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IMAGE-FIELD FOCUSING OF INTENSE ULTRA-RELATIVISTIC ELECTRON BEAMS IN VACUUM*

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A new method of focusing and transporting intense electron beams is presented. The technique is most applicable for intense beams in the current regime $I < 0.2I_A$ where $I_A = 17\beta\gamma$ kiloamperes is the usual Alfven current. The net focusing force is applied by using foils to periodically alter the radial electric field in the drift tube. The result is to increase the net force on the beam particles near the foil by γ^2, and give a net inward impulse to all particles. The requirements for lens spacing are discussed, as well as aspects of solid and hollow beams. The stability of focusing to the image displacement instability is briefly analyzed. Finally, a proof of principle experiment for a beam of lower kinetic energy is proposed.

I. INTRODUCTION

A new class of accelerators, which will produce electron beams with ten's of MeV energies and currents in the 1 kA-1 MA parameter regime, is now under construction. These beams result from the linear induction accelerator technology which has previously been realized in the Livermore Astron,[1] Berkeley ERA,[2] Cornell LIA,[3] Russian LIU-10,[4] and AFWL-Sandia RADLAC[5] accelerators. The typical accelerating gradient on these machines is less than 2 MV/m, necessitating ~ 50 m for a 100 MeV accelerator. At present, the beams are usually focused by solenoids between accelerating gaps. The most important defocusing force in the solenoid is the radial electric field. Because the net outward force scales as $1/\gamma^2$ [$\gamma^2 = (1 - v^2/c^2)^{-1}$], this force rapidly decreases at higher energies.[6] If the beam is extracted from the magnetic field, defocusing occurs because of the resulting beam rotation. The beam must rotate where $B_z = 0$ because the canonical angular momentum is conserved, and is non-zero at the electron source (cathode). The large size of the accelerator requires a large investment in magnetic field energy—in most cases, much larger than the total beam energy. In addition, the placement of field coils complicates the design of the accelerator considerably. A typical accelerator beam transport section is shown in Fig. 1(a).

Focusing the beam by neutralizing the radial electric field in a gas is one obvious method of transporting the beam. There are a number of difficulties with this approach including two-stream type instabilities, thermalization due to phase-mixing of betatron oscillations, and gas pumping.

In this report an alternate focusing system, depicted in Fig. 1(b), is proposed. An intense beam, after extraction from the uniform field, is periodically interrupted by a thin, conducting foil. The radial electric field is zero at this surface, resulting in a net inward or "focusing" force due to the azimuthal magnetic field (or the radial electric field in the hollow beam case).

Early RF drift tube linacs used foils or grids to shape the radial RF fields in accelerating gaps. In the thin-lens approximation, the net radial force in the gap of an induction accelerator is zero, and requires no shaping. In the high-current linear induction accelerator, defocusing forces in the drift sections can be compensated by the image charges on conducting foils (resulting in the term "Image-Field Focusing"). In fact, it may close the chain of parallels to note that a similar effect (radial oscillations) in induction linac gaps results because of solenoidal focusing. When the solenoidal field is removed, the radial oscillations disappear, but we produce a new defocusing force which can be treated by a similar solution: conducting foils with focusing resulting from the beam image charge!

The paper is arranged as follows: the radial impulse for a solid beam is discussed in Section II along with relevant approximations, beam dy-

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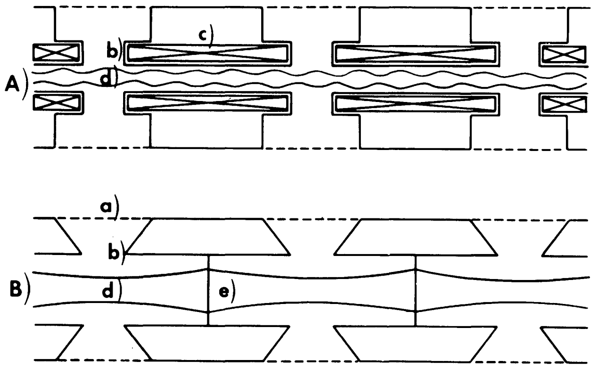

FIGURE 1 Transport systems for high-current, multi-gap accelerators using A) solenoidal focusing and B) foil focusing. The components shown are a) the power feed, b) the field-shapers in the accelerating gap, c) solenoid, d) conceptual particle orbits and e) conducting foil.

namics considerations leading to the required lens spacing are discussed in Section III, hollow beam focusing is discussed in Section IV, and an $m = 1$ transverse impulse is calculated in Section V.

