

EQUILIBRIA FOR FOIL-FOCUSED RELATIVISTIC ELECTRON BEAMS

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High-current relativistic electron beams can be focused effectively using a periodic array of transverse conducting foils. Image charges on the foils reduce the average radial electric fields acting on the beams so they can be confined by self-magnetic fields. This type of transport has potential application in linear induction accelerators. Calculations are presented which allow the determination of the foil geometry necessary for radial-force balance in electron beams of any emittance.

The space-charge electric fields of charged-particle beams propagating in vacuum cause them to expand. When the beams are relativistic, expansion is counteracted by self-magnetic-field forces. The magnetic force is a factor of $-\beta^2$ times the electric force¹, where $\beta = v_z/c$. The forces balance to within a factor of $1/\gamma^2$. Various methods have been used to balance the missing factor to transport constant-radius electron beams. The most common is to apply an additional focusing force, usually by axial magnetic fields or periodic fields from solenoidal magnetic lenses. Another approach is to reduce the radial electric field. This can be accomplished with a dilute background of positive ions with density $> 1/\gamma^2$ times that of the electron beam.²

An alternative method, suggested by Adler,³ is the use of a periodic array of conducting foils transverse to the beam axis, as shown in Fig. 1.⁴ In this case, the positive image charge on the foils reduces nearby radial electric fields. The average electric force on electrons passing through the array is reduced, and the electrons may be confined magnetically. Two characteristics of relativistic electron beams make this

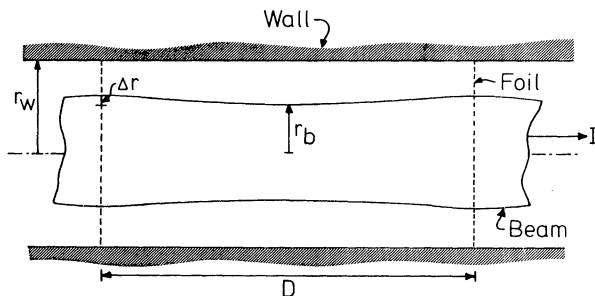


FIGURE 1 Geometry for calculating foil-focusing of a relativistic electron beam. The system has cylindrical symmetry; one focusing cell is shown.

focusing method feasible. First, at high γ , focusing and defocusing forces are almost equal, so only a small electric-field reduction is needed. This can be provided with widely spaced foils. Second, electrons have relatively small energy loss in materials. This means that substantial foils can be used without significant degradation of the beam energy or emittance. Furthermore, at low duty cycle, high-current pulsed beams can propagate with little problem of foil heating. Foil focusing has the potential for long-distance transport of multi-kiloampere electron beams above 10 MeV. It could replace magnetic focusing in high-current linear induction accelerators. The method is more predictable than plasma focusing. It has the further advantage that shaped foils may be used to steer the beam and stabilize it on axis.

The calculations of Adler³ treat the specific case of an emittance-dominated beam extracted from an immersed cathode. In this paper, general calculations of transport limits as a function of foil geometry will be presented for beams of any emittance, including the laminar-flow case. The approach will be to seek a global balance by matching forces on the beam envelope. Reference 3 shows that for the common case in which the distance between foils is large compared with the beam radius ($D \gg r_b$), the average electric force is approximately linear for a uniform beam; the envelope approach therefore gives good estimates. The following assumptions will be made. Electron motion in the beam is paraxial and all electrons move with the same β . The average beam density does not vary axially; this is true if there is no acceleration between foils, or if the beams are highly relativistic. The beam has cylindrical symmetry with radially uniform space charge and current density. Oscillations of the beam envelope (see Fig. 1) are small; limits of validity of this assumption will be investigated. The foils are perpendicular to the system axis and radially uniform; the problem of transverse stability will not be addressed.

To begin, conditions for a laminar-flow equilibrium are derived. The assumptions above permit calculation of the average electric fields acting on the beam by determining the radial electric field of a uniformly charged cylinder of radius r_b in a conducting pipe of radius r_w and length D with conducting ends. The fields are averaged over the length of the pipe at the envelope radius to estimate the average defocusing force acting on peripheral particles. The ratio of this force to the radial electric force on the envelope of a beam of infinite length will be denoted by Δ . Since magnetic fields are unaffected by the presence of foils, a laminar-flow force balance will occur when $(1 - \Delta) = 1/\gamma^2$.

