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A NOVEL APPROACH TO BEAM OPTICS DESIGN FOR PARTICLE ACCELERATORS

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A new computational method is proposed for optimal design of beam lines. The method is based on a dynamic-programming recursive algorithm that minimizes an additively or multiplicatively expressed function of the desired parameters.

A formidable amount of effort has been made to create computer programs capable of optimizing the number, length, position and strength of beam optics elements (multipole lenses, bending magnets, etc.) used in particle accelerators, storage rings, beam-transport systems, etc., when various constraints are imposed on the beam design.[1–3] The available software is very powerful in beam calculations with fixed parameters and is also capable of finding local optima if the number of variable parameters and the number of constraints are not too high. The initial values of the parameters must be chosen from some approximate solutions or rough physical intuition. If the starting values are too far from a local optimum or the problem is over-constrained, no solution can be found. An additional problem arises in connection with the weighting factors that can only be determined by experience and strongly depend on the nature of the particular problem. Totally different results may appear if these factors are changed; they can therefore be considered as additional unknown parameters. The results usually also fluctuate strongly if the order of variation of the parameters is changed. The most important problem, however, is that even if a local optimum is found, we have no guarantee that there are not other much better solutions.

This situtation is quite similar to that in the field of low-energy beam optics, where there is an additional complication that we are usually also required to find electrode or pole-piece configurations that can be expected to have certain predefined electron-optical properties. Electron-optical synthesis was of a trial-and-error nature in the past. The usual procedure was to change the dimensions of electrodes or pole pieces until the desired optical properties were obtained.

The author has proposed an entirely different approach to electron optical synthesis.[4] The novel computational method involves restricting the possible electrostatic or magnetic field distributions to those connecting the intersection points of a rectangular grid by straight lines and gives a recursive algorithm for choosing distributions that minimize a given combination of aberration coefficients. This method was successfully applied to both magnetic[5] and electrostatic[6] lens synthesis.

The aim of this paper is to show that the above method can also be applied to beam-optics design for particle accelerators, at least for cases where the requirements can be expressed in an additive or multiplicative way. This is certainly the case when some aberration must be minimized because the aberration coefficients can always be expressed in the form of a definite integral. For the sake of simplicity, however, we choose an illustrative example.

Let us try to solve the following problem. Given a long straight channel with definite dimensions and some spaces occupied by rf cavities, the task is to find a suitably situated system of quadrupole lenses that minimizes the overall sum of vertical (β_v) and horizontal (β_h) betatron functions measured at the medium planes of each quadrupole along the whole channel

$$G = \min \sum_k (\beta_{vk} + \beta_{hk}), \qquad (1)$$

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where k is the ordinal number of a given quadrupole in the system.

The values of the quadrupole excitations are naturally constrained by saturation and energy considerations. Another constraint is imposed by the fact that the cavities and quadrupoles cannot overlap each other. Therefore, for the first try one may suppose that the lengths and positions of the quadrupoles are fixed. (Of course, this is not a general requirement for the application of the method.) If we accept these constraints, the problem is reduced to that of finding a set of excitation values h_k in the given limits for the quadrupoles placed along the channel at arbitrary (not constant) distances from each other (Fig. 1).

As a further simplification, for illustrative purposes one can use the thin-lens approximation, so it is assumed in Fig. 1 that the lenses are concentrated in their mid-planes and therefore that