II. RADIAL IMPULSE OF A SOLID BEAM

We begin by analysing the focusing mechanism for a solid beam. In general, we assume that the betatron wavelength λ_b is much longer than the beam radius at a foil (a) so that the beam radius is assumed fixed near the foil. This condition is given by

$$\frac{a}{\lambda_b}\left(\frac{2I}{\gamma^2 I_A}\right)^{1/2} \ll 1, \tag{1}$$

where I is the beam current, $I_A = mc^3\beta\gamma/e$ is the Alfven current, m is the electron mass, e is the electron charge, c is the speed of light, β is the electron velocity, and $\gamma = (1 - \beta^2)^{-1/2}$. Approximation (1) above allows us to model the beam dynamics as a change in radial momentum δp_r without a change in radial position (this is the thin lens approximation). Assuming that $\beta \sim 1$, we have

$$\begin{aligned}\delta p_r &= -e\int E_r - \beta B_\theta\, dt \\ &= -\frac{e}{c}\int_{-L/2}^{L/2} (E_r - \beta B_\theta)dz,\end{aligned} \tag{2}$$

where the foil is positioned at $z = 0$, E_r is the radial electric field, L is the distance between foils (lenses), and B_θ is the beam magnetic field. The B_θ field is not affected by the foil and is assumed to be dependent on z only through changes in beam radius. The radial electric field is made up of a term which is dependent on z only through the beam radius, and one due to the image charge on the foil. The βB_θ and E_r fields which depend only on radius cancel to order $1/\gamma^2$, and in the limit $L \ll \gamma^2 a$ we assume they cancel exactly. Self-field focusing results from the positive image charges on the foil which lower E_r near $z = 0$. Thus, although the focusing force in a solid beam is magnetic, it is evaluated by calculating the z − dependent part of E_r. The radial electric field was evaluated by the Green function method, and equation (2) was applied. The result is

$$\begin{aligned}\frac{\delta p_r}{p_z} &= \frac{16I}{I_A}\sum_{n=1}^{\infty}\left(\frac{b}{a}\right) \\ &\times \frac{J_1(\chi_{on}a/b)J_1(\chi_{on}r/b)}{\chi_{on}^3 J_1(\chi_{on})^2},\end{aligned} \tag{3}$$

where p_z is the axial particle momentum, b is the drift tube radius, and χ_{mn} is the nth root of J_m. The result of Eq. (3) is shown in Fig. 2 for three values of b/a. The axial dependence of E_r is shown in Fig. 3 for typical parameters—it is, in general exponentially decreasing.

In general the focusing impulse (δp_r) increases

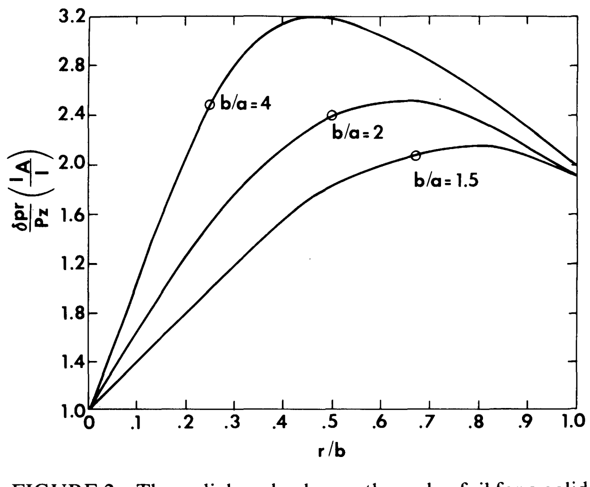

FIGURE 2 The radial angle change through a foil for a solid beam as a function of r/b for three values of beam radius. Circles indicate the outer beam boundary.

