s + \langle \delta\Omega \rangle.$$

In an approximation linear with respect to the magnetic field $\langle \Delta E/E_s \rangle = 0$, the width of the central line is

$$\langle \delta\Omega \rangle_\gamma = \tfrac{1}{2}\alpha\nu_s\left(\frac{\Delta E}{E_s}\right)^2 \omega_s,$$

where the factor of one-half comes from averaging over the synchrotron oscillations. For real storage rings, taking into account the nonlinearity of a magnetic field gives rise to a difference of the average energy $\bar{E}$ of the particle from its equilibrium value E_s. One can show (see Appendix) that in the next approximation the spread of spin frequencies is mainly associated with the occurrence of the quadratic nonlinearity $\partial^2 H_z/\partial x^2$ of a magnetic field H_z and, after averaging over the azimuth θ, the width is given by

$$\langle \partial\Omega \rangle_\beta = \frac{\nu_s \omega_s}{2\langle \kappa\psi_x \rangle}\left[|C_x|^2\left\langle \frac{\partial^2 H_z}{\partial x^2}\,\psi_x\,|f_x|^2\right\rangle + \left(\frac{\Delta E}{E_s}\right)^2\left\langle \frac{\partial^2 H_z}{\partial x^2}\,\psi_x^3\right\rangle\right], \tag{3}$$

where κ is the curvature, $|f_x|$ is the absolute value of the Floquet function, ψ_x is the dispersion function, and $|C_x|$ is the rms amplitude of radial betatron oscillations.

An estimation of $\langle \delta\Omega \rangle_\gamma$ and $\langle \delta\Omega \rangle_\beta$ for various existing storage rings shows that the spread of spin frequencies does not exceed $\langle \delta\Omega \rangle \sim 10^{-5}\omega_s$ and provides the accuracy of energy calibration by the method of resonance depolarization. However, in an attempt to measure directly such as small spread, there arise a number of experimental difficulties. The present paper describes an experiment in which the quantity $\langle \delta\Omega \rangle$ has been found from measurements of the depolarization time on the VEPP-2M storage ring during a coherent spin flip by an external rf field $\tilde{H}$.[2]

2. SPIN FLIP

It is known[3,4] that when a spin resonance, characterized by the precession frequency **w** around a field $\tilde{H} \perp \Omega$, is crossed with a velocity $\dot{\varepsilon}$, the polarization degree changes by the quantity $\delta S = 2S(e^{-J} - 1)$, where $J = \pi w^2/2\dot{\varepsilon}$ determines the angle of spin rotation around $\tilde{H}$ in an effective resonance region $\varepsilon_{\text{eff}} \lesssim [\max(w, \dot{\varepsilon})]$. The quantity $\varepsilon = \Omega - \Omega_{\text{ext}}$ is the detuning of the resonance frequency from that of an external rf field, and $\dot{\varepsilon}$ is the rate of change of detuning. In the case of $w^2 \gg \dot{\varepsilon}$, there takes place an adiabatic reversal of polarization with an exponentially small reduction of its degree.

For electrons in a storage ring, the energy diffusion caused by quantum radiation fluctuations needs to be taken into account.[5]

If the time of slow crossing of the effective resonance zone is of the order of, or longer than, the radiation damping time τ. for which the energy and hence the precession frequencies inside the distribution are mixed, a characteristic depolarization time τ_d can be estimated from the formula[5]

$$\tau_d^{-1} = \frac{\pi}{2} \frac{\langle \delta\Omega \rangle^2}{\tau_0 w^2}. \tag{4}$$

The occurrence of depolarization limits from below the rate of resonance crossing, and the complete condition for adiabatic spin flip is of the form

$$w^2 \gg \dot{\varepsilon} \gg \dot{\varepsilon} = \frac{\pi}{2} \frac{\langle \delta\Omega \rangle^2}{w\tau_0}. \tag{5}$$

Thus, the magnitude of the spin-frequency spread can be found from Eq. (5) if the value of $\dot{\varepsilon}_{\min}$ is established experimentally.

3. MEASUREMENTS

The possibility of adiabatic spin flip in a storage ring was demonstrated in Ref. 2. An rf device for spin reversal (the so-called "flipper") provides, on the orbit, a high-frequency longitudinal magnetic field of up to 100 Gs on a 40-cm length, with a frequency

$$f = \frac{\Omega - 2\omega_s}{2\pi} = 7.9\,\text{MHz}.$$

The control system allows the frequency of a generator to be varied within the $\pm 3 \times 10^{-3}$ range for times from 10^{-3} to 10^2 s.