The electrostatic potential can be calculated by expanding the potential and space-charge density in a Fourier-Bessel series of the form

$$\phi = \sum_m \sum_n A_{mn} J_0(x_m r/r_w) \cos(n\pi z/D), \quad (1)$$

and

$$\rho(r, z) = \sum_m \sum_n \left\{ \frac{2en_0(r_b/r_w) J_1\left(\frac{x_m r_b}{r_w}\right) \sin\left(\frac{n\pi}{2}\right)}{\epsilon_0 \pi^2 x_m n J_1^2(x_m)} J_0\left(\frac{x_m r}{r_w}\right) \cos\left(\frac{n\pi z}{D}\right) \right\} \quad (2)$$

$(x_m = 2.405, 5.520, \dots)$.

The potential of Eq. (1) satisfies the boundary conditions $\phi = 0$ at $r = r_w, z = -D/2$ and $z = D/2$. The coefficients in Eq. (2) apply to a uniform charge density. The

coefficients A_{mn} can be determined by substituting Eqs. (1) and (2) into Poisson's equation. Thus

$$A_{mn} = \frac{8(r_b/r_w)J_1(x_m r_b/r_w) \sin(n\pi/2)}{\pi\epsilon_0 x_m n J_1^2(x_m) \left\{ \left(\frac{x_m}{r_w}\right)^2 + \left(\frac{n\pi}{D}\right)^2 \right\}}. \quad (3)$$

Taking $-\partial\phi/\partial r$ at the envelope, averaging over the length of the focusing cell and dividing by $r_b e n_0 / 2\epsilon_0$ (the radial electric field of an infinite-length beam) gives

$$\Delta = \frac{\langle E_r \rangle}{(r_b e n_0 / 2\epsilon_0)} = \sum_{m=1,2} \sum_{n=1,3} \left(\frac{32}{\pi^2}\right) \frac{[J_1(x_m r_b/r_w)/J_1(x_m)]^2}{n^2 \left\{ x_m^2 + \left(\frac{r_w}{r_b}\right)^2 \left(\frac{n\pi r_b}{D}\right)^2 \right\}}. \quad (4)$$

This expression was evaluated numerically with 12 radial terms and 10 axial terms as a function of (D/r_b) for various choices of (r_w/r_b) . Accuracy of the series expansion deteriorates with large D/r_b , so it was used for field variations within $2r_b$ of the foil, and the uniform field approximation used in between. Results are plotted in Fig. 2, showing $(1 - \Delta)$ versus D/r_b . The second vertical axis shows the value of γ corresponding to a laminar-flow equilibrium. For instance, a 4-MeV electron beam of 2 cm radius in a 4-cm radius pipe can propagate with a foil spacing of 1.7 m.

The results above are independent of the beam current. Nonetheless, the current must be included to determine if the conditions of the model are satisfied. It is required that oscillations of the beam envelope between foils be small compared with the beam radius. The change in beam radius away from a beam neck (the midpoint in Fig. 1) to

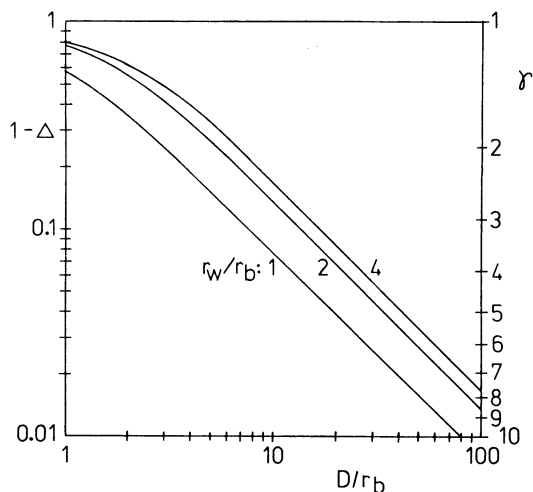


FIGURE 2 Reduction of average radial electrical fields as a function of foil geometry. The parameter Δ is the ratio of the longitudinal average of radial fields at the beam envelope to the field on the periphery of an infinite length beam. The vertical axis on the right shows the value of electron γ for a laminar flow equilibrium.

the foil is approximately

$$\frac{\Delta r}{r_b} \cong \frac{eI(D/2r_b)^2}{4\pi\epsilon_0 m_0 c^3 \gamma} \tag{5}$$

Equation (5) is based on the assumption that $\beta \cong 1$ and $\Delta r/r_b \ll 1$. For $\Delta r/r_b = 0.1$, Eq. (5) becomes

$$\frac{D}{r_b} < 82 \sqrt{\frac{\gamma^3}{I}}, \tag{6}$$

where I is expressed in amperes. The limit is plotted in Fig. 3 as a function of d/r_b for choices of γ . The model is valid for parameters in the region to the left of the lines.