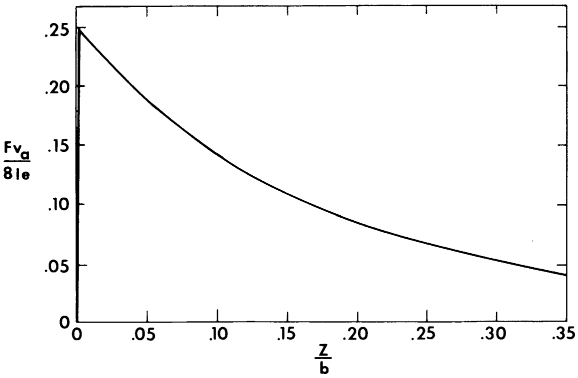

FIGURE 3 The focusing force F as a function of z/b for $r = a$. This is normalized with respect to $8Ie/va$.

approximately linearly with radius to a value of $\sim 2p_z(I/I_A)$ at the beam radius. It is also important to note that δp_r peaks more than $0.2b$ outside the beam. This means that particles for which $a + 0.2b > r > a$ will receive a larger focusing force than those for which $r < a$. This is desirable to assure particle confinement.

III. LENS SPACING

The problem of beam extraction from the field region is complicated. It will be particularily affected by the position of the first foil, solenoid diameter, etc. We will assume that the beam comes to some quasi-equilibrium characterized by the beam radius (a) as a free parameter.

The required distance between foils for optimal focusing is determined by solving the equation of motion between foils. This is related to the focusing force through the initial conditions at the foil (r, p_r, p_θ, p_z).

A particle "born" in an axial magnetic field of cyclotron frequency $\omega_c = eB/mc$ at a radius r_0 experiences a pseudo-force of magnitude $\gamma m r\dot{\theta}^2$ (θ is the angular position). Because the system is assumed to be azimuthally symmetric, the canonical angular momentum

$$p_\theta = \gamma m r^2\dot{\theta} - \frac{er^2B}{2c} \qquad (4)$$

is conserved, and equal to the value at the cathode (B is the external magnetic field). After the beam is extracted from the magnetic field ($B = 0$) we have

$$\gamma m r^2\dot{\theta} = er_0^2B_0/2c, \qquad (5)$$

where the subscript zero indicates the value of a quantity at the cathode. Using Eq. (4), the radial equation of motion becomes (between foils)

$$r'' = \left(\frac{I}{\gamma^2 I_A}\right)\frac{r}{a^2} + \frac{r_0^4\omega_c^2}{\gamma^2 r^3 c^2}, \qquad (6)$$

where the prime indicates differentiation with respect to z, and ω_c is the cyclotron frequency of the beam electrons at the cathode. In the equation above, the first term is due to self fields, and the second is due to conservation of cannonical momentum. In the thin lens approximation, the focusing force results in an abrupt change in r given by

$$r'(z > 0) - r'(z < 0) = \frac{\delta p_r}{p_z}. \qquad (7)$$

The relative importance of the two defocusing terms in equation (12) can be assessed by evaluating the quantity

$$\xi = \left(\frac{I}{I_A}\right)\left(\frac{r^4c^2}{r_0^4a^2\omega_c}\right).$$

We will assume $\xi \ll 1$, and set the first term to zero. Solving Eq. (6) for the case $-r'(-\epsilon) = +r'(\epsilon) \equiv r'(0)$ (ϵ is a small positive length), we have

$$z_1 = \frac{a}{r'(0)\left(1 + \dfrac{\alpha^2 r_0^2}{a^2 r'(0)^2}\right)}, \qquad (8)$$

where $\alpha = r_0\omega_c/2\gamma c$ and z_1 is the position where $r' = 0$ (the turning point). The distance between lenses should be $L \leq 2z_1$.

It is important to demonstrate that the same lens spacing is appropriate to all beam particles.

First we consider the variation of L as a function of the position r inside the beam. Assuming that the particle radius in the foil focused region is linearly proportional the particle radius at the cathode, this ratio is the same as r_0/a. Noting that both r' and α vary linearly with radius, we see that L is fixed as a function of radius.