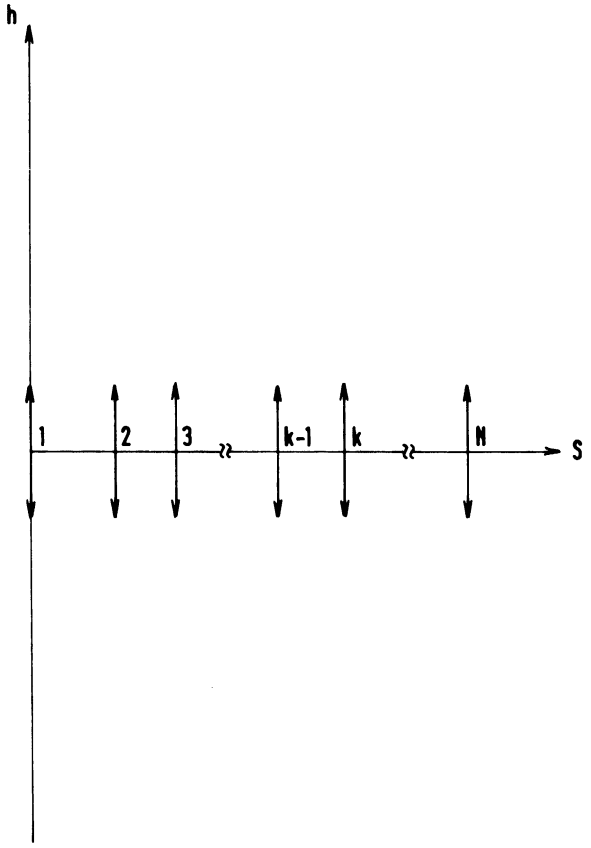

FIGURE 1 The arrangement of N quadrupoles along a channel (s is the axial coordinate, h is the excitation). The arrows represent that we do not know anything a priori about the values and signs of the excitations.

the betatron functions do not change inside the lenses. The lens action is taken into account by the changes in the values of the derivatives of the betatron functions.

We do not know anything a priori about the solution. Even the "signs" of the lenses are not given. (We consider a lens "positive" if it is focusing in the horizontal plane and defocusing in the vertical one.)

The solution of this simple problem is quite difficult. Indeed, let us suppose that we have N lenses and every lens can have M different excitation values. This is already a simplification because the excitation is changed in a continuous way by changing the currents in the coils, but if M is large enough it is quite a practical assumption. Then there are M^N different possibilities available. If we want to try all these, we must calculate the vertical and horizontal betatron functions and their derivatives ($M + M^2 + M^3 + \cdots + M^N$) times for the lenses and do the same amount of calculation for the drift spaces between them in order to get all the information necessary to compare the results and choose the best among them. If we have only 10 lenses and suppose that $M = 100$ (which is not a very large number to approximate a continuous variation of the excitation between the negative and positive saturation limits) we arrive at 10^{20} different possibilities and an even larger number of calculations and comparisons. It is easy to agree that this "brute-force" approach is absolutely out of the question. If we try to use some existing optimization software, we will certainly arrive at a locally "optimal" solution but only if the number of lenses is strongly limited. In case of $N = 100$, for example, there is no hope at all.

But the hope is here! We shall try now to apply our method to the solution of this problem. First, we define a "domain of existence" of the problem. This means that the distances along the axis are limited by the actual length of the channel and the quadrupole excitations are limited by the constraints. Then we design a rectangular "computational grid" which will restrict the excitations to $M + 1$ different values along the ordinate (zero excitations must not be excluded) while fixing the situations of the quadrupoles along the abscissa. The computational grid is shown in Fig. 2. Our problem is now reduced to that of finding out the intersection points of this grid that determine the optimum distribution of quadrupole excitations h_{ik}, where i is the value of the exci-