When the electric field is reduced below the laminar-flow conditions, beams with non-zero emittance can be propagated. The emittance force on the envelope of a beam with an elliptical distribution is given by¹

$$F_\epsilon = \epsilon^2 \gamma m_0 c^2 \beta^2 / r_b^3. \tag{7}$$

The emittance ϵ is defined as the area of the distribution ellipse divided by π . The net focusing force is given by

$$F_f = \frac{e^2 n_0 r_b}{2\epsilon_0} [\Delta - \beta^2] \tag{8}$$

for a foil-focused beam, where the quantity Δ can be found from Fig. 2. Force balance

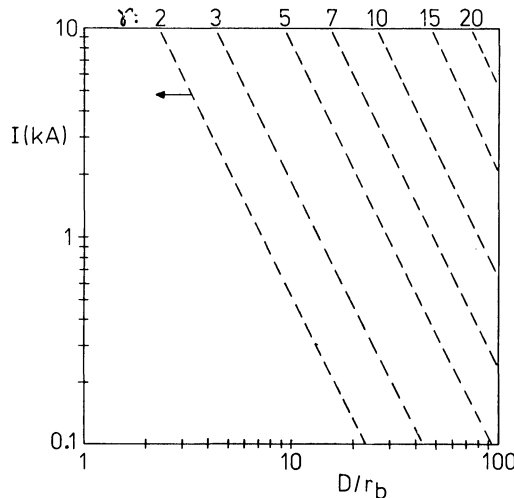


FIGURE 3 Limits of applicability of the model as a function of foil geometry, beam current, and electron energy. In the region to the left of the dashed lines, envelope oscillations are less than 10 per cent of the beam radius.

gives the condition

$$\epsilon = \sqrt{\frac{2r_b^2 I}{\beta^2 I_A} \{\beta^2 - \Delta\}}, \quad (9)$$

where I_A is the Alfven current,

$$I_A = (4\pi\epsilon_0 m_0 c^3 / e) \beta \gamma. \quad (10)$$

The following example illustrates the determination of the propagation characteristics of a foil-focused beam. Assume a 10-kA beam with radius $r_b = 5$ cm and energy 9.5 MeV ($\gamma = 20$). The distance between foils is 2.5 m. Under these conditions, the beam can travel a distance of 42 m through 0.005 cm thick (2 mil) aluminum foils with a 5 per cent energy loss. An accelerating gradient of only 0.01 MV/m can balance these losses. With a wall radius of 10 cm, Fig. 2 indicates that the average electric field of the beam is reduced by a factor of 0.973. The magnetic force is 0.9975 times the electric force of an infinite-length beam. Since $\beta^2 > \Delta$, the beam is confined. The Alfven current for the beam is 168 kA. Thus, $I/I_A = 0.06$. (This condition also guarantees that particle motion is paraxial.) Substituting the quantities in Eq. (9), a beam with an emittance of 270 mrad-cm will be matched. Finally, reference to Fig. 3 shows that the relative beam envelope oscillations are less than 10 per cent.

I would like to thank R. Adler for his suggestions on this material.

REFERENCES

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2. R. J. Briggs, J. C. Clark, T. J. Fessenden, R. E. Hester, and E. J. Lauer, in *Proc. 2nd Int'l. Topical Conf. High Power Electron and Ion Beam Research and Technology*, edited by J. A. Nation and R. N. Sudan (Laboratory of Plasma Studies, Cornell University, 1977), 319.
3. R. J. Adler, *Part. Accelerators*, **12**, 263 (1980).
4. The type of focusing discussed in this paper differs from the process of "foil focusing" used in some low-current linear ion accelerators. In the low-current case, grids are placed across the downstream drift tubes at acceleration gaps to modify applied accelerating fields for increased electric-field focusing.