The degree of beam polarization is obtained by observing elastic electron scattering inside the bunch.[6,7,9] Under our conditions, the counting rate for this effect is $\dot{n} \simeq 20I^2(1 - 0.12S^2)$, where I is the beam current and S is the polarization degree. Due to radiative polarization,[8] the polarization degree of the electron beam varies according to the law $S = S_m(1 - \epsilon^{-t/\tau_p})$ and achieves $S_0 = 0.95S_m$ for the time $t = 3$ h (for the storage ring VEPP-2M, the polarization time is $\tau_p = 1$ h, at an energy of 650 MeV). After that, a new, unpolarized bunch with approximately the same current and shifted by π relative to the first beam, is injected into the storage ring. Comparison of the counting rates for intrabeam scatterings in the polarized and unpolarized bunch makes it possible to measure the polarization degree of the first beam for $t \simeq 30$ s. To increase the accuracy, these measurements are made repetitively. After one of the measurements, the flipper is switched on. Varying with a constant velocity $\dot{\varepsilon}$, its frequency intersects the value of $f = (\Omega - 2\omega_s)/2\pi$, corresponding to the spin resonance. The next measurement of the polarization degree S_1 and the comparison of it with the initial S_0 give information about depolarization during resonance crossing. Figure 1 (curve 1) presents several cycles of these measurements made at different rates of resonance crossing.

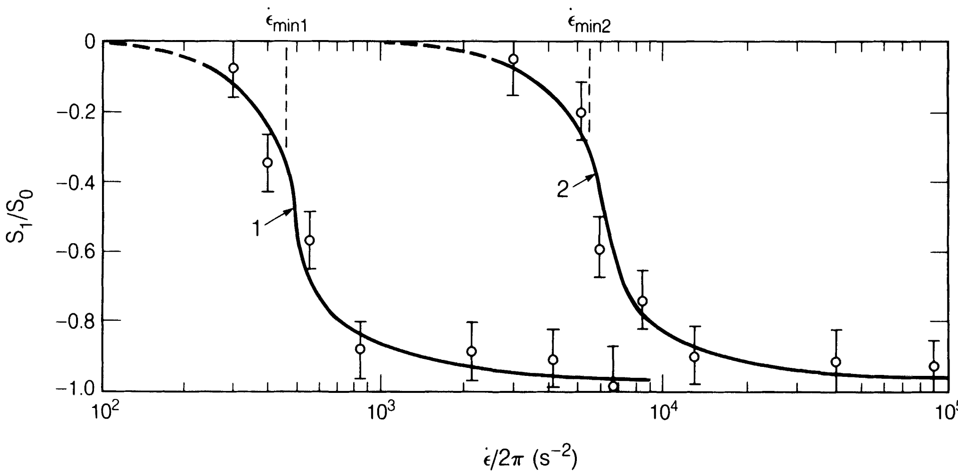

FIGURE 1 Depolarization measurements made at several values of the rate of resonance crossing, $\dot{\varepsilon}$, with and without sextupoles excited (curves 1 and 2, respectively).

To demonstrate the depolarization time versus the nonlinearity of the storage ring magnetic structure, analogous measurements were made in the regime with sextupole corrections off (Fig. 1, curve 2). It is seen that the values of $\dot{\varepsilon}_{\min 1,2}$ differ approximately by one order of magnitude.

It follows from Eq. (5) that to find the value of the spin-frequency spread, the value of the resonance harmonic w needs to be known. This quantity was preliminarily found from radiotechnical measurements of the flipper parameters and of the rf voltage on the section providing a longitudinal magnetic field. In