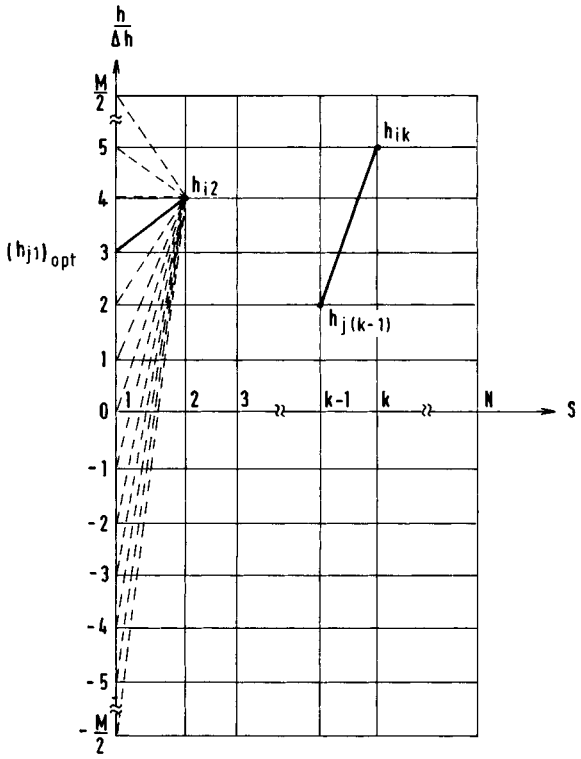

FIGURE 2 The computational grid.

tation expressed in the elementary units $\Delta h = 2h_{max}/M$ and k is the ordinal number of the lens under consideration. For any given value of h_{ik} there are exactly M + 1 possible values in the previous column of the grid. We shall denote the actual value of the excitation of the previous lens by $h_{j(k-1)}$ (see Fig. 2). Then we can assign a value

$$F_{ijk} = \beta_{V\,ijk} + \beta_{H\,ijk} \tag{2}$$

to the linear segment connecting these two points in the computational grid. It is quite clear that the values of the betatron functions at the output of the k-th lens must depend on the excitation i of the given lens and also on the previous history of the beam, i.e., the excitations of the preceding lenses, which are taken into account by the single index j. We are able to use this single index by applying R. E. Bellman's optimality principle[7], which can be formulated in the following way: "The optimal procedure in a multistage decision process has the property that, whatever the final decision is, the prior decisions must constitute an optimal procedure leading from the initial state to the state preceding the final decision." This very deep mathematical principle is widely used in economics and control engineering but, unfortunately, almost unknown among physicists.

We shall express the optimality principle by the recursion relation

$$G_{ik} = \min_j [F_{ijk} + G_{j(k-1)}], \tag{3}$$

where G_{ik} is the result of the optimal procedure leading from the initial state to the k-th state, $G_{j(k-1)}$ is the result of the optimal procedure leading from the initial state to the $(k-1)$-th state and F_{ijk} is the contribution of the k-th lens. Of course, j is a variable and the aim of the procedure is to find out that particular, optimal value of j for which the value of the sum of $F_{ijk} + G_{j(k-1)}$ is minimal. The initial conditions G_{i0} must be determined from the given initial values of the betatron functions and their derivatives. In our case $G_{i0} = \beta_{H0} + \beta_{V0}$ for every possible value of i. It is evident that $F_{ij0} = 0$ for any values of i and j. Because in our case F_{ij1} does not depend on j, we can simply calculate the values of G_{i1} for every i and then begin the optimization procedure by calculating F_{ij2} for a fixed value of i, but for every possible value of j (see Fig. 2). Varying the value of j we find

$$G_{i2} = \min_j (F_{ij2} + G_{j1}), \tag{4}$$

where, in general, the values of G_{j1} are the same as G_{i1} at the first lens because the beginning of the k-th region of the computational grid obviously coincides with the end of the $(k-1)$-th region (what was "i" before becomes "j" now). In this simplified model we assume that the betatron functions are calculated at the inputs of the lenses, thus combining the drift space and lens calculations into one step. We store the value of G_{i2} together with h_{i2} and the corresponding optimal value of $h_{j1\,opt}$, as well as the betatron functions and their derivatives. All the other possibilities have no interest for us anymore (dotted lines in Fig. 2). After evaluating M + 1 possibilities, we choose the best of them and abandon the others.

Now we change the value of i and find the corresponding values of G_{i2} and $h_{j1\,opt}$. Doing that for every possible value of i, we shall have a set of M + 1 data for each of the following parameters: h_{i2}, G_{i2}, $h_{j1\,opt}$, β_{Vi2}, β_{Hi2}, α_{Vi2} and α_{Hi2} where $\alpha = -\beta'/2$ (the prime represents differentiation with respect to the axial coordinate s).