Time variations of γ and I could result in time-dependent scaling of L. We assume that the impedance $\gamma mc^2/eI$ is fixed in time. Thus, I/I_A and $r'(0)$ are also fixed. For fixed values of a we must have

$$\left(\frac{\alpha r_0}{ar'(0)}\right)^2 \ll 1$$

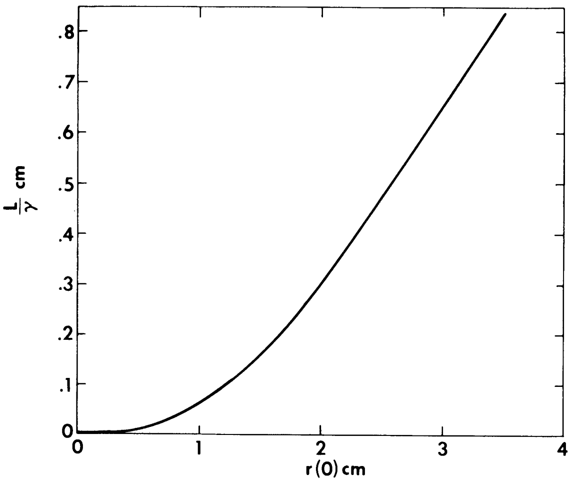

FIGURE 4 Focusing length L for a beam with $r_0 = 1.5$ cm, $B_z = 20$ kG, and $I = 100$ kA, as a function of $r(0)$.

for fixed foil distance. This sets an upper bound on the initial p_θ or a lower bound on γ.

If $L < 2z_1$, the beam is over focused, and we must use the exact solution of (6) and (3) to follow the beam radius through the accelerator.

The ideal inter-lens spacing is shown as a function of a for sample parameters $r_0 = 1.5$ cm, $B_z = 20$kG, $I = 100$kA and $b/a \sim 3$ in Fig. 4. For example, a $\gamma = 50$ beam of radius 3 cm requires foils spaced ~35 cm apart.

Given the foil spacing above, for 25 − 100 MeV of foil focusing we can estimate the foil scattering. Using $L = 2z$, from Eq. (8), 2μm Kapton foils, and a total length of 38 m, a total thickness of 100 μm of Kapton is traversed. The rms scattering angle is ~1.5 mR.

IV. HOLLOW BEAM FOCUSING

Hollow electron beams are of particular interest for linear induction accelerators because of net space-charge considerations at the injector.[8–10] For beams close to the wall, the space-charge limit is much higher for a hollow beam.

The analysis of hollow-beam focusing can be made in the same manner as for solid beams. Taking an infinitessimally thin beam of radius a, the change in radial momentum through the foil is

$$\frac{\delta p_r}{p_z} = \frac{8I}{I_A} \sum_{n=1}^{\infty} \frac{J_0\left(\frac{\chi_n a}{b}\right) J_1\left(\frac{\chi_n r}{b}\right)}{\chi_n{}^2 J_1(\chi_n)^2} \quad (\chi_n \equiv \chi_{on}). \qquad (9)$$

It is important to note that $\delta p_r/p_z$ is continuous across $r = a$. This is demonstrated by applying the asymptotic expansions for Bessel functions to showing that Eq. (9) is absolutely convergent. It is well known that an absolutely convergent series of continuous functions is continuous, and so the focusing force is continuous.

The physical nature of the focusing force merits some comment. B_θ is zero on the inside of a hollow beam of small but finite thickness. The focusing force on the inside of the beam results from electric field lines which connect the beam inner diameter to the foil. Thus the focusing force is electrostatic on the beam inside diameter, and magnetic on the outside diameter. This is of particular interest when alternatives to foils are considered.

Because the focusing force is continuous, particles in and outside the beam will have the same value of z. Thus, focusing of hollow beams by foils is also feasible.

The radial angle change of a hollow beam is shown as a function of radius in Fig. 5. Although the focusing impulse falls off with radius, the defocusing force falls off as $1/r^3$, leading to net confinement. A comparison of Fig. 2 and Fig. 5 indicates that hollow and solid beam focusing angles are comparable. One can show analytically that $(\delta p_r/p_z)(I_A/I)$ converges to the value $8/\pi$ for both hollow and solid beams in the limit $b/a \to \infty$.

V. $m = 1$ IMPULSE

A simple analysis indicates that foil focusing may have some advantages over solenoidal focusing

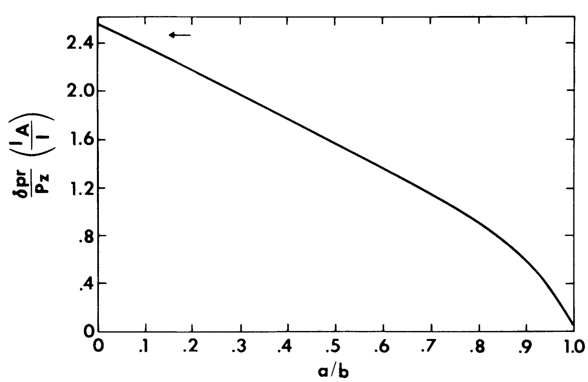

FIGURE 5 Angle change through a foil for an infinitesimally thin hollow beam of radius a.