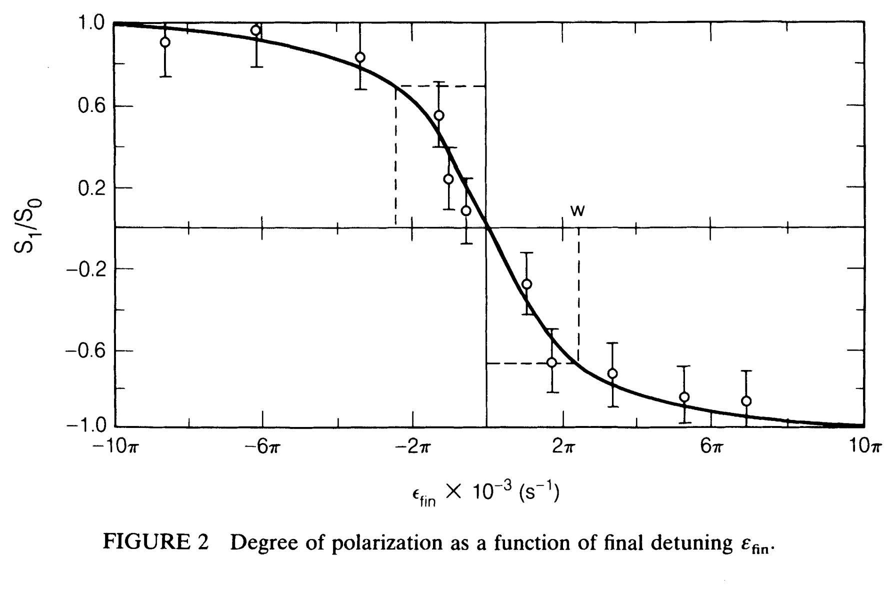

FIGURE 2 Degree of polarization as a function of final detuning ε_{fin}.

addition, owing to the fact that the VEPP-2M storage ring has a system for automatic stabilization[9] of the average energy of particles with an accuracy of $\Delta E/E \simeq 10^{-5}$, it turned out to be possible to determine, by direct measurements, the boundaries of the resonance zone and hence the value of the resonance harmonic w. For this purpose, in a series of runs at a maximum amplitude of the rf field, the adiabatic spin flip was made by stopping the scanning and switching off the rf field at different frequencies near the resonance. The dependence of the residual polarization degree S_1 on the magnitude of the final detuning ε_{fin} is depicted in Fig. 2. The analytical expression of this dependence is of the form

$$\frac{S_1}{S_0} = \pm \frac{1}{\sqrt{1 + w^2/\varepsilon_{\text{fin}}^2}}.$$

The value of the resonance harmonic, measured in this way, proved to be close to the value obtained from radiotechnical measurements: $w_{\text{max}} = 7.5 \times 10^{-5}\omega_s = 2\pi \cdot 1250\,\text{s}^{-1}$.

4. RESULTS

In conclusion we present some concrete experimental data and results:

Particle energy $E = 650\,\text{MeV}$ ($\nu = 1.47$)
Revolution frequency $\omega_s = 2\pi \cdot 16.7 \times 10^6\,\text{s}^{-1}$
Radiation damping time $\tau_0 \simeq 5 \times 10^{-3}\,\text{s}$
Operating magnitude of the resonance harmonic $w = 3 \times 10^{-5}\omega_s = 2\pi \cdot 500\,\text{s}^{-1}$.

The initial and final detunings relative to the resonance were equal to $\varepsilon_{\text{in}} = |\varepsilon_{\text{fin}}| = 7.5 \times 10^{-4}\omega_s = 2\pi \cdot 12{,}500\,\text{s}^{-1}$.

The measured magnitude of the spin-frequency spread in the operating version of the magnetic structure (Fig. 1, curve 1) was

$$\langle \delta\Omega \rangle_{\text{exp}} = 1 \cdot 5 \times 10^{-6}\omega_s = 2\pi \cdot 35\,\text{s}^{-1}.$$

This quantity is somewhat higher than the calculated magnitude $\langle \delta\Omega \rangle = 1 \times 10^{-6}\omega_s$, obtained from Eq. (3) for the operating structure of the storage ring. It is apparent that this can be explained by inaccurate knowledge of the quadratic nonlinearity of the storage ring and by the contribution from the higher-order nonlinearities.

In the regime without sextupoles (Fig. 1, curve 2),

$$\langle \delta\Omega \rangle_{\text{exp}} \simeq 5 \times 10^{-6}\omega_s.$$

Comparison of the measured magnitude with the calculated value for a linear case, $\langle \delta\Omega \rangle_\gamma \simeq 10^{-7}\omega_s$, shows that the contribution of the storage ring nonlinearities plays a decisive role for the spin-frequency spread. This contribution can, in principle, be considerably reduced by a special choice of the corrections of nonlinearities.

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APPENDIX

Association of spin-frequency spread with Quadratic Nonlinearities

Let us find the difference between the average particle energy $\bar{E}$ and its equilibrium value E_s. For simplicity, we consider the particle motion without vertical deflections from the equilibrium orbit and make use of the following exact equations:[11]

$$\begin{aligned}\left(\frac{d\theta}{dt}x'\right)' &= (1+\kappa x)\left(\frac{eH_z}{\gamma mc}+\kappa\frac{d\theta}{dt}\right),\\ \frac{d\theta}{dt} &\equiv \omega = [(1+\kappa x)^2+(x')^2]^{-\frac{1}{2}},\end{aligned} \tag{A-1}$$

where $\kappa = H_z/\langle H_z\rangle$ is the orbit curvature and $\gamma = E/mc^2$.