A graph of M + 1 lines connecting the corresponding points h_{i2} and $h_{j1\,opt}$ for every possible value of i will correspond to these sets of data (see the first region of Fig. 3).

After completing the calculations at the second lens, we move further to the third lens, repeating the whole procedure. We must only remember that the index "i" from the previous region always becomes "j" in the new region. Thus the initial conditions for the third lens are those given as final values after the second drift space. Proceeding in the same way, we shall have again a set of M + 1 data for each of the parameters that must be stored. Our graph can be continued (see second region in Fig. 3).

Repeating this procedure N − 1 times, we arrive at the end of our calculations. We have N sets of the values of h_{ik}, G_{ik}, $h_{j(k-1)opt}$, β_{Vik}, β_{Hik}, α_{Vik} and α_{Hik} (k = 1, 2, ···, N) and the whole graph of all M + 1 possible optimal excitation distributions can be completed. In Fig. 3 a simple illustrative example is given for the case of M = 8, N = 5. We can choose that particular value of i at the end which is best suited to our special needs or which corresponds to the smallest value c′ G. Then we follow the optimum-value distribution in the graph starting from the given point (i, N) and proceeding in the backward direction step by step. The thick line in Fig. 3 represents

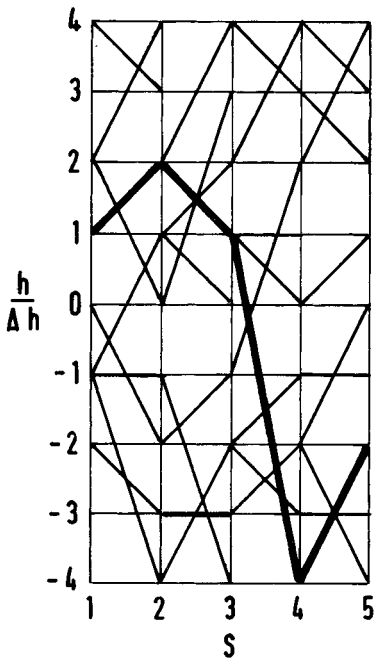

FIGURE 3 The completed optimum-value graph.

such an "optimal" distribution if $i = -2$ is chosen as the final excitation value. For the completion of the whole procedure, the betatron functions and their derivatives must be evaluated only $(M + 1) + (M + 1)^2 (N - 1)$ times, which makes a big difference in comparison with the brute-force approach. It means that the necessary computer time increases with the number of elements only linearly!

The advantages of the method are obvious. It is a global search with a linear dependence on the number of elements as far as computational time is concerned. Any constraint that reduces the number of possibilities at a particular stage simplifies the computing procedure. (For example, any of the quadrupoles can be forced to zero excitation or a given excitation value). After completing the calculations we automatically also have solutions for many sub-problems with a smaller number of elements. A system of lenses with variable lengths can also be treated, as well as many other combinations.

Practical problems can be formulated in quite different ways. For example, it might be desirable to obtain given values of the betatron functions at some places and simultaneously fulfill some more requirements at other steps, as is the case when a low-beta insert has to be designed.[8] Then it is convenient to formulate the requirement so that the sum of differences between the actual and desirable values must be minimized. We are well aware of the fact that not all optimization processes are like this, but our method certainly can be applied to many practical problems of beam-line optimization. We can minimize emittance, momentum-compaction factor, phase advance, chromaticity, etc. There is also a possibility to reduce the nonlinear effects of the correcting sextupoles. The minimization of the sum of betatron functions is important in rf-cavity regions. It is desirable to reduce the aberrations in beam-transport systems, etc.

Our aim in this short theoretical communication has been to present only the method. Its practical application to cases where previous methods proved to be very difficult to use has begun now. The results will be reported separately.

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