for suppression of the image displacement instability.[11,12] We will again use the thin lens approximation and $\beta \sim 1$ to estimate the transverse ($m = 1$) momentum change through the foil. If the simplifying assumption $a \ll b$ is used, the transverse (x) momentum change is

$$\delta p_x \approx \frac{-2Ie}{c^2} \sum_{n=1}^{\infty} \frac{J_1\left(\frac{\chi_{1n}x}{b}\right)}{J_2(\chi_{1n})^2 \chi_{1n}^2}$$

$$\sim \frac{-2Iex}{bc^2}\; x \ll b.$$

This can be compared with the defocusing "kick" which results from the image displacement instability of

$$\delta p_x \sim \frac{2Iel\eta}{b^2c^2},$$

where l is the accelerating gap length, and $\eta < 1$ is a correction factor which gives the exact image force for a gap. The beam will be stable to image displacement for

$$\frac{b}{L} > \frac{\eta l}{z_2},$$

where z_2 is the distance between accelerating gaps.

Other $m = 1$ stability issues such as the beam breakup instability must also be considered, however, the anticipated image displacement stability is encouraging in view of the possibility of the image displacement instability in a solenoidal field.

VI. CONCLUSION

Transport of high-current, high-energy electron beams by use of periodic focusing foils has been proposed. A preliminary study indicates that no fundamental difficulty with the concept exists. The required focusing length has been determined as a function of input parameters. The defocusing force considered is the azimuthal motion due to the conserved angular momentum. Foil focusing is also shown to have attractive stability properties for the image displacement instability.

Since no accelerators presently exist in the 25 MeV, 100 kA parameter regime used as an example in this work, a proof-of-principle experiment is proposed for lower energies and currents. Taking a 4 MeV, $b/a = 1.5$ and $\delta p_r/p_z = 0.2$ as reasonable figures, we have $I \sim 34$ kA from Fig. 2. Further, for $L = 20$ cm, and assuming $r_0 = 0.7$ cm, $r(0) = 2$ cm, a diode magnetic field of 13 kG results. The parameter ξ is then ~0.5, which means that $L \sim 15$ cm allowing for the beam radial electric field.

Because ξ is finite, a solid beam experiment is desirable. A hollow beam experiment would have different focusing lengths on the inside and outside of the beam due to the net radial electric force ($\xi > 0$). Limiting current considerations indicate that for the above parameters, we must have $\ln b/a < 1$. In particular, an experiment would address important issues which are beyond the scope of this report. These include the beam energy spread, the effect of overfocusing, instabilities, extraction from a magnetic field, and violations of the thin lens approximation.

For applications requiring high repetition rates, foils which may be destroyed each shot are not feasible. The obvious alternatives are various types of plasma sources, or gas puffs.

The plasma requirements are different for hollow and solid beams. In particular, for a plasma which neutralizes only the beam space charge (as opposed to the current) a magnetic focusing force will exist inside the plasma slab. For a solid beam, the focusing force is proportional to r and so the enhanced focusing power results in an inter-lens distance which is the same for all particles of all radii (L is independent of r_0). For a hollow beam, however, there will be no focusing force enhancement on particles on the beam inner diameter. This will result in an effective particle dispersion, transverse heating and possibly, beam expansion since the focusing force will be effectively random. For hollow beams, the plasma slab must provide both charge and current neutralization. Experimental evidence indicates that a significant net current exists in a plasma generated by a beam in a gas.[9-11] In addition, a gas puff would also present severe pumping and timing problems.

An attractive alternative scheme is the use of a flashboard. Here a flash (surface flash-over plasma source) powered by coupling to the beam pulsed power source. Using this scheme, the relative timing of plasma production and beam propagation is provided by passive delay lines. Desirable parameters for the plasma slab are $n_e > 10\ n_b$ where $n_{e(b)}$ is the plasma (beam) electron density, and $T_e > 5$ eV. The flashboard plasma will probably persist for less than 100 μsec, due to collisions with walls, recombination, etc. Flashboards have been used to enhance such electron transport in collective-acceleration experiments,[14] and for ion transport in linacs.[15]

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