The equation for x motion in second order with respect to x and x' will be of the form

$$x'' = (1+2\kappa x)\left[\frac{eH_z}{\gamma mc}+\frac{\kappa}{\sqrt{(1+\kappa x)^2+(x')^2}}\right]+(\kappa x)'x'. \tag{A-2}$$

Expanding in a power series of x, for a particle with the energy $E = E_s + \Delta E$ we obtain

$$x'' + g_x x = \kappa\frac{\Delta\gamma}{\gamma} + F_2. \tag{A-3}$$

Accurate up to the terms of order x^2, the addition to the linear equation is as follows:

$$F_2=\left(\kappa^2-2\kappa g_x-\frac{1}{2}\frac{\partial^2 H_z}{\partial x^2}\right)x^2+(2\kappa^2+n)x\frac{\Delta\gamma}{\gamma}-\frac{\kappa x'^2}{2}+(\kappa x)'x'. \qquad \text{(A-4)}$$

In Eq. (A-4) the term with the second field derivative $\partial^2 H_z/\partial x^z$ is the strongest because, usually, $K\sim 1$ and

$$\frac{\partial^2 H_z}{\partial x^z}\bigg/ g_x\simeq\frac{R}{a}\gg 1,$$

where a is the aperture.

Retaining the leading term and solving Eq. (A-3) by the method of successive approximations, we obtain the forced solution

$$x_1=\frac{1}{2i}f_x e^{i\nu_x\theta}\int_{-\infty}^{\theta} f_x^* e^{-i\nu_x\theta}\left(\frac{1}{2}\frac{\partial^2 Hz}{\partial x^2}x_0^2\right)d\theta+\text{c.c.}, \qquad \text{(A-5)}$$

where c.c. devotes the complex conjugate and

$$x_0=x_\beta+x_\gamma=C_x f_x e^{i\nu_x\theta}+C_x^* f_x^* e^{-i\nu_x\theta}+\psi_x\frac{\Delta\gamma}{\gamma}$$

is the solution of Eq. (A-3) without F_2 (free oscillations).

From the second equation in (A-1) one can derive, in particular, the law of phase variation for a particle passing through an accelerating cavity:

$$\frac{d\phi}{dt}=\omega-\omega_s\simeq -\kappa x.$$

Substituting into the above expression $x=x_0x_1$ and averaging over the time $(\overline{d\phi/dt}=0)$, we have

$$\kappa\psi_x\frac{\Delta\gamma}{\gamma}=\langle\kappa\psi_x\rangle\frac{\overline{\Delta\gamma}}{\gamma}=\frac{1}{T}\int_{-T}^{T}\kappa\left[\frac{1}{2i}f_x e^{i\nu_x\theta}\int_{-\infty}^{\theta} f_x^* e^{-i\nu_x\theta'}\left(\frac{1}{2}\frac{\partial^2 H_z}{\partial x^2}x_0^2\right)d\theta'+\text{c.c.}\right]dt. \qquad \text{(A-6)}$$

After integration by parts, we introduce

$$\psi_x=\frac{1}{2i}f_x\int_{-\infty}^{\theta}\kappa f_x^*\, d\theta+\text{c.c.}$$

and proceed to the limit $T\to\infty$.

In averaging, the terms containing the fast frequency ν_x and $2\nu_x$ give zero, and we eventually have

$$\frac{\overline{\Delta\gamma}}{\gamma}=\frac{1}{\langle\kappa\psi_x\rangle}\lim_{\tau\to\infty}\left(\frac{1}{4T}\int_{-T}^{T}\frac{\partial^2 H_z}{\partial x^2}x_0^2\psi_x\, dt\right)$$

$$=\frac{1}{2\langle\kappa\psi_x\rangle}\left[C_x^2\left\langle\frac{\partial^2 H_z}{\partial x^2}\psi_x\,|f_x|^2\right\rangle+\left(\frac{\Delta\gamma}{\gamma}\right)^2\left\langle\frac{\partial^2 H_z}{\partial x^2}\psi_x^3\right\rangle\right]. \qquad \text{(A-7)}$$

It is known that the occurrence of the quadratic nonlinearity of a magnetic field gives rise also to the dependence of betatron oscillation frequencies on the particle energy. Separating the term with $x_\beta x_\gamma$ in Eq. (A-5) and averaging over fast oscillations, we obtain

$$\gamma \frac{\partial \nu_x}{\partial \gamma} = \frac{1}{2} \left\langle \frac{\partial^2 H_z}{\partial x^2} \psi_x \, |f_x|^2 \right\rangle.$$

This quantity is experimentally measurable, and its inspection can be used to compensate the chromaticity.